Lesson: A Review of Angles and Degrees

Draw, label, and define how to make an angle:

Show how to create an angle of positive versus negative measure:

Degree is generally how we measure angles.

Draw the complete rotation of a ray. How many degrees is this?

Define:

- Acute angle

- Right angle

- Obtuse angle

- Straight angle

Define:

- Complementary angles

- Supplementary angles

Example Find the complement and supplement of a $55°$ angle.

Example Find the measures of the marked angles shown in the video.

Examples: Converting Between Minutes and Seconds

<u>Examples</u> Convert the following to degrees.

a. $23°15'32''$

b. $38°27'15''$

<u>Examples</u> Convert the following to degrees, minutes, and seconds.

a. $32.713°$

b. $72.17°$

Examples: Adding and Subtracting Degrees, Minutes, and Seconds

Add or subtract.

a. $45°15' + 23°57'$

b. $23°18'29'' + 14°19'47''$

c. $94°16' - 27°29'$

d. $72°18'15'' - 15°20'30''$

Examples: Applications of Minutes and Seconds

<u>Example</u> A turntable makes 45 revolutions in a minute. How many revolutions is that per second?

<u>Example</u> A windmill makes 80 revolutions per minute. How many revolutions is that per second?

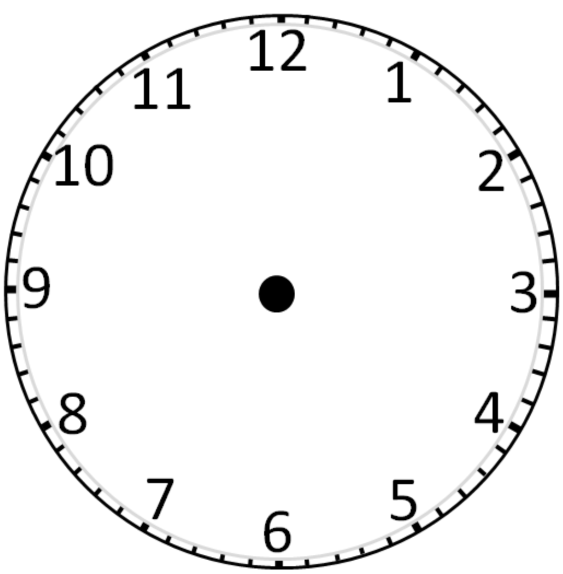
<u>Example</u> Wind power is caused by a turbine spinning 10 to 20 revolutions per minute. How many revolutions does it make per second?

Lesson: Angle Measure with a Clock

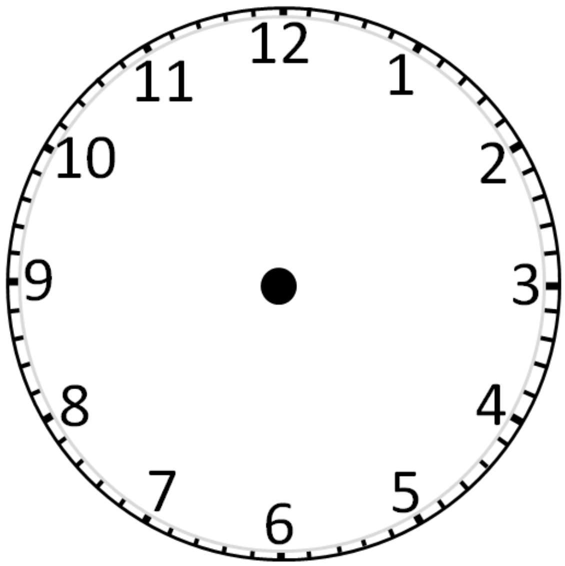
Explain the basics of angle measure, ie where is $90°, 180°$?

Each number represents how many degrees? SHOW how to actually determine this?

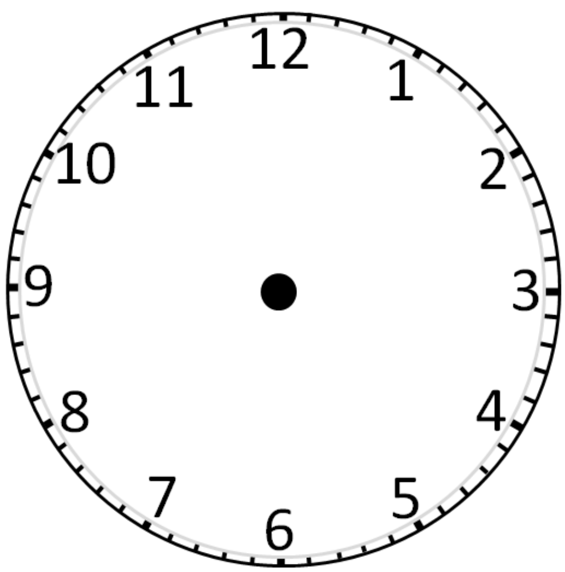
Example Find in the clock below, and show how to find the angle measure for 5:00. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



Example Find in the clock below, and show how to find the angle measure for 8:20. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



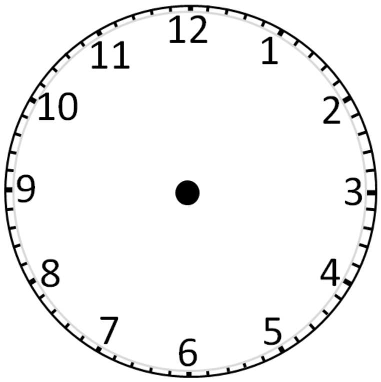
Example Find in the clock below, and show how to find the angle measure for 10:05. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



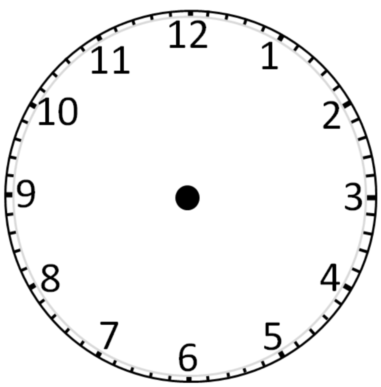
Examples: Angle Measure from a Clock

Find the angles associated with each time.

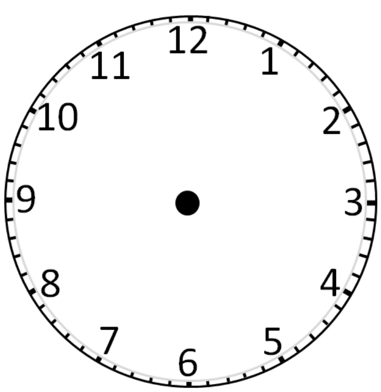
a. 6:25



b. 3:15



c. 2:38



Lesson: Standard Position and Coterminal Angles

When is an angle in standard position? Draw an example.

Show some examples of how we can determine angle measure by quadrant.

What are quadrantal angles? Draw a picture to illustrate.

What makes two angles coterminal? Draw a picture to explain.

<u>Example</u> Find the angle of least positive measure that is coterminal with each angle:

a. $715°$

b. $-80°$

<u>Example</u> How can you find all angles coterminal with $45°$? Prove it by filling in the following table.

Value of n	Angle coterminal with $45°$
2	
0	
-1	

Examples Fill in the table below for which angles are coterminal with the quadrantal angles

Quadrantal Angle	Coterminal Angles
$0°$	
$90°$	
$180°$	
$270°$	

Lesson: Angle Relationships

Draw and explain what vertical angles are:

