Lesson: Applications and Modeling with Linear Equations

We will revolve a few different types of applications for linear equations.

Example Two sides of a triangle have the same length. The third side measures 8 inches less than twice the common length. The perimeter of the triangle of 20 inches. What are the lengths of the three sides?

Example Dan ran the 100-m dash with a time of 9.92 seconds. If this pace could be maintained for an entire 26-mile marathon, what would his time be? (Note: 1 m = 3.281 ft, 1 mi = 5280 ft)

Example Sam earned $25,000 from royalties on his music. He set aside 20% of this for a down payment on a new home. The balance will be used for a new car (eventually). He invested a portion of the money in a CD account (bank certificate of deposit) that earns 2.25%, and the remainder in another CD account that earns 1.75%. If the total interest earned after 1 year is $405, how much money was invested in each account?

Lesson: A Review of Imaginary Numbers

Please note: Knowing imaginary numbers if one of the "prerequisite" topics for this course. Ie – you should already know this (though I also realize you probably need a review!). If you have *never* seen imaginary numbers before, then I strongly suggest that you watch my series on imaginary numbers (because, yes, I have one). So if you felt like this went too fast and you want more details, please take responsibility for your education and go back to review this!

How do we represent an imaginary number?

What is i^2?

What is a complex number?

When are 2 complex numbers equal?

What does $\sqrt{-a}$ mean?

Examples Write using the definition of $\sqrt{-a}$

a. $\sqrt{-12}$

b. $\sqrt{-25}$

Examples Complete the following operations.

a. $\sqrt{-5} * \sqrt{-3}$

b. $\frac{\sqrt{-12}}{\sqrt{-18}}$

c. $\frac{\sqrt{-50}}{\sqrt{9}}$

To add or subtract complex numbers, we can add/subtract the real parts and the imaginary parts similar to working with like terms.

Examples Find the sum or difference.

a. $(2 - 6i) + (8 + 2i)$

b. $(5 - 7i) - (-4 + 2i)$

When multiplying complex numbers, recall that $i * i = i^2 = -1$!

Examples Multiply.

a. $(2 + 4i)(3 - 5i)$

b. $(4 - 5i)(3 - 2i)$

c. $(3 + 2i)(3 - 2i)$

Recall, the conjugate is found by using the opposite side for the second term!

Examples What is the conjugate of the following?

a. $4 + 2i$

b. $-3 - 4i$

c. i

To find the quotient of 2 complex numbers, we multiply by the conjugate of the denominator.

Examples Find each quotient. Write your answer in *standard form,* ie $a + bi$ form

a. $\frac{3-4i}{2+i}$

b. $\frac{5}{i}$

How do the powers of i cycle through?

Examples Show how to simplify the following powers of i. Write your work in great detail so that you remember how to do this!

a. i^{17}	c. i^{-14}
b. i^{28}	d. i^{-5}

<u>Examples</u> The following quadratic equations have imaginary solutions. Show how to solve them.

a. $x^2 = -25$ using the **square root property**

b. $x^2 + 75 = 0$ using the **square root property**

c. $2x^2 - 3x = -6$ using the **quadratic formula**

d. $x^4 - 16$ using **factoring** and also the **square root property**

Lesson: A Quick Review of How to Solve Quadratic Equations

What is the form of a quadratic equation?

How is this different from a quadratic function?

What are the 4 ways you can solve a quadratic equation?

Which technique should you use when solve a quadratic equation?

We will review these 4 techniques on the next page!

What type of answers are possible when you solve a quadratic equation?

Example Show how to use each of the 4 different techniques for solving quadratic equations by using each one at most once for the following examples.

a. $5x^2 + 6x + 1 = 0$

b. $(x + 4)^2 = 12$

c. $x^2 - 4x + 8 = 0$

d. $x^2 - 5x + 1 = 0$

Optional extra help: If you need a more thorough (and more complete) review of the methods we discussed today, watch these:

- A review of ALL the different types of factoring (just factoring, nothing about solving quadratic equations): https://youtu.be/nrI7dJRXe5I and https://youtu.be/yGY5eqZkJcw
- How to solve a quadratic equations by factoring: https://youtu.be/5uof6OEyod0
- How to solve quadratic equations by the square root property: https://youtu.be/a1-smI3Xx5w
- How to solve quadratic equations by completing the square: https://youtu.be/6jXpMTozmhI
- How to solve quadratic equations by the quadratic formula: https://youtu.be/ryp2rdkhks8
- A review that compares all the techniques on the same problem: https://youtu.be/2ny9HbQ8doY

A Quick Refresher of the 4 Ways to Solve Quadratics

For all of these examples, we will solve $x^2 + 6x + 5 = 0$.

Solving by Factoring

Solving by Completing the Square

Solving by Using the Quadratic Formula

The quadratic formula is: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

One final way we can solve quadratics is by using the square root property.
Use the square root property to solve $(x+3)^2 = 12$

Choosing the Best Method to Solve a Quadratic

There are 4 ways we can solve quadratics:

1.	3.
2.	4.

Using each of these methods only once, determine the best way to solve these quadratics:

a. $x^2 + 6x = 2$
b. $x^2 + 3 = 2x$
c. $(2x + 1)^2 = 49$
d. $3x^2 + 2 = 3x$

Examples Using each of these methods only once, determine the best way to solve these quadratics:
a. $(3x-2)^2=-24$
b. $2x^2+5x-1=0$
c. $x^2-12=x$
d. $x^2-2x+5=0$

Lesson: A Quick Review of The Discriminant

What is the discriminant?

Fill in the table below:

If the discriminant is …	How many solutions are there?	What type of solution?
Positive and a perfect square		
Positive (not perfect square)		
Zero		
Negative		

<u>Example</u> Use the discriminant to determine how many solutions there are to: $x^2 - 4 = 0$.

<u>Example</u> Use the discriminant to determine how many solutions there are to: $x^2 + 6x + 9 = 0$.

Lesson: Solving for Quadratic Variables

<u>Example</u> Solve for t: $s = \frac{1}{2}gt^2$

<u>Examples</u> Solve for x: $kx^2 - tx = m$

Lesson: Applications of Quadratic Equations

Example Find the problem of 2 pairs of 2 consecutive integers whose product is 156.

Example You want to buy a rug for a room that is 18ft x 26ft. You want to have a uniform strip of floor around the rug, and you can afford 384 square feet of carpeting. What dimensions should the rug have?

Example The area of a square is 32 more than its perimeter. Find the length of a side.

Lesson: Solving Rational Equations

Rational equations are equations that look like:

$\frac{2x}{x+1}-\frac{3}{x}=\frac{5}{x+1}$ or $\frac{3}{x^2-4}+\frac{2}{x^2+4x+4}=\frac{5}{x-2}$

What is the general strategy for solving a rational equation?

What do you have to watch out for with rational equations?

<u>**Examples**</u> Solve each equation.

a. $\frac{x}{x-4}-4=\frac{4}{x-4}$

b. $\frac{3}{x+2}+\frac{10}{x^2-6x-16}=\frac{5}{x-8}$

Examples Solve each equation.

a. $-\frac{14}{3x^2+15x-18}=\frac{x}{x-1}+\frac{2}{3x+18}$

b. $\frac{x}{x^2+4x-32}+\frac{2}{x^2-14x+40}=\frac{6}{x^2-2x-80}$

Lesson: A Review of Solving Radical Equations and Equations with Rational Exponents

What is the general strategy with solving radical equations?

What do you have to watch out for when solving radical equations?

Examples Solve each equation.

a. $x - 3\sqrt{x+2} = 2$

b. $\sqrt{6x+7} - 9 = x - 7$

Examples Solve: $\sqrt{3x+4}-5=\sqrt{3x-11}$

Example Solve each equation.

a. $\sqrt[3]{2x+1} = \sqrt[3]{3x+5}$

b. $\sqrt[4]{x+3} = 2$

Example Solve each equation.

a. $x^{\frac{5}{4}} = 32$

b. $(2x+1)^{\frac{2}{3}} = 36$

Example Solve for x: $y = k\sqrt{x}$

Lesson: A Review of Solving Equations Quadratic in Form (Using Substitution)

Examples Solve each equation.

a. $x^4 - 10x^2 + 9 = 0$

b. $(x+1)^{\frac{2}{5}} - 3(x+1)^{\frac{1}{5}} + 2 = 0$

c. $x^{-\frac{2}{3}} - x^{-\frac{1}{3}} - 6 = 0$

Lesson: A Review of Basic Inequalities

Please check out the examples video on how to graph on a number line, use interval notation, or use set notation in case you forgot!

The Inequality Signs

$<$ Less than

$\leq$ Less than or equal to

$>$ Greater than

$\geq$ Greater than or equal to

The Properties of Inequalities

If a, b, and c are real numbers then:

1. If $a < b$, then $a + c < b + c$

2. If $a < b$ and $c > 0$ then $ac < bc$

3. If $a < b$ and $c < 0$, then $ac > bc$

These properties also hold if you switch to $>, \leq, \geq$

<u>Examples</u> Solve the following. Put your final answer in interval notation.

$-2x + 8 < 10$

Examples Solve the following. Put your final answer in interval notation.

a. $-\frac{1}{3}(x+3)+\frac{1}{3}x+\frac{2}{5}x \leq \frac{1}{10}$

b. $-5 \leq 1+2x < 13$

c. $2 \leq -6x-3 < 9$

Lesson: Quadratic Inequalities

A quality inequality is an inequality in the form: $ax^2 + bx + c < 0$ where a, b, and c are real numbers. You can interchange $<$ with $\leq, >, \geq$.

There are 3 steps to solving a quadratic inequality. Write out each step as you solve the next example.

Example Write out the steps that you use to solve the following: $x^2 - x - 6 > 0$. Write your answer in interval notation.

Explain Step 1:

Explain Step 2:

Explain Step 3:

Example Solve: $3x^2 - 4 \leq x$. Write your answer in interval notation.

Example Solve $x^2 \geq 25$. Write your answer in interval notation.

Lesson: Rational Inequalities

Rational inequalities look like this:

$\frac{5}{x+3} > 3$ or $\frac{2x+5}{4x-1} \leq 2$

We will out the steps to solve a rational inequality by using the following example.

Example Solve $\frac{4}{x+2} \geq 6$ Put your final answer in interval notation.

Explain Step 1 and show how to do it:

Explain Step 2 and show how to do it:

Explain Step 3 and show how to do it:

What do you have to watch out for with rational inequalities?

<u>Example</u> Solve $\frac{3}{x+6} \leq 2$ Put your final answer in interval notation.

<u>Example</u> Solve $\frac{x+3}{x-5} \geq 1$ Put your final answer in interval notation.

Lesson: A Review of Absolute Value Equations and Inequalities

We can summarize how to solve each case with the following chart:

Case (k is a number)	How to Setup
$\lvert x\rvert = k$	
$\lvert x\rvert < k$ (or $\leq k$)	
$\lvert x\rvert > k$ (or $\geq k$)	

<u>**Examples**</u> Solve the following. Put your answer on a number line and in interval notation when you have an inequality.

a. $\lvert x+3\rvert = 5$

b. $\lvert 2x-4\rvert \geq 6$

Examples Solve the following. Put your answer on a number line and in interval notation when you have an inequality.

a. $|4x - 2| = |6x + 8|$

b. $|5x - 2| + 1 < 9$

c. $|2 - x| - 1 > 9$

Special Cases!

Example Explain what is special about each case below and how to solve the problem.

a. $|4x - 1| = -7$

b. $|4x - 1| = 0$

c. $|4x - 1| < -7$

d. $|4x - 1| > -7$

Example Explain what is special about each case below and how to solve the problem.

a. $|4x - 1| \geq 0$

b. $|4x - 1| \leq 0$

c. $|4x - 1| < 0$

d. $|4x - 1| > 0$

Lesson: A Quick Review of Some Things About Graphing

Remind yourself how to graph the following points. Draw your own graph.

Point 1: $(-3, 4)$ Point 2: $(0, -2)$

Draw another rectangular coordinate system below and label the quadrants and origin.

Often when graphing, we use equations in two variables.

<u>Example</u> Find the ordered pair associated with the equation $3x + y^2 = 7$ when $x = 1$

Example Find at least 3 ordered pairs for the following equations.

a. $y = \sqrt{x+2}$

b. $2x + 3y = 6$

Finding 3 points can be a strategy for graphing.

Example Sketch a graph for the previous 2 examples.

What is the x-intercept? And how do you find it?

What is the y-intercept? And how do you find it?

<u>Examples</u> Find the x- and y-intercepts for the following equations.

a. $y = 3x - 1$

b. $x = \sqrt{y - 1}$

c. $x^2 - 3x + 2 = y$

Lesson: The Distance Formula

Graph the points $(2, 4)$ and $(-4, -6)$. Explain how you can find the distance between these points by summarizing the explanation given in the video (make a decent effort to explain this – you will have an easier time remembering the formula if you understand WHY it works).

State the distance formula:

Example Find the distance between the following points: $(4, -1)$ and $(7, 3)$

There are 2 ways we can use the distance formula.

Example Show how to use the distance formula to determine whether the following points are the vertices of a right triangle.

a. $(0, -2), (-4, 3)$ and $(5, 1)$

b. $(-2, -8), (0, -4)$ and $(-4, -7)$

What does it mean for 3 points to be collinear?

<u>Example</u> Show how to use the distance formula to determine if the 3 points are collinear.

a. $(-1, 4), (-2, -1), (1,14)$

b. $(-2, -4), (-6, 11), (0, -12)$

Lesson: The Midpoint Formula

What is the midpoint formula?

So what is the midpoint formula effectively doing?

Example Find the midpoint between the points $(-5, -6)$ and $(-3, 2)$

Example The midpoint of a line segment is $(5, 8)$ and the endpoint of that line segment is $(11, 10)$. What is the OTHER endpoint on this line segment?

Example The midpoint of a line segment is $(3, 1)$ and the endpoint of that line segment is $(6, 8)$. What is the OTHER endpoint on this line segment?

Example The midpoint of a line segment is $(2a, 5b)$ and the endpoint of that line segment is $(-1a, 9b)$. What is the OTHER endpoint on this line segment?

Lesson: Equations of Circles and How to Graph Circles

Draw the circle in the video and label the relevant parts of it.

Using your sketch above, explain how the distance formula can be used to help us determine the equation of a circle (you will need to summarize the explanation in your own words).

What is the **Center-Radius Form** of the equation of a circle?

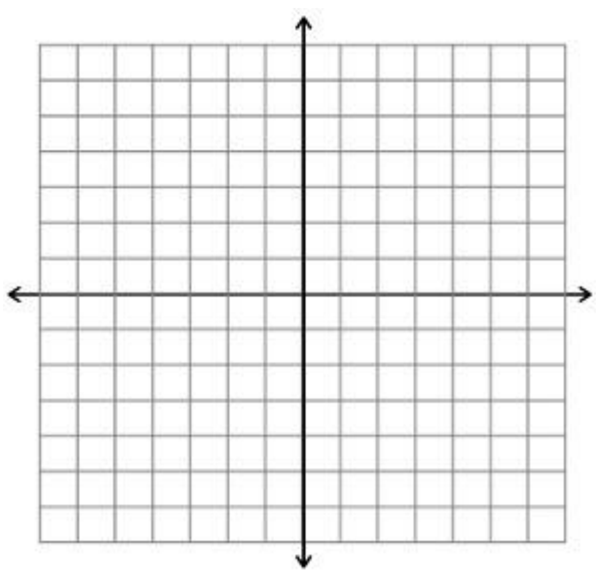
Example Find the center radius form of a circle using the given information.

a. center $(-2, 3)$ and radius 4

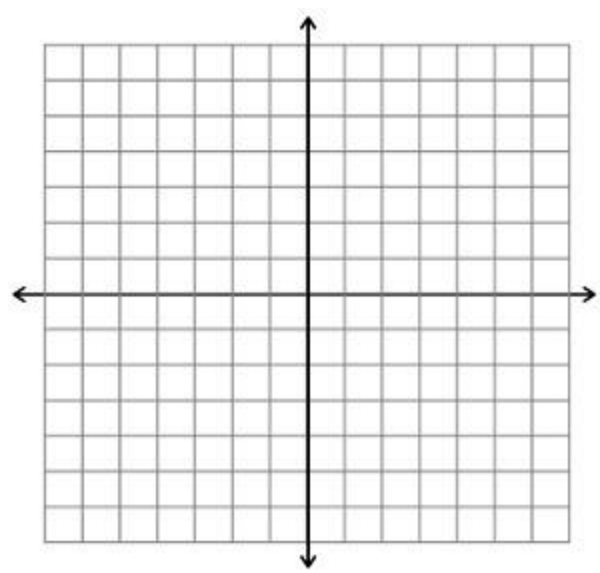
b. center $(0, -1)$ and radius 2

Example Graph the following circles.

a. $(x-1)^2 + (y+3)^2 = 4$



b. $x^2 + y^2 = 9$



Expand $(x-1)^2+(y+3)^2=4$ and show how this creates the general form.

What is the **General Form** of the equation of a circle?

Example Review how to use completing the square with the equation: $x^2-6x+4=0$.

Example Show how to use completing the square to write each in the center-radius form, and then sketch a graph of the circle.

a. $x^2 - 4x + y^2 + 8y - 44 = 0$

b. $x^2 + y^2 + 2x - 6y + 14 = 0$

Examples: Working with the General Form Equation of a Circle

Example Show how to use completing the square to write in the center-radius form, and then sketch a graph of the circle if possible.

a. $4x^2 + 4y^2 + 4x + 16y - 19 = 0$

b. $x^2 + y^2 - 4x - 4y + 8 = 0$

c. $x^2 + y^2 + 2x - 4y + 18 = 0$

d. $9x^2 + 9y^2 + 12x - 18y - 23 = 0$

Examples: Graphing Circles Using Technology

You can use this page to take notes for how to solve the following 2 examples:

a. Find all coordinates of points whose distance from (-3, 2) and (-2, 1) is $\sqrt{5}$.

b. There's an earthquake! There are 2 receiving stations with the following coordinates:

$$A = (-1, 1), B = (-2, 1), C = (-3, -1)$$

The epicenter of the earthquake is 2 units from stations A and C, and $\sqrt{5}$ units from B. Where is the epicenter?

Lesson: Review of Functions

How do you represent a function?

What is a function often interchangeable with? How can you rewrite $f(x) = 3x + 8$?

So what IS a function? What does it actually represent?

What's another handy feature of functions?

What are the domain and range?

Does domain and range only work for functions?

Examples Are the following functions, and what is the domain and range?

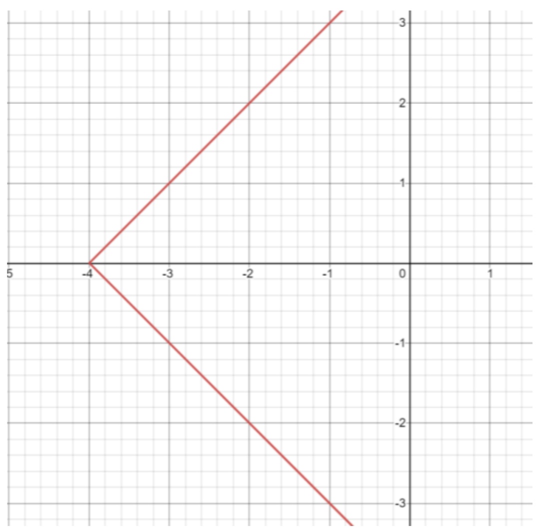
a. $\{(2,3),(5,-1),(-3,7)\}$

b. $\{(-1,5),(-8,3),(-1,6)\}$

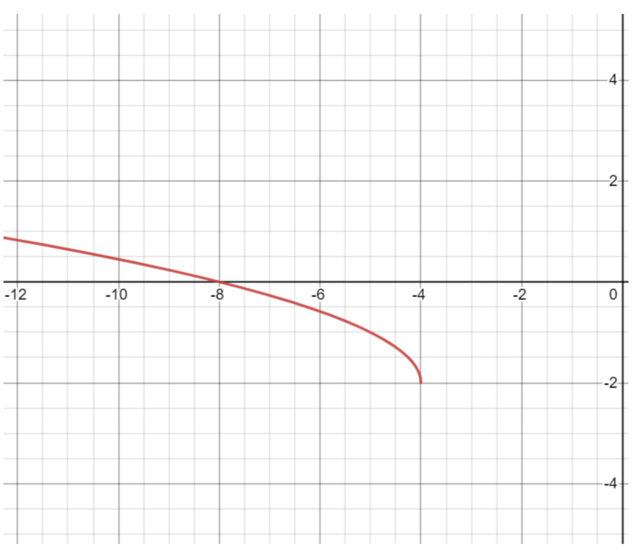
What is the Vertical Line Test, and how does it help you determine if you have a function?

<u>Example</u> State the domain and range of the following, and use the vertical line test to determine if the graph represents a function.

a.



b.



Examples Determine if the following represent functions.

a. $y = 3x^2$

b. $x = y^2$

c. $y = \frac{5}{x^2}$

<u>Example</u> Write the following in function notation: $3x + 2y = 6$

<u>Example</u> Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x - 1$ and $h(x) = \{(2,5), (6,-3), (-1,-4)\}$. Find the following:

a. $f(7) =$

b. $g(3) =$

c. $f(k) =$

d. $g(x + h) =$

e. $h(6) =$

Lesson: Increasing, Decreasing, and Constant Functions

Below are the “formal math” definitions of whether a function is increasing, decreasing, or constant. Let I be an open interval and let x_1 and x_2 be two points in that interval.

- A function is **increasing** over I if when $x_1 < x_2, f(x)_1 < f(x_2)$
- A function is **decreasing** over I if when $x_1 < x_2, f(x)_1 > f(x_2)$
- A function is **constant** over I if when $x_1 < x_2, f(x)_1 = f(x_2)$

While these are the formal definitions, what we really want to know are the intuitive meanings of these definition.

Sketch a graph that is increasing:

Decreasing:

And constant:

Example Sketch the graph in the video, and then identify where the graph is increasing, decreasing, or constant.

a.

b.

Lesson: Review of Lines and Linear Functions

What makes a function **linear?** What are 2 forms of a linear function? How do you interpret this equation?

Examples What is the domain and range of $f(x) = 4x + 1$. Show how to graph it.

Examples What is the domain and range of $f(x) = 5$. Show how to graph it.

Examples What is the domain and range of $x = -4$. Show how to graph it.

What is the **standard form** of a line?

__Examples__ Give the domain and range of $3x - y = 6$. Show how to graph it.

How do you find the **slope** of a line?

What is the slope of a vertical line?

What is the slope of a horizontal line?

Examples Find the slopes between the following sets of points.

a. $(5, 6)$ and $(-3, 2)$

b. $(-4, 7)$ and $(2, 7)$

c. $(6, 2)$ and $(6, -1)$

Example Find the slope of the following lines:

a. $4x - 2y = 8$

b. $-5x + 3y = -15$

Lesson: Review of Writing Equations of Lines

What is the point-slope form of a line?

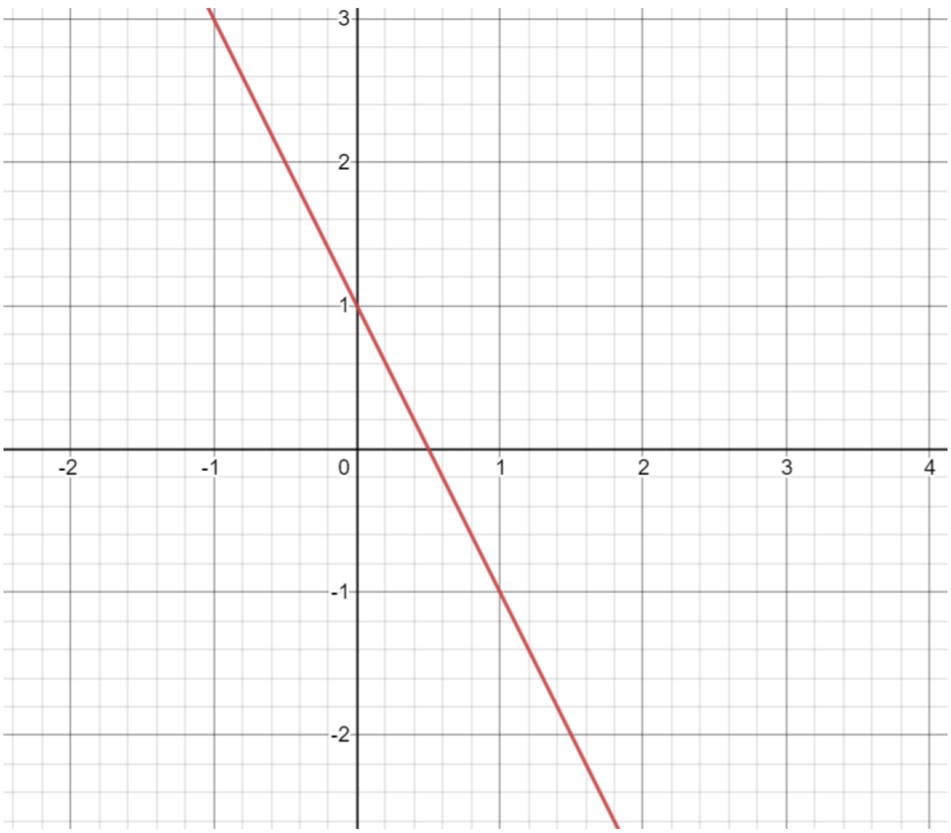
<u>Example</u> Write the equation of a line through the point $(4, 3)$ with slope $m = -2$.

<u>Example</u> Write the equation of a line that goes through the points $(2, 4)$ and $(-4, 10)$. Put your answer in standard form.

What is the slope-intercept form of a line?

Example Show how to write $4x - 6y = 12$ in slope-intercept form.

Example Find the slope, x- and y-intercept, and equation of the line:

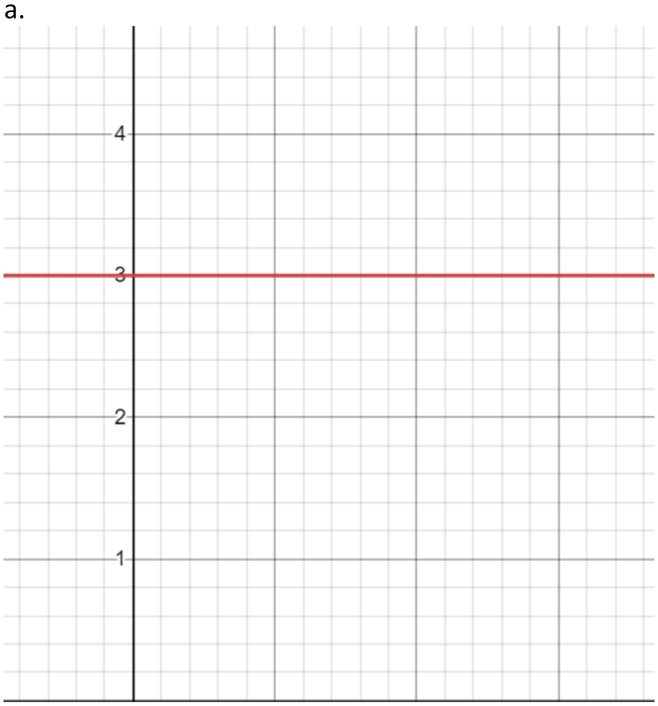


What is the form of a vertical line?

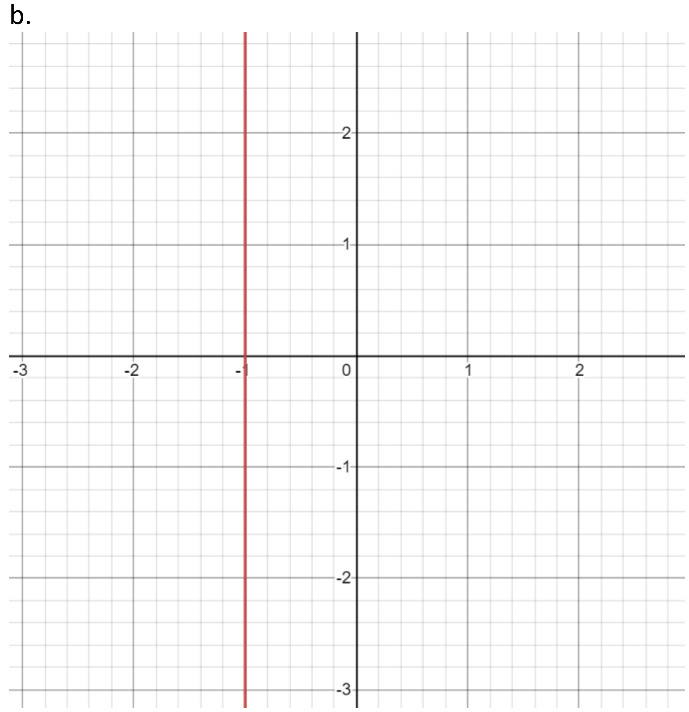
What is the form of a horizontal line?

<u>Example</u> Determine the equation of the following lines.

a.



b.



What makes 2 lines parallel?

What makes 2 lines perpendicular?

Examples Write the equation of a line that goes through $(-2, -5)$ that satisfies the conditions:

a. parallel to the line: $3x - 2y = -6$

b. perpendicular to the line: $x - 2y = -4$

Quick Reference of Line Formulas

Slope-Intercept Form

Standard Form

Point-Slope Form

Slope Formula

Form of a Horizontal Line

Form of a Vertical Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

Examples: Writing the Equation of a Line Through 2 Points

Example Write the equation of a line through the given 2 points. Put your answer in standard form, and your standard form should not include any fractions or decimals.

a. $(4, 5)$ and $(2, 9)$

b. $(2, 7)$ and $(-4, 7)$

c. $(-4, 2)$ and $(2, 5)$

d. $(3, 5)$ and $(3, 8)$

Examples: Writing the Equation of a Line Through 2 Points

Example Write the equation of a line through the point $(2, 3)$ that satisfies the given conditions. Your final answer should be in standard form with no fractions or decimals.

a. Parallel to $5x - 2y = 10$

b. Perpendicular to $4x - 2y = 1$

c. Parallel to $y = 5$

d. Perpendicular to $x = 7$

Lesson: Graphs of Basic Functions

In this section, we will review some basic graphs that are vital to know for this class and calculus. Knowing the “basic shapes” of certain graphs will make your life much easier and give you greater intuition when graphing.

Show hot to graph the following and highlight the **key points** and **key patterns.** This will come in handy for the next lesson. Take your time to sketch your own NEAT CLEAN CLEAR graphs.

$f(x) = x$

$f(x) = x^2$

$f(x) = x^3$

$f(x) = \sqrt{x}$

$f(x) = \sqrt[3]{x}$

$f(x) = |x|$

Lesson: Piecewise Functions

What is a piecewise function? Draw an example of what they look like.

<u>Example</u> Given the function: $f(x) = \begin{cases} 3x - 1, & x \geq 2 \\ \frac{1}{2}x - 4, & x < 2 \end{cases}$, evaluate the following:

a. $f(3) =$

b. $f(-1) =$

c. $f(2) =$

<u>Example</u> Given the function: $f(x) = \begin{cases} 4, & x > 3 \\ 4x - 2, & -2 < x \leq 3 \\ x^2 - 5, & x \leq -2 \end{cases}$ evaluate the following:

a. $f(4) =$

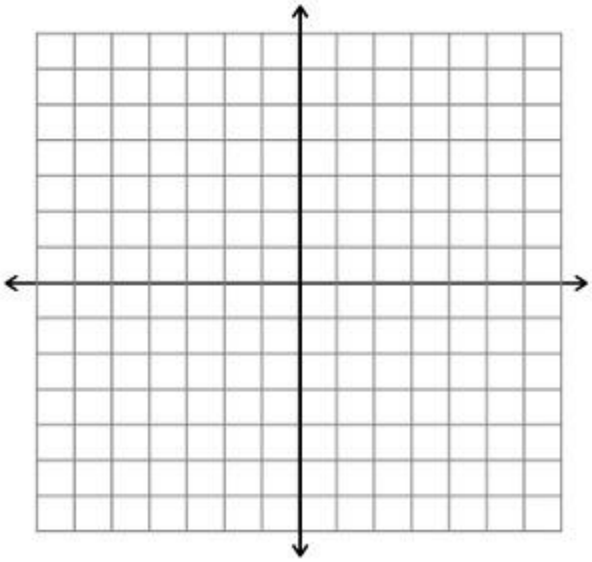
b. $f(-5) =$

c. $f(-2) =$

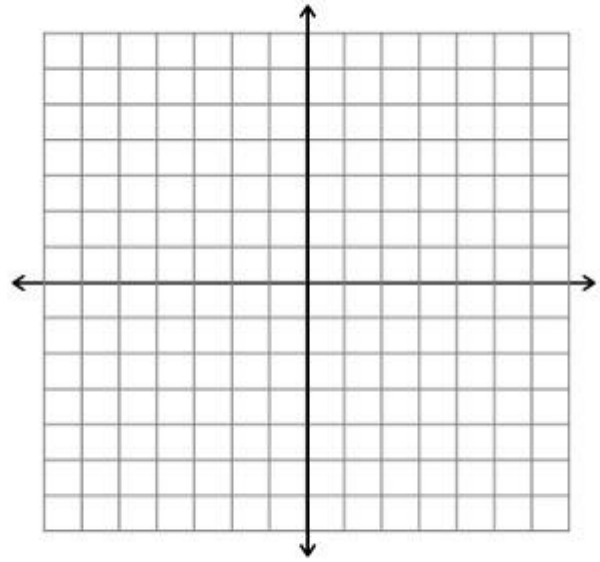
d. $f(3) =$

Example Show how to graph the following piecewise functions.

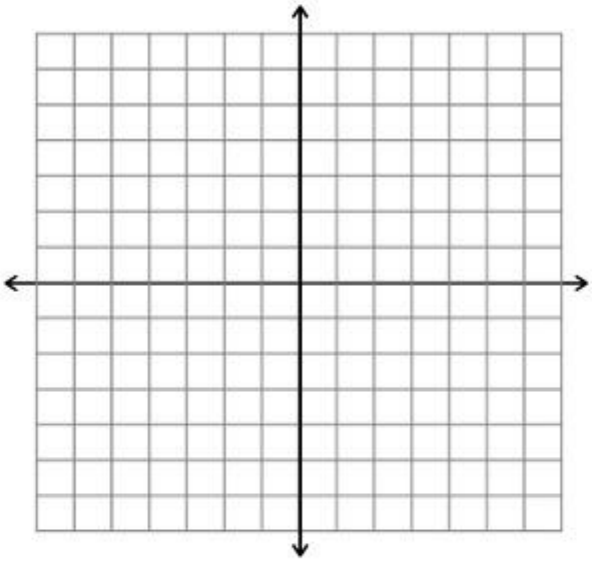
a. $f(x) = \begin{cases} 4, & x > 5 \\ -1, & x \le 5 \end{cases}$



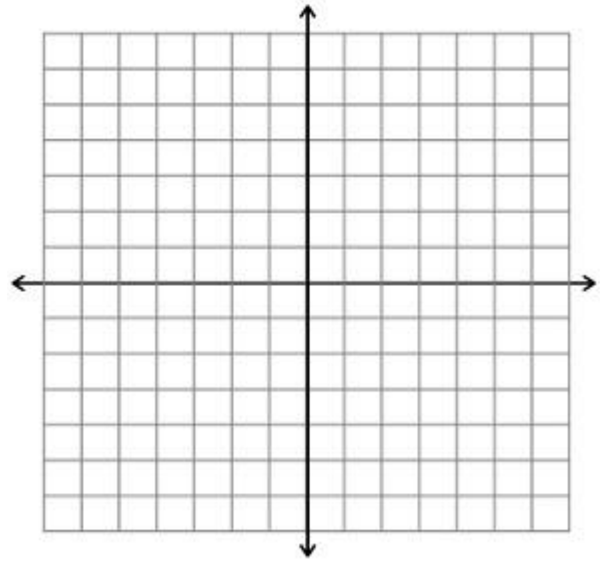
b. $f(x) = \begin{cases} 3x + 1, & x > -3 \\ -2x, & x \le -3 \end{cases}$



c. $f(x) = \begin{cases} 3, & x > 2 \\ 2x, & -1 < x \leq 2 \\ -x+3, & x \leq -1 \end{cases}$



d. $f(x) = \begin{cases} 3x+1, & x \geq 1 \\ 6, & -1 \leq x < 1 \\ x^2, & x < -1 \end{cases}$



Lesson: The Greatest Integer Function

Show how to graph the greatest integer function $f(x) = [\![x]\!]$

<u>Example</u> Graph $f(x) = \left[\!\!\left[\frac{1}{2}x - 2\right]\!\!\right]$

Lesson: Translations and Reflections of Functions

Using the graph $f(x) = |x|$ as the "base function," sketch how vertical translations work (summarize what was discussed in the video).

How do you describe vertical translations in function notation?

Using the graph $f(x) = x^2$ as the "base function," sketch how horizontal translations work (summarize what was discussed in the video).

How do you describe horizontal translations in function notation?

Use the graph $f(x) = \sqrt{x}$ to show how reflections over the x- and y-axis work (summarize what was discussed in the video).

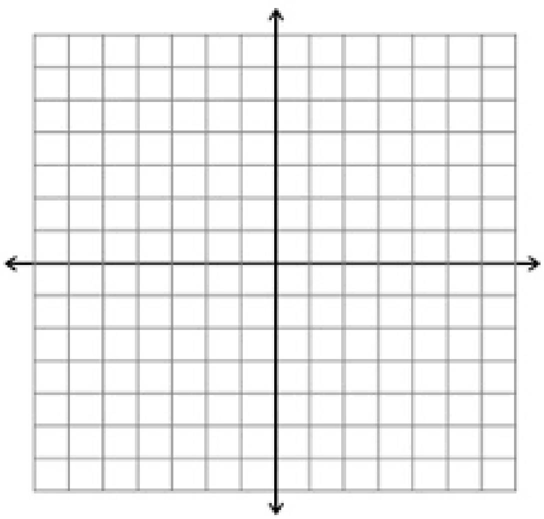
How do you describe reflections over the x-axis and y-axis using function notation.

Use the graph $f(x) = |x|$ to show how stretching and shrinking works (summarize what was discussed in the video).

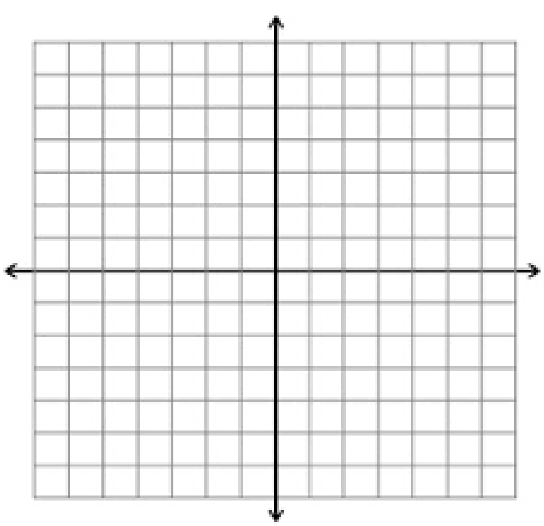
Explain how stretching and shrinking works with function notation.

Example Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

a. $f(x) = -x^2 + 3$

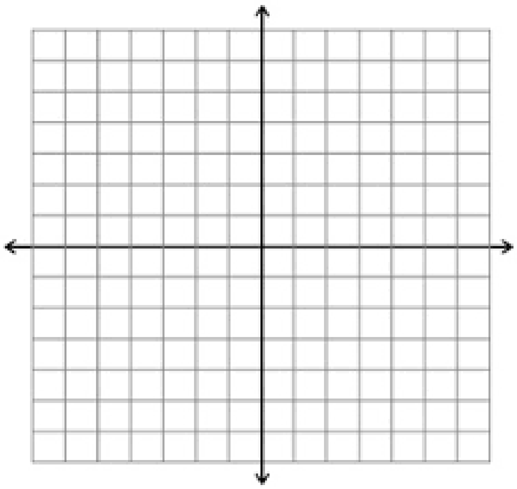


b. $g(x) = (x + 2)^3 - 3$

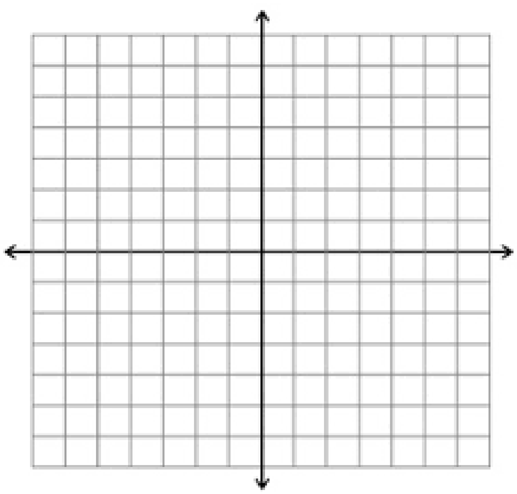


Example Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

a. $g(x) = -2(x+1)^2 - 5$



b. $h(x) = 2\sqrt{-x} + 1$



Examples Write the equation of the given functions with the following characteristics.

a. The function $f(x) = x^2$ moved 3 units left and 5 units down.

b. The function $g(x) = \sqrt[3]{x}$ reflected over the y-axis, translated 2.5 units right and 1.75 units up.

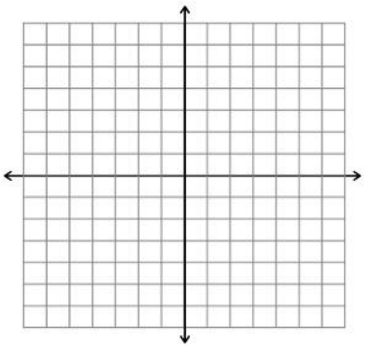
Transformations of Functions

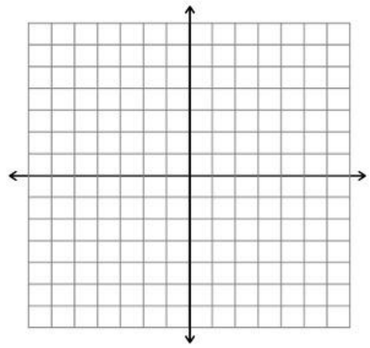
These are looking at graphs of things like:

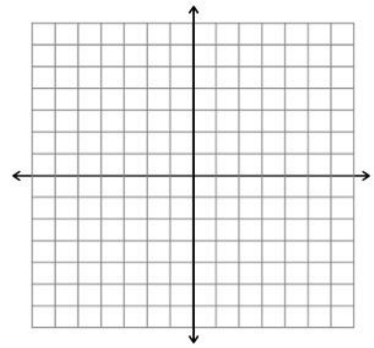
$$f(x) = x^2 - 5 \quad \text{or} \quad f(x) = |x+3| - 2 \quad \text{or} \quad f(x) = \sqrt{x-3}$$

First we need to discuss the shape of a few basic functions. Write these down and note the important points.

a. $f(x) = x^2$ b. $f(x) = |x|$ c. $f(x) = \sqrt{x}$



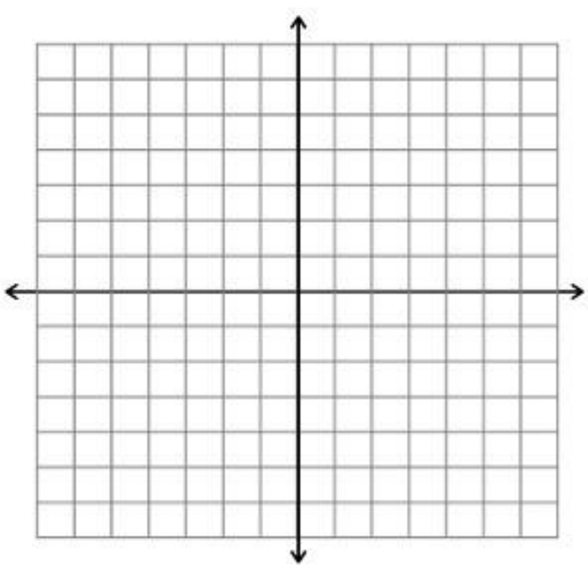




Using a few different colors, sketch out what the graphs of

a. $f(x) = x^2$ versus b. $g(x) = x^2 + 3$ versus c. $h(x) = x^2 - 1$

look like all on the same graph below:

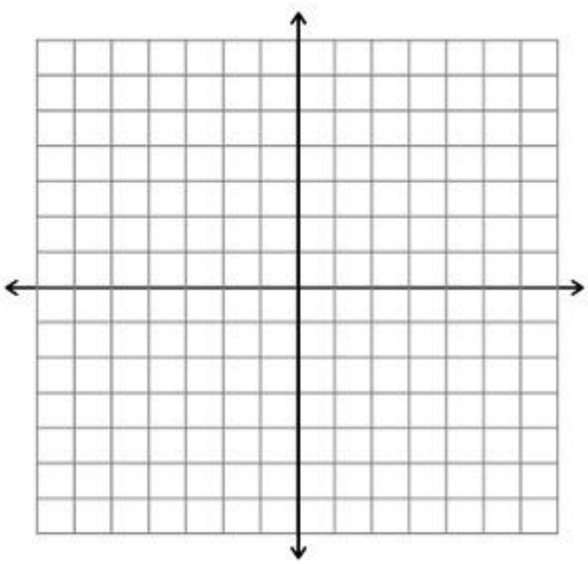


This is an example of a vertical shift. Explain what this means in mathematical terms:

Using a few different colors, sketch out what the graphs of

a. $f(x) = |x|$ versus b. $g(x) = |x - 1|$ versus c. $h(x) = |x + 2|$

look like all on the same graph below:

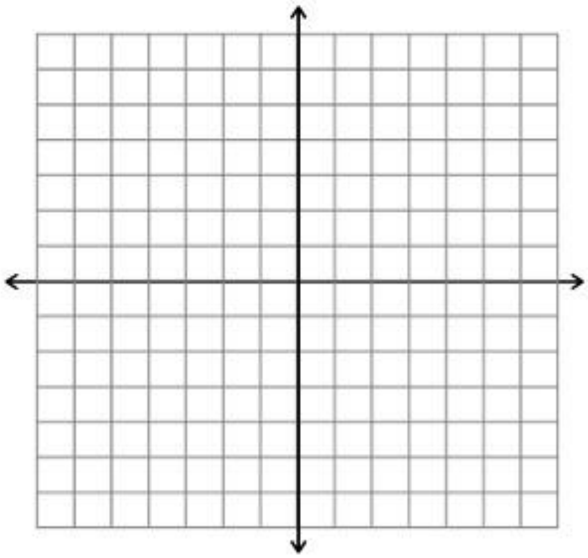


This is an example of a horizontal shift. Explain what this means in mathematical terms:

Using a few different colors, sketch out what the graphs of

a. $f(x) = |x|$ versus b. $g(x) = -|x|$

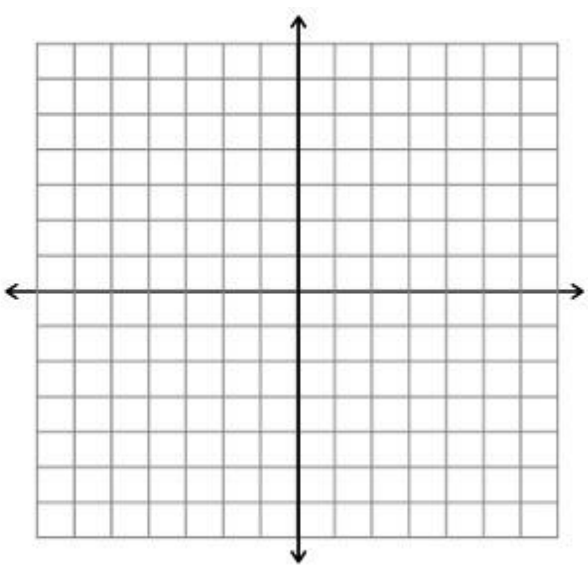
look like all on the same graph below:



This is an example of a reflection over the x-axis. Explain what this means in mathematical terms:

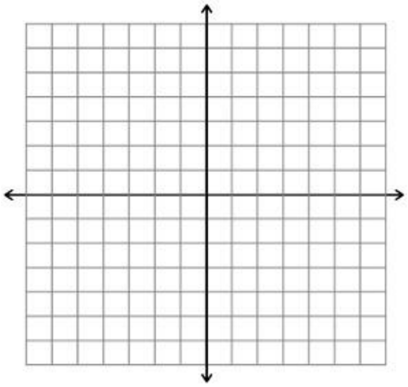
<u>Example</u> Explain in plain language what types of shifts/reflections are occurring, and then graph the function.

$$f(x) = |x| + 3$$

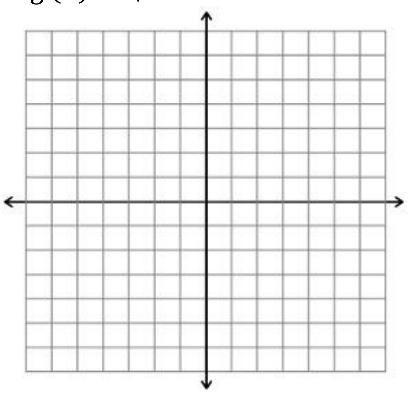


Examples Explain in plain language what types of shifts/reflections are occurring, and then graph the function.

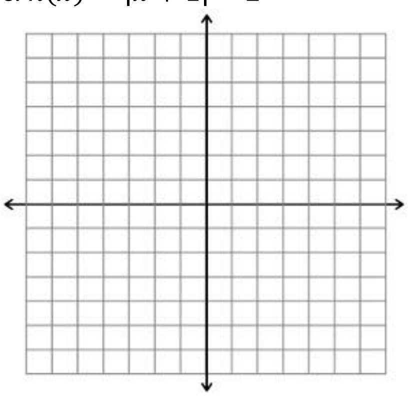
a. $f(x) = (x-1)^2$



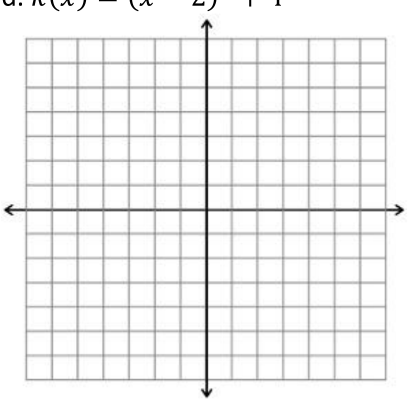
b. $g(x) = \sqrt{x} - 2$



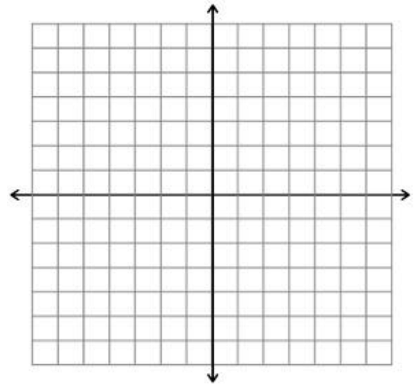
c. $h(x) = |x+1| - 2$



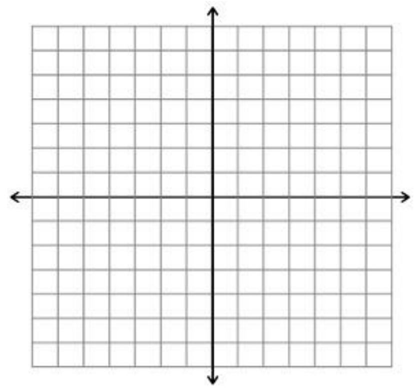
d. $k(x) = (x-2)^2 + 4$



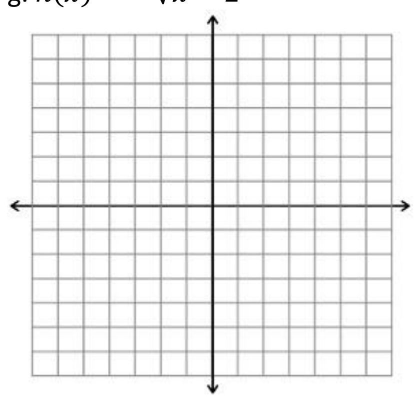
e. $f(x) = -(x+3)^2 - 2$



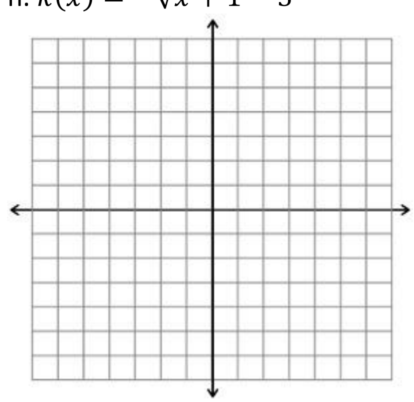
f. $g(x) = -|x+3| - 1$



g. $h(x) = -\sqrt{x-2}$



h. $k(x) = -\sqrt{x+1} - 3$



Examples Write the equations that represent the listed transformations.

a. A function $f(x) = \sqrt{x}$ is shifted 7 units down.

b. A function $g(x) = x^3$ is shifted 3 units left and 2 units up.

c. A function $h(x) = |x|$ is shifted 2.5 units right and reflected over the x-axis.

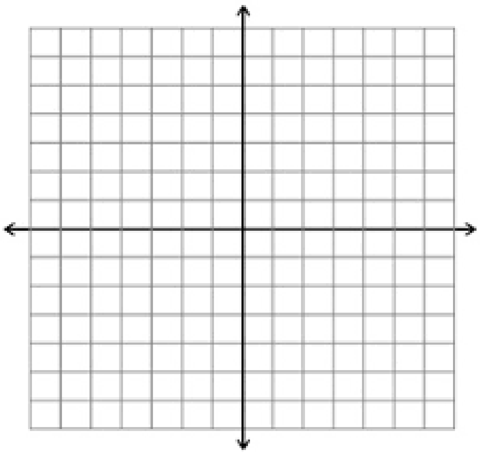
d. A function $f(x) = e^x$ is shifted 4 units up and reflected over the x-axis.

e. A function $g(x) = \sqrt[3]{x}$ is shifted 3.2 units right, 5.7 units up, and reflected over the x-axis

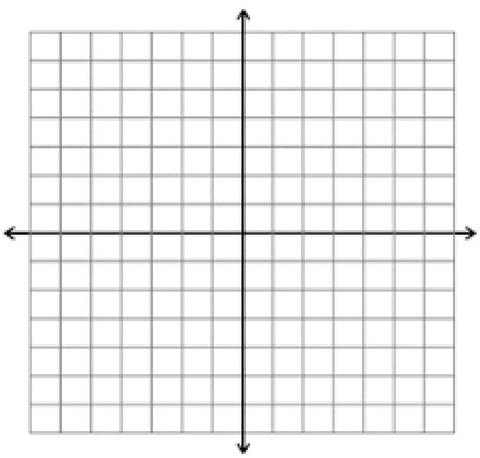
Examples: Translations and Reflections of Functions

Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

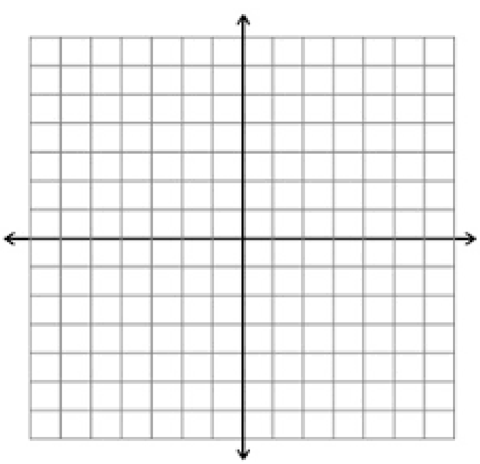
a. $f(x) = -(x+2)^2 - 3$



b. $g(x) = \frac{1}{2}(x-1)^3 + 2$



c. $h(x) = -\frac{1}{3}\sqrt{x+1} + 1$



Lesson: Symmetry and Even or Odd Functions

Explain when a graph is symmetric with respect to the y-axis:

Explain when a graph is symmetric with respect to the x-axis:

Explain when a graph is symmetric with respect to the origin:

What makes a function odd?

What makes a function even?

So what is the test for even or odd functions?

<u>Examples</u> Determine what type of symmetry the following functions have:

a. $y = x^2 + 4$

b. $x^2 + y^2 = 25$

Examples Determine what type of symmetry the following functions have:

a. $y = x^3 - 5$

b. $y = 7x^2 + 2x - 4$

c. $y = 2x^{\frac{1}{3}} - 3x$

Lesson: Review of Function Operations

Given that f and g are functions, how do we define the function operations?

$(f+g)(x) =$

$(f-g)(x) =$

$(fg)(x) =$

$\left(\frac{f}{g}\right)(x) =$

<u>Example</u> Let $f(x) = 3x+1$, $g(x) = x^2-4$, $h(x) = 3x^2+7x+2$, and $k(x) = x^2+6x+9$. Find each of the following:

a. $(f+g)(x) =$

b. $(f+g)(3) =$

c. $(g-k)(1) =$

Example Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = 3x^2 + 7x + 2$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $\left(\frac{h}{f}\right)(x) =$

b. $(gh)(x) =$

c. $\left(\frac{k}{h}\right)(1) =$

Lesson: Review of Composition of Functions

For two functions f and g, what is the composition $(f \circ g)(x)$?

Example Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(f \circ g)(x) =$

b. $(g \circ f)(x) =$

Example Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(k \circ f)(1) =$

b. $(g \circ h)(4) =$

c. $(h \circ k)(x) =$

Lesson: The Difference Quotient

Explain what the difference quotient is and draw some sketches to explain how it works. In other words, watch the video explanation and then explain it in your own words on this page.

State the formula for the difference quotient here:

Example Calculate the different quotient for the following.

a. $f(x) = 4x + 5$

b. $f(x) = x^2 - 1$

Lesson: The Difference Quotient

Find the difference quotient.

a. $f(x) = \frac{1}{x+3}$

b. $f(x) = x^2 - 4x + 1$

c. $f(x) = -x^2 + 3$

Lesson: Review of Quadratic Functions

A polynomial function of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Where $a_n, a_{n-1}, \dots, a_1, a_0$ are any complex or real number, $a_n \neq 0$, and n is a nonnegative integer.

What do we call n in this equation?

What do we call a_n in this equation?

What do we call $a_n x^n$ in this equation?

<u>**Example**</u> What is the degree, leading coefficient, and dominating term in the following?

a. $f(x) = 4x^4 - 5x^2 + x^7 + 100$

b. $g(x) = x\sqrt{3} - 5$

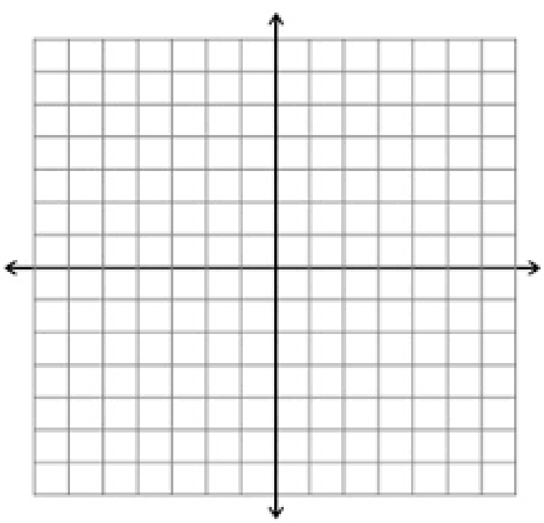
c. $h(x) = 7$

What is a quadratic function?

What do we call the graph of a quadratic function?

Draw and label the parts of a parabola.

Example Sketch the graph of $f(x) = -(x+4)^2 - 2$



What is the **vertex form** of the equation of the parabola?

Example Show how to use completing the square to put $f(x) = x^2 + 6x - 4$ in vertex form. Write out clear directions.

Example Use completing the square to write $f(x) = 3x^2 + 5x + 2$ in vertex form, then sketch the graph.

Example Find the intercepts of $f(x) = 3x^2 + 5x + 2$

How many x-intercepts can a parabola have? Draw a little sketch of each situation.

How can you tell how many intercepts a parabola will have if the parabola is in the form: $f(x) = ax^2 + bx + c$

What is the vertex formula for a parabola in the form $f(x) = ax^2 + bx + c$?

<u>Example</u> Use the vertex formula to find the vertex of the following: $f(x) = 2x^2 - 4x + 1$

Examples: Finding Intercepts, Vertices, Etc of Quadratic Functions

Find the vertex, axis of symmetry, domain, range, intercepts, and intervals of increasing and decreasing. If the function is in $f(x) = ax^2 + bx + c$ form, put it in vertex form. Sketch the graph. If the function is in $f(x) = a(x - h)^2 + k$ form, put it into standard form.

a. $f(x) = -\frac{1}{2}x^2 - 3x - \frac{1}{2}$

b. $f(x) = 2(x - 1)^2 + 3$

Lesson: Synthetic Division and the Remainder Theorem

The outcome of division can be restated using the division algorithm.

State the division algorithm:

Show how we use polynomial long division with $(x^2 - 3x - 1) \div (x + 2)$, and then restate the result using the format of the division algorithm.

Now show how to do the operation $(x^2 - 3x - 1) \div (x + 2)$ with synthetic division:

Restate this result using the division algorithm:

<u>Example</u> Use synthetic division to perform: $\frac{4x^3-3x^2-7}{x+1}$. Rewrite the result using the division algorithm.

So what are 3 keys about using synthetic division to remember?

What is the remainder theorem?

Example Use the remainder theorem to find the desired functional values: $f(x) = -3x^4 + 5x^3 - 30x + 20, f(2)$. What is the equivalent division problem associated with this? Write the result of your division using the division algorithm.

Example Use the remainder theorem to find the desired functional values: $f(x) = 3x^2 + 5, k = 3 - i$

Lesson: Synthetic Division and Potential Zeros of Polynomials

What is a “zero” of a polynomial?

How does the remainder theorem help us to determine if we have a zero?

What are some other terms that we use for “zero” of a polynomial?

Example Determine if the given k is a zero of the polynomial. If it is not, then what is the value of $f(k)$?

a. $f(x) = x^2 - 4x - 5; k = 5$

b. $f(x) = 2x^3 - 3x^2 + 5; k = 1$

c. $f(x) = x^2 - 2x + 2; k = 1 + i$

Lesson: The Factor Theorem

What does the factor theorem say?

<u>Example</u> Determine whether $(x-1)$ is a factor of each polynomial:

a. $f(x) = 9x^4 - 13x^2 + 4$

b. $f(x) = 4x^3 + 10x^2 + 2x - 4$

Given the above, what can we say is also a zero for example a?

Example Factor $f(x) = 3x^3 + 4x^2 - 17x - 6$ into linear factors given that -3 is a zero.

Example Factor $f(x) = 2x^3 - x^2 - 8x + 4$ into linear factors given that $\frac{1}{2}$ is a zero.

Lesson: The Rational Zeros Theorem

The rational zeros theorem is a way to determine *possible* candidates for *rational* zeros of a polynomial function.

State the Rational Zeros Theorem:

Example Find all the possible rational zeros for $f(x) = 2x^3 - x^2 - 8x + 4$ and then factor into linear factors.

Example Find all the possible rational zeros for $f(x) = 5x^4 - 19x + x^2 + 31x + 6$ and then factor into linear factors.

Lesson: The Fundamental Theorem of Algebra and the Number of Zeros Theorem

What does the Fundamental Theorem of Algebra state?

What is the number of zeros theorem?

<u>Example</u> What is the maximum number of zeros that each polynomial could possibly have?

a. $f(x) = x^2 - 3x + 1$

b. $f(x) = x^{17} - 3x + 1$

c. $f(x) = 9$

<u>Example</u> Find a polynomial $f(x)$ that has zeros $-3, 3$, and 1, where $f(2) = -10$

<u>Example</u> Find a polynomial $f(x)$ that has zeros $6, -3$, and -2, where $f(1) = 20$

Lesson: Conjugate Zeros Theorem

Before we can discuss the theorem, let's review a few properties of conjugates.

What is the conjugate of the following:

a. $3 + 2i$ b. $5 - 3i$ c. $-7 + i$

What is another way that we denote conjugates? Summarize the explanation in the video. Give an example of how this notation works.

What are some properties of conjugates?

What is the conjugate zeroes theorem?

Example Find a polynomial function of least degree with only real coefficients and zeros 2 and $(3 - i)$.

Example Suppose $k = -i$ is one zero of $f(x) = x^4 + 26x^2 + 25$. What are the other zeros?

Example Factor $f(x) = 2x^3 + (3 - 2i)x^2 + (-8 - 5i)x + (3 + 3i)$ into linear factors given $k = 1 + i$ is a factor.

Before you begin, what is unique about this problem?

Lesson: Finding Zeros of a Polynomial Function

In previous videos we discussed:

- The factor theorem
- The rational zeros theorem
- The fundamental theorem of algebra
- The number of zeros theorem
- The conjugate zeros theorem

Using all of these theorems, we can combine them to find the zeros of *any* polynomial.

What can we say about a polynomial function of odd degree with real coefficients?

What can we say about a polynomial function of even degree with real coefficients?

What if a polynomial doesn't have only real coefficients?

Example Find all zeroes of the following polynomials.

a. $f(x) = x^3 - 21x - 20$

b. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

Examples: Finding Zeros of a Polynomial Function

Find all the zeros of the polynomials.

a. $f(x) = x^4 + 13x^2 + 36$

b. $f(x) = 2x^4 - 11x^3 + 4x^2 + 29x + 12$

c. $f(x) = x^4 - 7x^3 + 17x^2 - x - 26$

Lesson: Descartes' Rule of Signs

Let $f(x)$ be a polynomial function with real coefficients and a nonzero constant term, with terms in descending powers of x. Explain how to use Descartes' Rule of Signs:

<u>Example</u> Determine the number of different possibilities for the number of positive, negative, and nonreal complex zeroes:

a. $f(x) = 3x^5 + 4x^4 - x^3 + 2x - 1$

b. $f(x) = x^6 + x^5 - x^4 + x^3 + x^2 - x + 1$

c. $f(x) = 3x^5 - 7x^3 + 6x + 8$

Lesson: Graphs of Basic Polynomial Functions and Translations of the Basic Graphs

Explain the graphs of $f(x) = x^2, f(x) = x^4, f(x) = x^6$, etc.

Explain the graphs of $f(x) = x^3, f(x) = x^5, f(x) = x^7$, etc.

If we multiply the polynomial like so: $f(x) = ax^n$, what happens to the graph?

What happens if we have $f(x) = a(x-h)^n + k$?

<u>Example</u> Sketch the graphs:

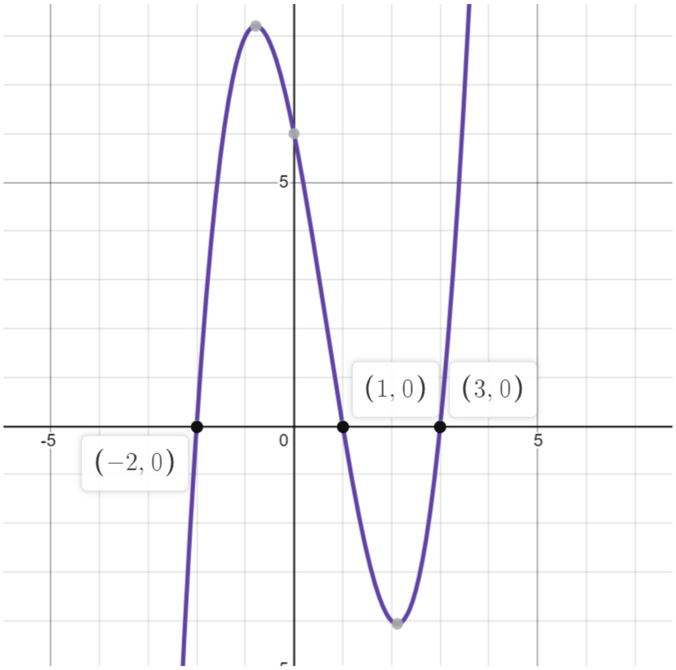
a. $f(x) = (x-5)^5 + 3$

b. $f(x) = -2(x+1)^6 - 4$

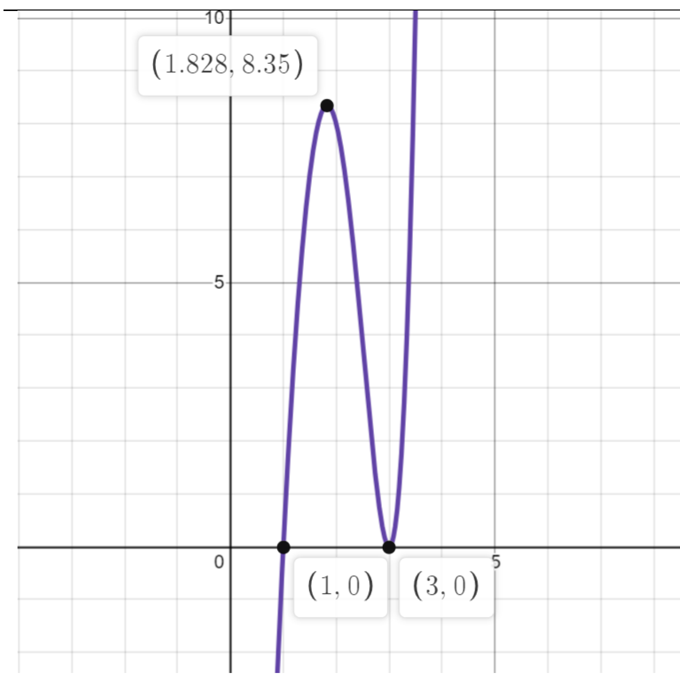
Lesson: Graphs of Polynomial Functions, Turning Points, and End Behavior

In the last video, we discuss the graphs of basic polynomials and translations. In this video, we will look at more complicated possibilities.

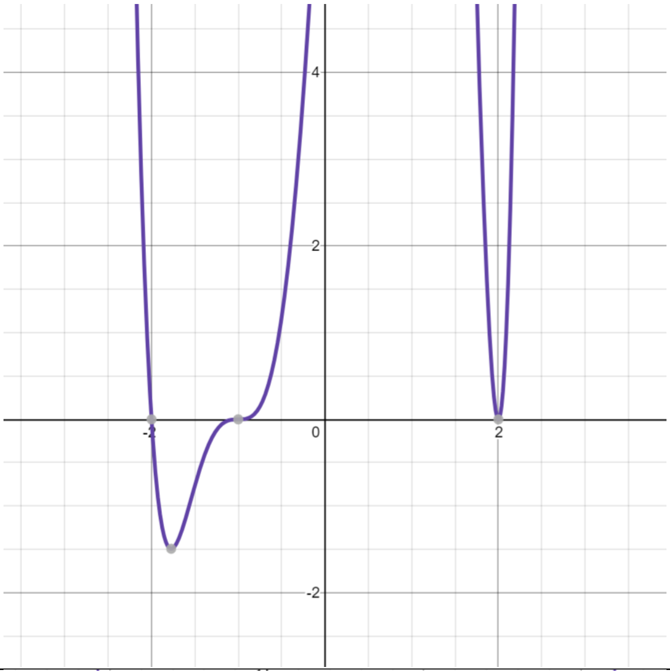
Below are the graphs of a few polynomials. What patterns do you notice?



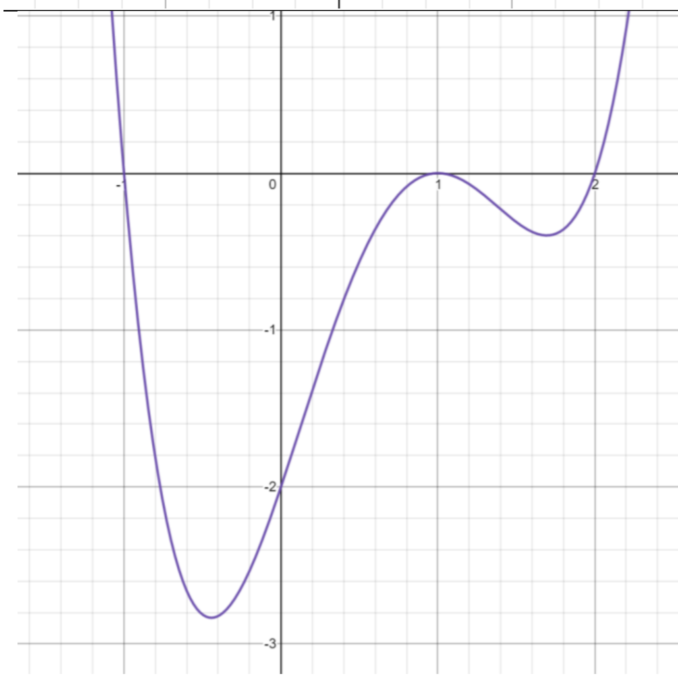
$f(x) = x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$



$$f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$$
$$= (x-3)^2(x-1)(x-2i)(x+2i)$$



$f(x) = (x-2)^2(x+1)^3(x+2)$



$f(x) = (x-1)^2(x+1)(x-2)$

What do you notice about the behavior of these graphs at zeros? Summarize this in your own words, it was discussed as a general pattern in the video.

What are turning points and how many will a degree n polynomial have?

What is the end behavior of a polynomial and how do you determine it?

Example What are the number of turning points of the following and what is the end behavior of each?

a. $f(x) = x^2 + 3x + 2$

b. $f(x) = (x+1)^2(x-2)$

c. $f(x) = -\frac{1}{2}x(x-3)^4(x+1)^3(x+2)$

List the steps required to graph a polynomial:

<u>Example</u> Graph $f(x) = x^3 - x^2 - 4x + 4$

Example Graph $f(x) = -(x+1)(x-2)^2(x-3)$

What is the relationship between x-intercepts, zeros, solutions, and factors of polynomials????

Examples: Graphs of Polynomial Functions, Turning Points, and End Behavior

Graph the following polynomials:

a. $f(x) = x^3 + 5x^2 + 2x - 8$

b. $f(x) = 3x^4 + 5x^3 - 2x^2$

c. $f(x) = (x-1)^3(x+2)^2$

Lesson: The Intermediate Value Theorem and the Boundedness Theorem

State the Intermediate Value Theorem

Example Using synthetic division to show $f(x) = -x^3 + 2x^2 - 4x + 5$ has a real zero between 1 and 2.

Example Using synthetic division to show $f(x) = x^4 - 4x^3 - x + 3$has a real zero between 0.5 and 1.

State the Boundedness Theorem

Example Show that $f(x) = x^4 + 5x^2 + 3x - 7$ has no real zero less than -3.

Example Show that $f(x) = 2x^5 - x^4 + 2x^3 - 2x^2 + 4x - 4$ has no real zero greater than 2

Lesson: The Basic Graphs of Rational Functions

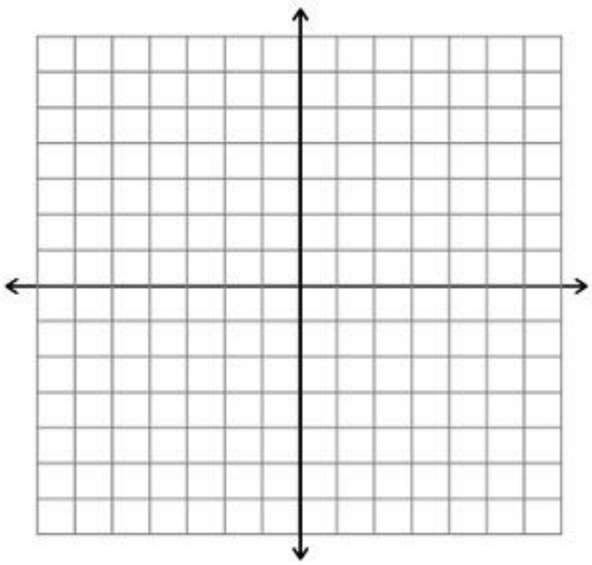
A rational function is a function of the form:

$\frac{f(x)}{g(x)}, g(x) \neq 0$

Where $f(x)$ and $g(x)$ are polynomials.

There are 2 graphs that we view as the most basic version of rational functions.

Make a table and graph $f(x) = \frac{1}{x}$



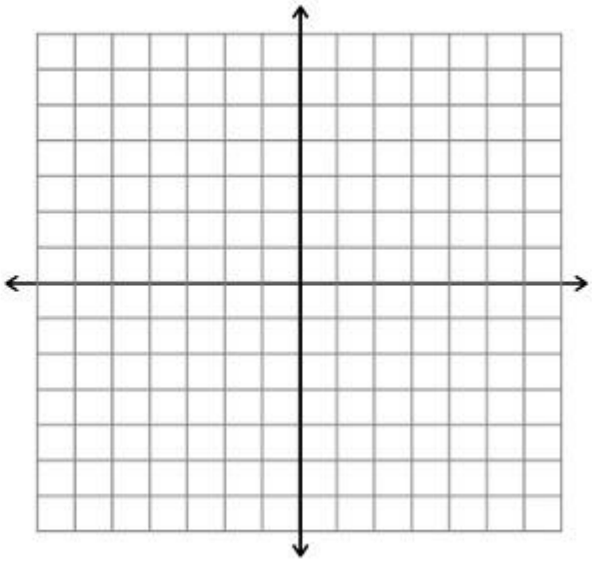
What is the domain and range?

Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?

Make a table and graph $f(x) = \frac{1}{x^2}$



What is the domain and range?

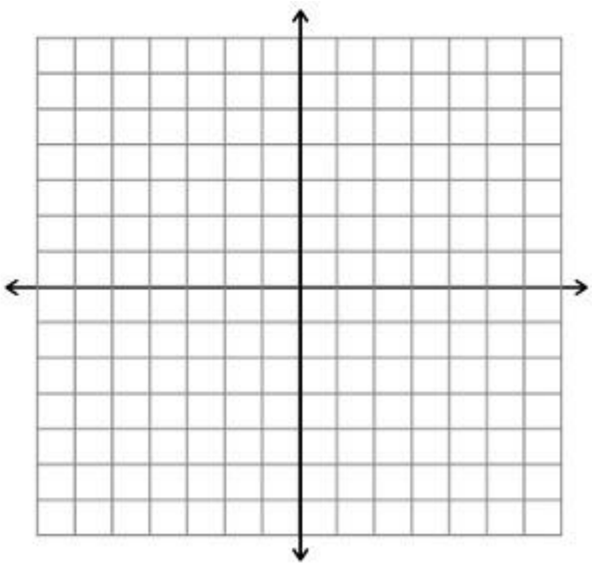
Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

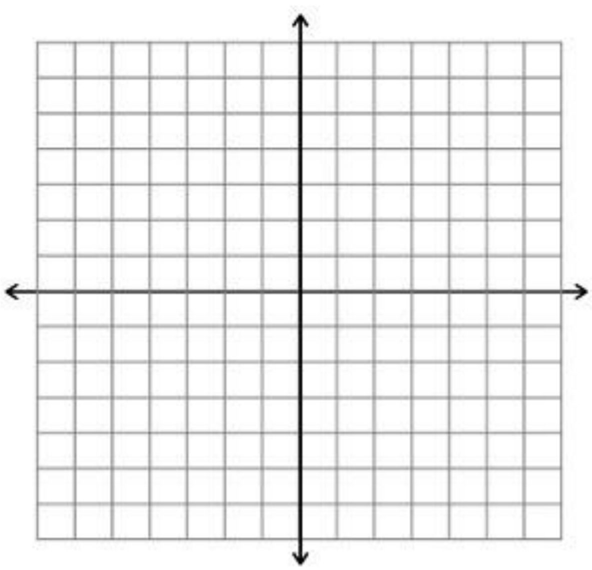
What kind of symmetry does this graph have?

Example Describe the translations for the following graphs, and then draw an approximate sketch of the graph.

a. $f(x) = \frac{1}{(x+1)^2} - 2$



b. $g(x) = -\frac{2}{x} + 1$



Sketch what happens for odd powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^3}, y = \frac{1}{x^5}, y = \frac{1}{x^7}$

Sketch what happens for higher even powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^4}, y = \frac{1}{x^6}, y = \frac{1}{x^8}$

Lesson: Asymptotes of Rational Functions

What is an informal definition of asymptotes?

Is it possible that a graph would graph an asymptote?

Here are the formal definitions of asymptotes:

A graph $f(x) = \frac{p(x)}{q(x)}$ $(q(x) \neq 0)$ has a

- **Vertical asymptote** if $|f(x)| \to \infty$ as $x \to a$. The asymptote is x=a.
- **Horizontal asymptote** if $f(x) \to b$ as $|x| \to \infty$. The asymptote is y=b.

What is super important about asymptote?

Where do you typically find vertical asymptotes?

How can you determine horizontal asymptotes? Describe the two ways, and also explain how oblique asymptotes. (In other words, explain what we talked about in the video).

<u>Examples</u> Find any vertical, horizontal, or oblique asymptotes.

a. $f(x) = \frac{x+2}{x^2+7x+12}$

b. $f(x) = \frac{3x+1}{4x-2}$

c. $f(x) = \frac{x^2+3x+2}{x^2-9}$

d. $f(x) = \frac{x^2+x}{x+3}$

Examples: Finding Oblique Asymptotes

Find all the asymptotes for the following functions.

a. $f(x) = \frac{x^2+2x}{2x+1}$

b. $f(x) = \frac{x^2+2}{x+1}$

Lesson: Graphing Rational Functions

To learn how to graph rational functions, we will outline the steps and then graph. **Clearly write directions for each step as we go through this process!!!**

Graph $f(x) = \frac{x+3}{x^2+7x+10}$. Outline all the relevant steps discussed in the video.

Graph $f(x) = \frac{x+1}{x-3}$. Outline all the relevant steps discussed in the video.

Graph $f(x) = \frac{x^2+1}{x-3}$. Outline all the relevant steps discussed in the video.

Examples: Graphing Rational Functions

Graph $f(x) = \frac{(x+2)^2}{(x-1)(x+3)}$. Outline all the relevant steps discussed in the video.

Graph $f(x) = \frac{x}{x^2-4}$. Outline all the relevant steps discussed in the video.

Graph $f(x) = \frac{2x^2-3x-1}{x-2}$. Outline all the relevant steps discussed in the video.

Examples: Equations of Rational Functions

Copy the graphs from the video and then describe how to find the equations of those graphs.

Lesson: A Review of Variation

Describe the 3 types of variation and outline any patterns that might help you remember.

Direct Variation

Inverse Variation

Joint Variation

Combined Variation

Example If y varies directly with x, and y=15 when x=3, find y when x=6

Example If a varies jointly with b and c, and a=40 when b=2 and c=5, find a when b=3 and c=2.

Example If p varies inversely with m and directly with the square of t, and p=32 when t=2 and m=4, find p when t=3 and m=9

Example Rewrite the following situations as variation.

a. $C = 2\pi r$, when C is the circumference of a circle and r is the radius.

b. $r = \frac{d}{t}$, where r is the speed when traveling d miles over t hours.

c. $f = \frac{mv^2}{r}$, where f is the centripetal force of an object of mass ma moving along a circle with radius r at velocity v.

Examples: Applications of Variation

The circumference of a circle varies directly as the radius. A circle of radius 8in has a circumference of 50.27in. Find the circumference of the circle if the radius is 5in.

The current in a simple electrical circuit varies inversely as the resistance. If the current is 30 amps when the resistance is 10ohms, find the current if the resistance if 5ohms.

The volume of a gas varies inversely as the pressure and directly as the temperature. If a gas occupies a 2.6L volume at 600 kelvins (kelvin is a temperature unit) and a pressure of 24 newtons, find the volume at 240 kelvins and a pressure of 32 newtons.

Lesson: A Review of Inverse Functions

What is a one-to-one function?

<u>Examples</u> Are the following one-to-one functions? If no, why not?

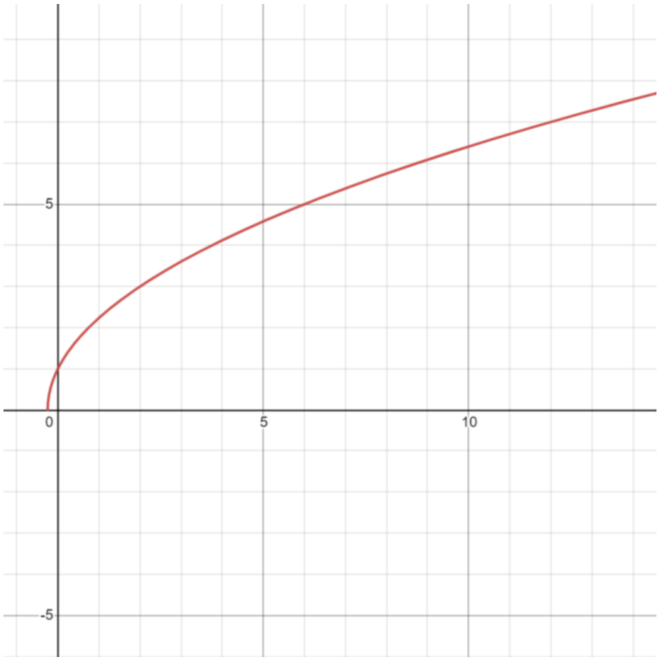
a. $f(x) = x^2$

b. $g(x) = 2x + 1$

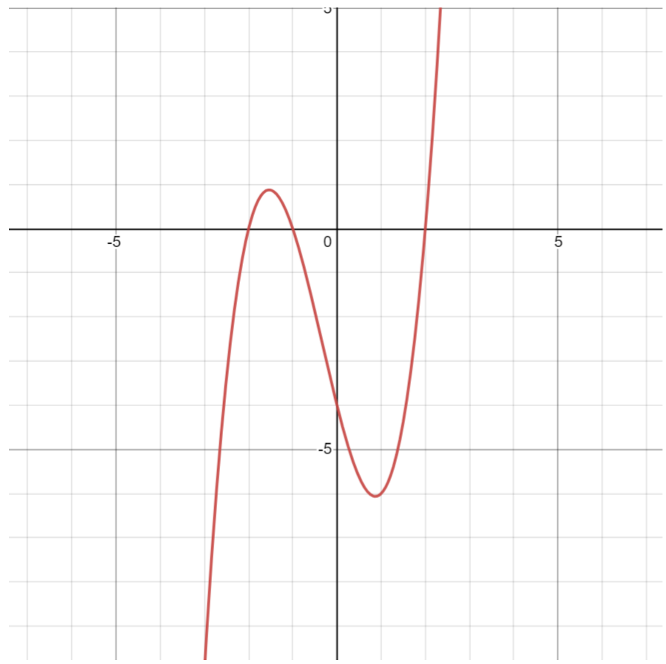
What is the horizontal line test?

Example Use the horizontal line test to determine if the functions are one-to-one:

a.



b.



What is an inverse function?

What does an inverse function have to do with one-to-one functions?

Describe how to use composition to determine if 2 functions are inverses of one another.

Example Use composition to determine if the functions $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x - 2$ are inverses.

Example Use composition to determine if the functions $f(x) = \frac{2}{x+6}$ and $g(x) = \frac{6+x}{2}$ are inverses.

Using $f(x) = 4x + 2$, outline the steps for how to find the inverse of a function.

Example Find the inverse of the following functions.

a. $f(x) = \sqrt[3]{3x + 1}$

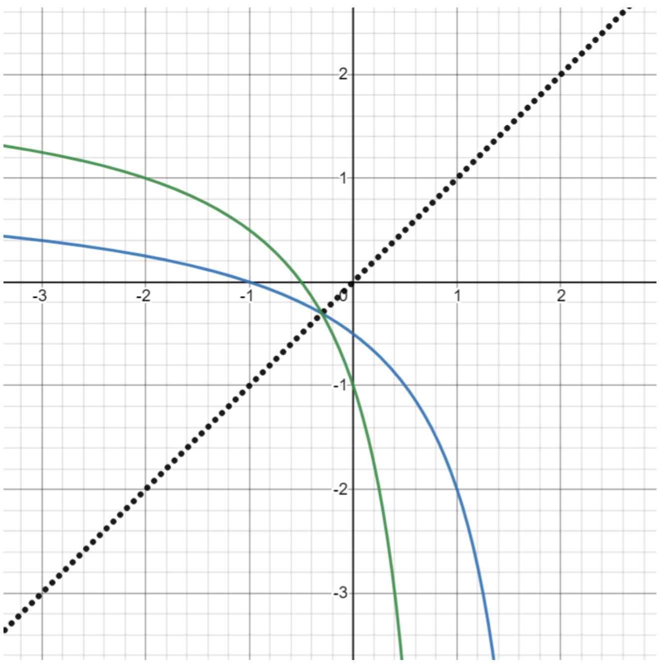
b. $f(x) = \frac{x+2}{x-1}$

c. $f(x) = \sqrt{x - 5}$

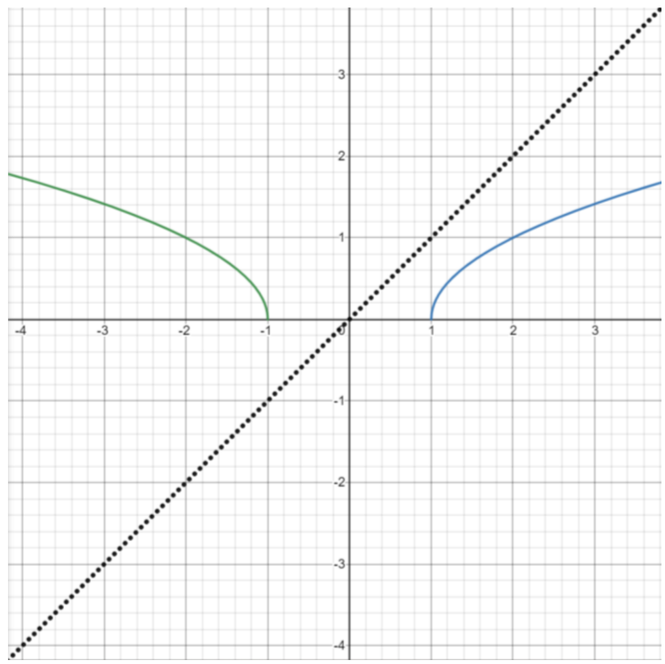
How can we tell on a graph if 2 functions are inverses of one another?

<u>Examples</u> Are the following graphs inverses of one another?

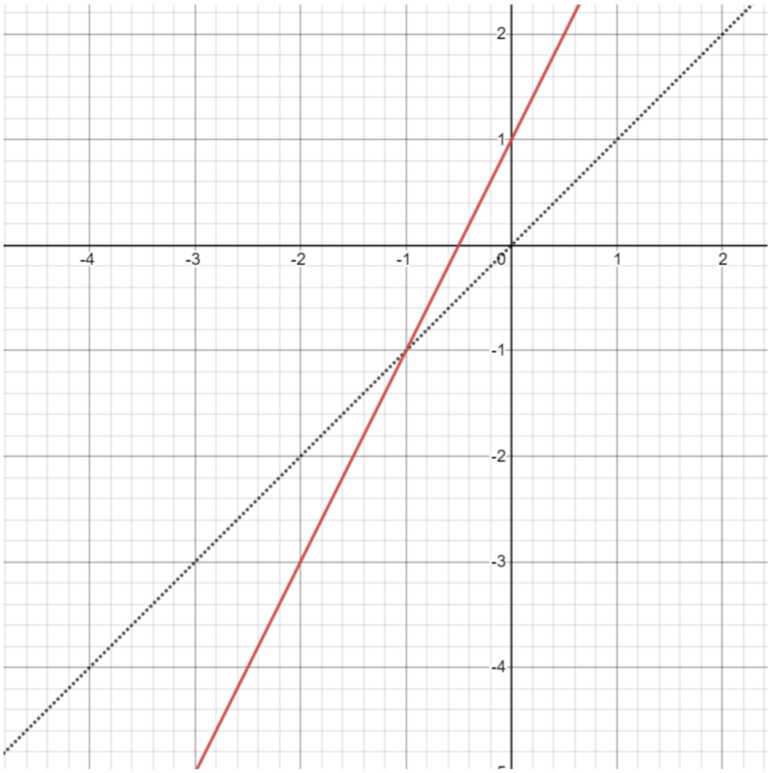
a.



b.



Example Draw the inverse of the following graph.



Lesson: A Review of Exponential Functions and their Graphs

What is the form of a general exponential equation?

<u>Example</u> Let $f(x) = 3^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Find the following.

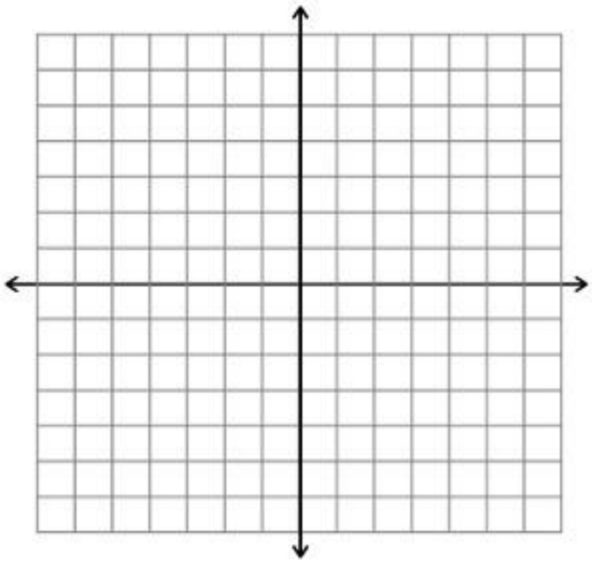
a. $f(-2)$

b. $f\left(\frac{3}{2}\right)$

c. $g(-3)$

d. $g(3.2)$

Sketch the graph of $f(x) = 2^x$ and create a table to go with it.



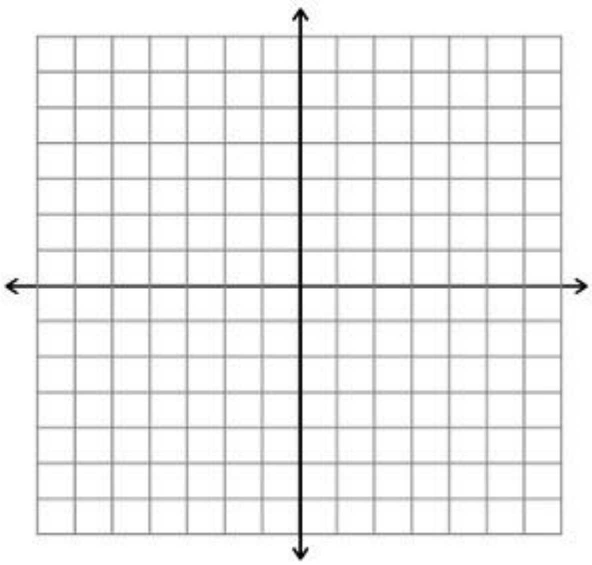
When will an exponential function $f(x) = a^x$ take this shape?

What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?

Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

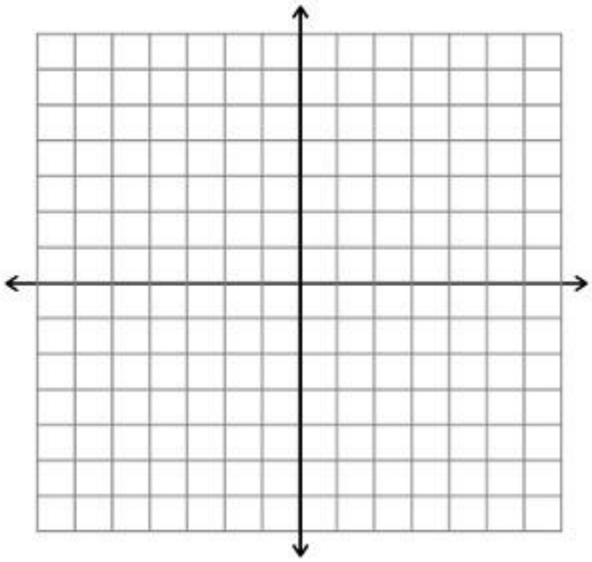
What is the x-axis for this graph?

What are 3 standard points to note here?

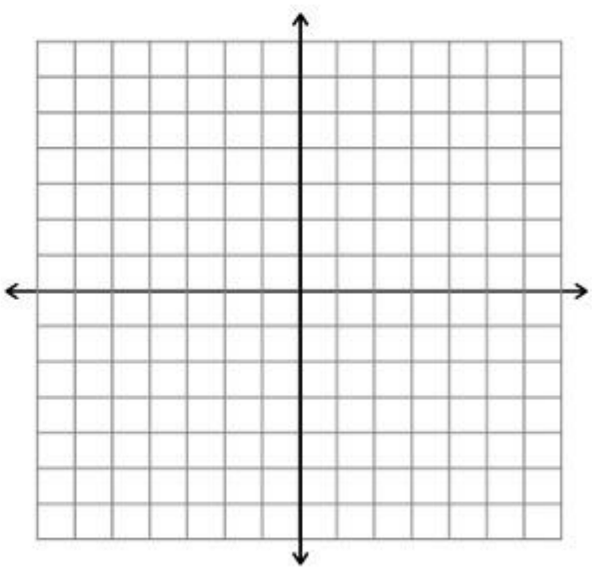
What is the domain and range?

Examples Describe the translations and sketch the following graphs.

a. $f(x) = -3^x + 1$



b. $f(x) = \left(\frac{1}{2}\right)^{x+1} - 2$



Lesson: Solving Basic Exponential Equations

Outline the general steps to solve $3^{x+1} = 81$.

Outline the general steps to solve $4^{x-2} = 8^{x+1}$.

Examples Solve the following.

a. $27^{2x} = 9^{x-1}$

b. $e^{3-x} = (e^2)^{-x}$

c. $\left(\sqrt{3}\right)^{x+3} = 9^x$

<u>Examples</u> Solve the following.

a. $x^{\frac{3}{2}} = 8$

b. $x^{\frac{5}{2}} = 4$

c. $x^{-4} = \frac{1}{16}$

Lesson: Compound Interest

What is the formula for compound interest and what does it mean?

<u>Example</u> Suppose $2000 is invested into an account paying 2.5% interest per year compounded monthly. How much is in the account after 10 years, and how much interest is earned in that time?

What else do we call the A from the compound interest formula?

What else do we call the P from the compound interest formula?

Example In 5 years, you want to have \$8000 in cash to buy a car (including taxes). Let's use savings to help us reach this goal.

a. You find an account with a 3.25% interest rate compounded biannually (2x a year). What amount would you have to deposit today to ensure you have \$8000 in 5 years?

b. Let's pretend that as of today, you have \$5000. What interested rate would you need to ensure that this will grow to \$8000 in 5 years?

What is the number e?

What is the formula for continuous compounding?

<u>**Example**</u> Suppose $2000 is invested into an account paying 2.5% interest per year compounded continuously. How much is in the account after 10 years, and how much interest is earned in that time?

Lesson: Review of the Basics of Logarithmic Functions

Explain how to find the inverse of $f(x) = 3^x$, and explain how this relates to logarithms.

Define a logarithm:

<u>Example</u> Write the following in logarithmic form.

a. $3^2 = 9$ b. $5^0 = 1$ c. $4^{-2} = \frac{1}{16}$

<u>Example</u> Write the following in exponential form.

a. $\log_3 1 = 0$ b. $\log_5 125 = 3$ c. $\log_3 81 = 4$

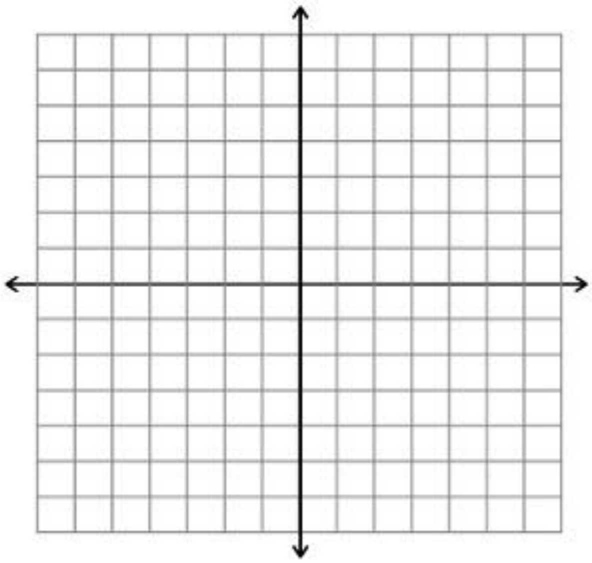
Example Show how to solve the following basic logarithmic equations.

a. $\log_x 8 = 3$

b. $\log_3 x = -2$

c. $\log_{49} 7 = x$

Create a table and show how to graph $f(x) = \log_2 x$



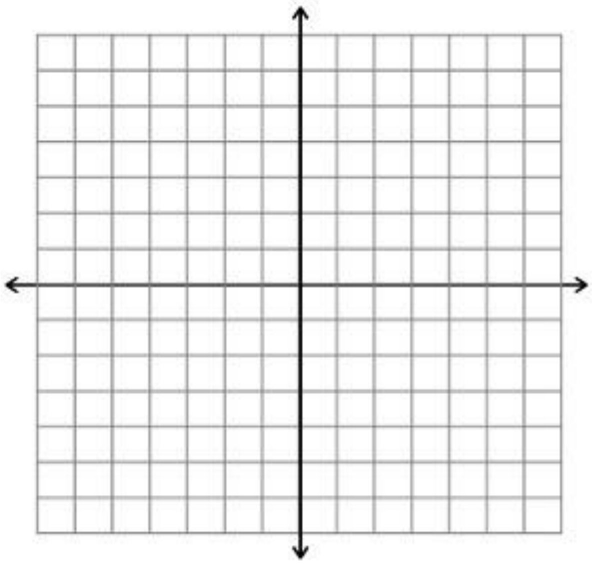
What 3 points can you easily find on the graph?

When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

Create a table and show how to graph $f(x) = \log_{\frac{1}{2}} x$



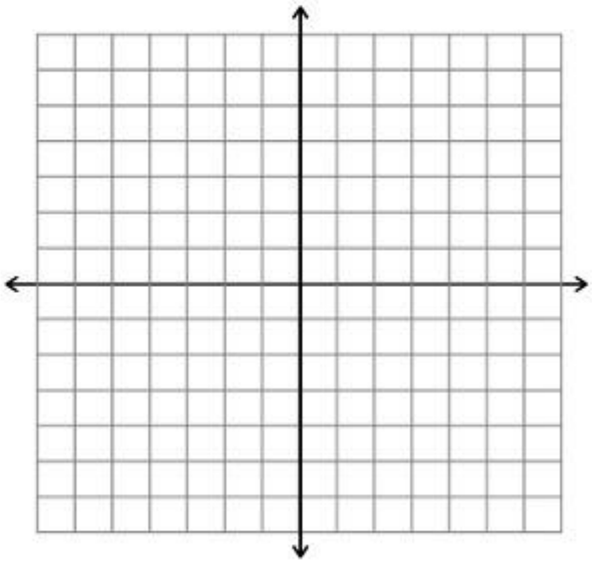
What 3 points can you easily find on the graph?

When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

Examples Graph the following: $g(x) = \log_2(x - 1) + 1$



Lesson: Review of the Basics of Properties of Logarithsm

List the major properties of logarithms.

Explain what the common logarithms mean:

$\log x =$

$\ln x =$

Examples Expand the following as much as possible.

a. $\log_3\left(\frac{3x^3y^7}{\sqrt{z}}\right)$

b. $\ln\left(\sqrt[3]{x^2y}\right)$

c. $\log_3 \sqrt{6}$

d. $\log_5 \frac{25x^3y^2}{p^7r^3}$

Example Write as a single logarithm.

a. $\log(x+2)+\log(x-2)-\log(x+3)$

b. $3\log_2 3+2\log_2 y-5\log_2 x$

c. $\frac{1}{2}\log_5 16+3\log_5 y^2-\log_5 x^7-\log_5 y$

d. $4\ln 2+\ln x^6-2\ln y^3-3\ln x^2$

Lesson: Change of Base Theorem

State the change of base theorem:

<u>Example</u> Show how to evaluate the following:

a. $\log_5 17$

b. $\log_2 3$

c. $\log_{\frac{1}{2}} 5$

Lesson: Solving Exponential Equations

Outline the steps to solve: $2^{x+3} = 7^{3x}$

Examples Solve the following.

a. $e^{x^2} = 100$

b. $e^{2x-3} * e^{3x} = 5e$

Examples Solve.

a. $3(2)^{x+2} + 6 = 105$

b. $e^{2x} + 8e^{x} + 15 = 0$

c. $3^{2x} + 2(3^{x}) = 24$

Lesson: Solving Logarithmic Equations

Outline the steps to solve: $\log(3 - x) = 0.75$

Compare these steps to: $\log(x + 20) = \log(x + 10) + \log(6)$. How are these problems different?

Examples Solve the following.

a. $\log_4(x^3 + 37) = 3$

b. $\log x + \log x^3 = 4$

Examples Solve the following.

a. $\log(x^2 - 4) - \log(x - 2) = \log(8)$

b. $\log(5 + 4x) - \log(3 + x) = \log 3$

Lesson: Solving Exponential and Logarithmic Formulas

<u>Examples</u> Solve for the desired variable.

a. for b: $p = a + \frac{c}{\ln b}$

b. for n: $A = \frac{Pr}{1-(1+r)^{-n}}$

c. for A: $\log A = \log B - C \log D$

d. for x: $D = 100 + 5 \log x$

Lesson: Models of Exponential Growth and Decay

Explain the equation for the general model:

Some examples of growth can come from populations or investments. Some examples of decay can come from half-life/radioactive decay, or carbon dating.

Define the following:

Doubling time:

Half-life:

What is the formula for continuous compounding?

Example A sample of 400g of radioactive lead decays to polonium according to the function: $A(t) = 400e^{-0.03t}$.

a. Find the amount of lead remaining after 5 years.

b. Find the half-life.

Example The amount of carbon-14 present in a paint after t years if given by $y = y_0 e^{-0.0002t}$. The paint contains 37% of its carbon-14. How old are the paintings?

Example Find the doubling time of an investment earning 5% interest if interest is compounded continuously.

Example During an epidemic, the number of people who have never had a particular virus and are not immune decreases exponential, where this decline is modeled by the function: $f(x) = 20{,}000e^{-0.025t}$, where t is time in days.

a. How many people were susceptible at the beginning of the epidemic?

b. How many people are susceptible after 2 weeks?

Lesson: A Quick Review of the Basics of Solving Systems of Linear Equations

Systems of equations are 2 or more equations in 2 or more variables. They can look like:

$\begin{cases} 2x + 3y = 6 \\ x - 3y = 3 \end{cases}$ or $\begin{cases} 2x - y + z = 8 \\ 5x + 2y - z = 9 \\ x - 4y + z = 10 \end{cases}$ or $\begin{cases} 3x + 2y - z = 7 \\ x - 4y + 2z = 8 \end{cases}$

A solution of a system is an ordered pair that is true in each equation of the system. Some systems have no solutions, and some systems have infinite solutions.

In this lesson, we will briefly review the main techniques to solve systems. We will focus on systems that have a solution.

Example Use the substitution method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

Example Use the elimination method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

Example Solve: $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 4 \\ \frac{3x}{2} + \frac{3y}{2} = 15 \end{cases}$

Example Solve: $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

Examples: Systems of 3 Linear Equations with 3 Variables

Solve: $\begin{cases} x+y+z=2 \\ 2x+y-z=5 \\ x-y+z=-2 \end{cases}$

Solve: $\begin{cases} x + 2y + 3z = -11 \\ 3x - y + z = 0 \\ -2x + 3y - z = 4 \end{cases}$

Lesson: Systems with No Solutions or Infinite Solutions

<u>Example</u> Solve: $\begin{cases} x + 4y - 3z = 2 \\ 2x - 5y + 2z = -8 \\ -3x - 12y + 9z = 7 \end{cases}$

<u>Example</u> Solve: $\begin{cases} 7x + 2y = 6 \\ 14x + 4y = 12 \end{cases}$

Example Solve: $\begin{cases} 5x - 4y + z = 9 \\ \quad y + z = 15 \end{cases}$

Example $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$

<u>Example</u> Solve: $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$

Lesson: Using Matrices to Solve Systems of Linear Equations

What is a matrix?

Show how to setup the associated **augmented matrix** for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

What are the dimensions of this matrix?

In general, how do we state dimensions of a matrix (circle): rows x columns columns x rows

<u>Example</u> What is the dimension of

a.

$$\begin{bmatrix} 3 & 2 & 4 & 1 \\ 7 & 8 & 1 & 2 \\ 9 & 5 & -2 & -3 \end{bmatrix}$$

b.

$$\begin{bmatrix} 2 & 4 \\ 5 & 3 \\ 8 & 3 \\ -1 & -4 \end{bmatrix}$$

With matrices, there are 3 **row transformations** that we can make. We are not allowed to do anything else to a matrix. Write down these 3 transformations.

Now, recall, our goal in this lesson is to use a matrix to solve systems of linear equations. What do we call this specific solving process?

What do we call the “ideal form” of a matrix in its solved form?

The Gauss-Jordan method has a goal of transforming a matrix into reduced-row echelon form or diagonal form. Draw a few examples of how the FINAL matrix should look after we transform it:

When we use the Gauss-Jordan method, which direction do we want to work on the matrix?

What is the general strategy to remember with this method?

Example Outline the steps for the Gauss-Jordan method for the system: $\begin{cases} -2x + 4y = -4 \\ x - y = 4 \end{cases}$

Example Use Gauss-Jordan method for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

Lesson: Using The Gauss-Jordan Method for a System with 3 Equations

Example Use the Gauss-Jordan method for the system: $\begin{cases} x+y+z=2 \\ 2x+y-z=5 \\ x-y+z=-2 \end{cases}$

Another page for you to show your work …

Lesson: Using Matrices to Solve Systems of Linear Equations when There is No Solution or Infinite Solutions

<u>**Example**</u> Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y - z = -2 \\ 4x - y + 2z = 1 \\ -6x - 10y + 2z = 0 \end{cases}$

<u>**Example**</u> Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y = -2 \\ 9x + 15y = -6 \end{cases}$

Example Use the Gauss-Jordan method to solve: $\begin{cases} x - 2y + 3z = 6 \\ 2x - y + 2z = 5 \end{cases}$

Example Use the Gauss-Jordan method to solve: $\begin{cases} 5x - 4y + z = 0 \\ x + y = 0 \\ -10x + 8y - 2z = 0 \end{cases}$

Examples: Using Matrices to Solve Systems of Linear Equations (any type of solutions)

Use the Gauss-Jordan method to solve the following.

a. $\begin{cases} x + 4y - z = 6 \\ 2x - y + z = 3 \\ 3x + 2y + 3z = 9 \end{cases}$

b. $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$

c. $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$

Lesson: How to Calculate the Determinate of a Matrix

Every n x n matrix has a real number associated with it called the **determinant.** What is the notation to represent this?

Example Show, in detail, how to calculate the determinants of the following matrices.

a. $A = \begin{bmatrix} 2 & 4 \\ -5 & 7 \end{bmatrix}$

b. $B = \begin{bmatrix} -5 & 1 \\ -3 & 4 \end{bmatrix}$

Example Show, with illustrations, how to calculate the determinant of $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & -3 \\ 3 & 4 & -2 \end{bmatrix}$

The 2x2 "sub" matrices that we calculate are called minors. The symbol M_{ij} represents the minor that results when row i and column j are eliminated. Using the matrix above, describe the following minors.

$M_{11} =$

$M_{32} =$

The number that we multiply the minor against is called the **cofactor.** Draw an illustration to show whether a cofactor will be positive or negative:

Example Using the matrix below, describe the minor associated with each cofactor AND determine whether the cofactor is positive or negative. You do not need to calculate the determinant.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ -3 & -5 & -1 \end{bmatrix}$$

a. 2

b. 3

c. -3

Example Show how to efficiently calculate the determinants of the following matrices. EXPLAIN YOUR REASONING FOR WHY YOU DID IT THE WAY THAT YOU DID.

a. $A = \begin{bmatrix} 2 & 7 & 8 \\ -1 & 0 & 1 \\ 3 & 5 & 1 \end{bmatrix}$

b. $B = \begin{bmatrix} 9 & -3 & 1 \\ 2 & 3 & 0 \\ 5 & 2 & 0 \end{bmatrix}$

Lesson: Determinant Theorems

Theorem 1: If every element (or column) of a matrix is 0, then the determinant will be 0.

<u>Example</u>: find the determinant of $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 2 & 8 & 9 \\ 3 & 2 & 8 & 4 \\ 9 & 3 & 1 & 7 \end{bmatrix}$

Theorem 2: If the rows of a matrix A are the corresponding columns of a matrix B, then the determinants are equal.

<u>Example:</u> Calculate both determinants: $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$

Theorem 3: If any 2 rows (or columns) of a matrix are interchanged, then the determinants are negatives of one another.

Example Find the determinants of: $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}$

Theorem 4: Suppose matrix B is formed by multiplying every element in a row or columns in a matrix A by a real number k. Then the determinant is $|B| = k|A|$.

Example: Find the determinants of: $A = \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 4 \\ 3 & 6 \end{bmatrix}$

Theorem 5: If two rows (or columns) of a matrix are identical, then the determinant will be 0.

<u>Example</u> Find the determinant of: $A = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

Theorem 6: Changing a row (or column) or a matrix by adding it to a constant times another row (or column) does not change the determinant of the matrix.

<u>Example</u> Find the determinant of $A = \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix}$. Then create a matrix B by taking adding $3 *$ (row 3) to row 1. Find the new determinant.

Theorem 7: If a matrix is in triangular form (all 0s either above or below the diagonal), then the determinant is found by multiplying the elements down the diagonal.

<u>Example</u>: Find the determinant of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 8 & 7 & 3 & 1 \end{bmatrix}$

<u>Examples</u> Use the determinant theorems to find the following determinants.

a. $\begin{vmatrix} -4 & 1 & 4 \\ 2 & 0 & 1 \\ 0 & 2 & 4 \end{vmatrix}$

b. $\begin{vmatrix} 1 & 0 & 8 \\ 4 & 0 & 2 \\ 0 & 0 & 8 \end{vmatrix}$

Lesson: Nonlinear Systems of Equations

Nonlinear systems of equations involve at least one equation that is not linear. Some examples look like this:

$$\begin{cases} x^2 - y = 8 \\ x + y = -4 \end{cases} \quad \text{or} \quad \begin{cases} x^2 + y^2 = 4 \\ |x| + y = 2 \end{cases}$$

Similar to systems of linear equations, we can still use substitution or elimination to solve the system. However, the best method to use tends to depend on the type of system you are trying to solve. Additionally, you might have to use both methods.

What is one potential issue that can happen with solving systems of nonlinear equations and what should you do about this?

When one equation is linear and the other is not, it is usually best to use the substitution method.

<u>Example</u> Solve the system: $\begin{cases} x^2 + y = 8 \\ x - y = -2 \end{cases}$

When both variables in both equations are squared, it's usually best to use elimination.

<u>Example</u> Solve the system: $\begin{cases} x^2 + y^2 = 25 \\ x^2 + 2y^2 = 9 \end{cases}$

Sometimes you will have to use both the substitution and elimination methods.

<u>Example</u> Solve the system: $\begin{cases} x^2 - xy + y^2 = 5 \\ 2x^2 + xy - y^2 = 10 \end{cases}$

Now let's look at a few other examples to get a sense of how to use these methods!

Example Solve the system: $\begin{cases} x^2 + y^2 = 9 \\ -|x| + y = 0 \end{cases}$

Example Solve the system: $\begin{cases} 3x^2 + y^2 = 3 \\ 4x^2 + 5y^2 = 26 \end{cases}$

Lesson: Properties of Matrices: Equality, Addition, Subtraction, and Scalar Multiplication

When are 2 matrices equal?

<u>Example</u> What values of x, y, z, and w will make the matrices equal?

$$\begin{bmatrix} 2+x & 3-w \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 2-y & 2+z \end{bmatrix}$$

When can you add 2 matrices?

What is the definition of matrix subtraction?

What is scalar multiplication with a matrix?

List the properties of scalar multiplication for matrices:

Example Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. $A + C$

b. $B - D$

c. $B - E$

d. $2B + E$

e. $2A - 3C$

Lesson: Matrix Multiplication

First, let's review dimension. What is the dimension of the following matrices?

$A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$

To understand how to multiply matrices, let's use the following 2 examples:

a. $\begin{bmatrix} 2 & 7 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 7 & 2 \end{bmatrix}$

Example Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. AC

b. CA

Using the above example, what can we say about matrix multiplication and commutativity?

List the properties of matrix multiplication:

Example Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. BC

b. ED

c. CB

Lesson: Inverse Matrices

What is the 2x2 identity matrix?

What is the 3x3 identity matrix?

How do we describe the general nxn identity matrix?

Example Multiply $A = \begin{bmatrix} 3 & 2 & 1 \\ 8 & 1 & 7 \\ 3 & 1 & 0 \end{bmatrix}$ by the identity matrix as follows:

a. AI

b. IA

What can we generalize about the identity matrix?

How do we describe a multiplicative inverse of a real number?

Some matrices have inverses. For a matrix A, what notation denotes its inverse?

What is the property of inverse matrices?

What are the steps for finding an inverse matrix?

Example Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 6 & -2 \\ 2 & 4 \end{bmatrix}$

Example Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

Lesson: Using Inverse Matrices to Solve Systems

We've previously talked about using matrices to solve systems. We will look at another way to do that in this lesson. Show another way to set up the system using matrices (as well as the relevant notation):

$$\begin{cases} x + y + z = 6 \\ 2x + 3y - z = 7 \\ 3x - y - z = 6 \end{cases}$$

Now, using our setup above, explain the reasoning for how inverse can help solve a system:

Example Use inverse matrices to solve: $\begin{cases} -x + y = 1 \\ 2x - y = 1 \end{cases}$

Example Solve the system: $\begin{cases} x + y + z = 6 \\ 2x + 3y - z = 7 \\ 3x - y - z = 6 \end{cases}$ using inverse matrices.

Lesson: An Intro to Conic Sections

What are the 4 conic sections we will discuss? Draw an illustration with it.

Lesson: Parabolas as Conic Sections

What is the previous equation we used for parabolas opening upwards or downwards? Draw a illustration to go with this.

How can we represent parabolas in similar notation for parabolas that open sideways?

Examples Give the domain and range of the following parabolas, then graph the parabola.

a. $x + 1 = (y + 1)^2$

b. $-x = 3y^2 + 6y + 2$

When dealing with conic sections, we shift how we think about things.

What is the geometric definition of a parabola? Draw an illustration to accompany this definition.

What equations do we use that align with the geometric definition?

Examples Give the focus, directrix, vertex, and axis of symmetry for each parabola. Then use this information to graph the parabola.

a. $x^2 = 4y$

b. $(y-2)^2 = 24(x-3)$

Lesson: Writing Equations of Parabolas

Example Write the equation of a parabola with the following characteristics.

a. focus $\left(0, -\frac{1}{2}\right)$ with the vertex at the origin

b. vertex (4, 3) and focus is (4, 5)

c. vertex (1, 2), directrix x=4

Lesson: Ellipses as Conic Sections

Give the geometric definition of an ellipse. Draw an illustration to accompany this.

What are the standard forms of an ellipse? Describe the parts and draw VERY CLEAR illustrations.

What is important to note with this equation?

Example Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

a. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

b. $9x^2 + y^2 = 81$

Example Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

c. $\frac{(x-1)^2}{9}+\frac{(y+2)^2}{4}=1$

Lesson: Writing Equations of Ellipses

<u>Example</u> Write the equation of an ellipse with the following characteristics.

a. major axis 6, focus at (0,2) and (0, -2)

b. center at (-2, 7), major axis vertical with length 10, and c=2

Lesson: Eccentricity with Ellipses

To discuss the concept of eccentricity, we need to discuss what the formal mathematical definition of conic sections is.

Define a conic:

The intuitive idea behind eccentricity can change slightly with each conic section, but it is generally a ratio of different measures within the conic section.

What does the eccentricity of an ellipse tell us?

How do we represent eccentricity?

How do you calculate the eccentricity of an ellipse?

How does the eccentricity of a parabola compare to the eccentricity of an ellipse? Summarize the explanation from the video.

<u>Examples</u> Find the eccentricity of each ellipse.

a. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

b. $x^2 + 25y^2 = 25$

<u>Example</u> Find the equation of an ellipse with $e = \frac{3}{4}$ with foci at $(-2,0), (2,0)$

Examples: Using Complete the Square to Write an Ellipse Equation in Standard Form

Rewrite the following equations in standard form.

a. $x^2 + 12y^2 - 14x - 48y = -85$

b. $9x^2 + 36x + 16y^2 - 160y + 292 = 0$

Lesson: Hyperbolas as Conic Sections

What is the geometric definition of a hyperbola? Draw an illustration with your definition.

State the equations for the standard form of the hyperbola. Draw illustrations with it.

Examples Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

b. $9x^2 - 25y^2 = 225$

Examples Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

c. $\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$

Lesson: Writing Equations of Hyperbolas

Examples Write the equation of the hyperbola with the following characteristics.

a. vertices at (0, 6) and (0, -6) with asymptotes $y = \pm\frac{1}{2}x$

b. foci at $\left(-3\sqrt{5}, 0\right), (3\sqrt{5}, 0)$ with asymptotes $y = \pm 2x$

Examples Write the equation of the hyperbola with the following characteristics.

c. center at (1, -2) and focus at (-2, -2) and vertex at (-1, -2)

Lesson: Eccentricity of Hyperbolas

How do you calculate the eccentricity of a hyperbola?

What can we say about the eccentricity of the hyperbola? What does it mean?

<u>Example</u> Find the eccentricity of the hyperbolas.

a. $\frac{x^2}{36} - \frac{y^2}{9} = 1$

b. $8y^2 - 16x^2 = 16$

Example Find the equation of a hyperbola with $e = \frac{25}{9}$ and foci at (9, -1) and (-11, -1)

Examples: Writing the Standard Form of a Hyperbola

Write each hyperbola in standard form.

a. $x^2 - 6x - y^2 + 4y = 1$

b. $y^2 - 14y - x^2 + 14x = 16$

Lesson: Summarizing the Equations and Graphs of Conic Sections

Re-summarize how the equations and graphs of parabolas work.

Re-summarize how the equations and graphs of circle and ellipses work.

Re-summarize how the equations and graphs of hyperbolas work.

Lesson: The General Form of a Conic Section

What is the general form of a conic section?

How do you interpret this equation to decide what type of conic section you have?

Example Identify the type of conic section and graph it (you will need to put each equation in standard form).

a. $x^2 - 6x + y = 0$

b. $2x^2 - 8x + 2y^2 + 20y = 12$

Example Identify the type of conic section and graph it (you will need to put each equation in standard form).

c. $y^2 + 6y - 4x^2 + 8x = -6$

d. $9x^2 + 9y^2 = 81$

Lesson: Eccentricity and Conic Sections

How can we define conic sections in terms of eccentricity? Draw an illustration to explain this.

Summarize the value of the e's based on the conic section

Conic Section	Eccentricity e
Parabola	
Circle	
Ellipse	
Hyperbola	

Examples Find the eccentricity of each conic section.

a. $\frac{x^2}{9} - \frac{y^2}{4} = 1$

b. $x^2 + 2x + y = 0$

c. $\frac{(x+3)^2}{25} + \frac{(y-1)^2}{9} = 1$

Lesson: Sequences

Define a sequence.

<u>Examples</u> Write the first five terms of each sequence.

a. $a_n = \frac{1}{n}$

b. $a_n = \frac{n \mp 1}{n+3}$

c. $a_n = (-1)^{n+1} * (2n + 1)$

What does it mean for a sequence to converge?

What does it mean for a sequence to diverge?

What is a recursively defined sequence?

<u>Examples</u> Find the first four terms in the sequence.

a. $a_1 = 3; a_n = 2a_{n-1} - 4$

b. $a_1 = 2; a_n = a_{n-1}^2 + n - 2$

Lesson: Series

What is the summation sign and what does it mean? What do we called the "i" in this formula?

Define a finite series.

Define an infinite series.

Examples Evaluate the following sums.

a. $\Sigma_{i=1}^{4}(3^i + 1)$

b. $\Sigma_{j=3}^{6} a_j$

c. $\Sigma_{k=1}^{4}(4x_k - 1)$ if $x_1 = 2, x_2 = 3, x_3 = 5, x_4 = 8$

d. $\Sigma_{i=1}^{3} f(x_i)\Delta x$, if $f(x) = 3x, x_1 = 1, x_2 = 4, x_3 = 7, \Delta x = 3$

What are the summation properties?

What are the summation rules for summing up terms?

Examples Use the summation properties and rules to evaluate each series.

a. $\Sigma_{i=1}^{50} 3$

b. $\Sigma_{k=1}^{25} 3k$

c. $\Sigma_{j=1}^{33} (4j^2 + 1)$

Lesson: Arithmetic Sequences

What is an arithmetic sequence?

Give some examples of an arithmetic sequence.

<u>Example</u> Determine the common difference, d, for the arithmetic sequence:

$$-7, -5, -3, -1, \ldots$$

Example Find the first five terms of each arithmetic sequence.

a. The first term is 3, and the common difference is -2.

b. $a_1 = -5, d = 7$

How do we define the general nth term of an arithmetic sequence?

Example Determine a_{11} and a_n for the arithmetic sequence $-2, 2, 6, 10, \ldots$

Example Determine a_n and a_{20} for the arithmetic sequence having $a_3 = 9$, and $a_4 = 16$.

Example An arithmetic sequence has $a_7 = -15$ and $a_{14} = -35$. Determine a_1.

Lesson: Arithmetic Series

When we sum up the terms of an arithmetic sequence, how can we define that sum? (sum of an arithmetic sequence)

<u>Example</u> Consider the arithmetic sequence $-7, -3, 1, 5, \ldots$

a. Evaluate S_{10}

b. Evaluate the sum of the first 50 positive integers.

Example The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find a_1 and d.

Examples Evaluate each sum.

a. $\Sigma_{i=1}^{19}(-3 - 5i)$

b. $\Sigma_{k=4}^{15}(2 + 4k)$

Lesson: Geometric Sequences and Series

Define a geometric sequence:

<u>Example</u> Determine a_6 and a_n for the geometric sequence 4, 12, 36, 108, ….

<u>Example</u> Determine r and a_1 for the geometric sequence with $a_2 = -18$ and $a_5 = 486$

How do we define the sum of the first n terms of a geometric sequence?

Example Evaluate: $\Sigma_{i=1}^{8} 3 * 2^{i}$

If we start with 2 and use a common ratio of $\frac{1}{2}$, what eventually happens as the series continues infinitely? Summarize the explanation below.

Summarize the Sum of the Terms of an Infinite Geometric Sequence

Examples Evaluate the following infinite sums.

a. The first term is 1 and the common ratio is $\frac{1}{3}$

b. $\Sigma_{i=1}^{\infty}\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{i-1}$

c. $\Sigma_{i=1}^{\infty}\left(\frac{3}{7}\right)^{i}$

Lesson: The Binomial Theorem

Below we can observe the pattern for binomial expansion:

$(x+y)^0 = 1$

$(x+y)^1 = x + y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Focusing on the VARIABLES x and y only, how can we generalize the exponents for the variables?

Pascal's Triangle can explain how to find the coefficient. Draw out Pascal's triangle below.

Having to draw out this pattern would be very difficult for larger expansions, so we will explore a few ways to simplify how to expand this pattern.

First, we need to understand factorials.

n-factorial: for any positive integer n,

$$n! = n(n-1)(n-2)\ldots(3)(2)(1)$$

By definition: $0! = 1$

<u>Examples</u> Evaluate:

a. $5!$

b. $8!$

Now that we understand factorials, we can discuss the formula to find binomial coefficients.

For nonnegative integers n and r, $r \leq n$, the binomial coefficient is defined as follows:

<u>Examples</u> Evaluate each binomial coefficient:

a. $\binom{5}{2}$

b. $\binom{6}{0}$

We can summarize how to expand $(x + y)^n$ as follows:

1. There are n+1 terms in the expansion

2. The first and last terms are x^n and y^n respectively

3. In each succeeding term, the exponent on x decreases by 1 and the exponent on y increase by 1

4. The sum of the exponents on x and y in any term is n.

5. The coefficient of the term with $x^r y^{n-r}$ or $x^{n-r} y^r$ is $\binom{n}{r}$

These observations are captured in the binomial theorem.

State **The Binomial Theorem:**

<u>Example</u> Find the first, last, and seventh term of the binomial expansion: $(3x + 2y)^{10}$

Example Use the binomial theorem to expand: $(a-b)^5$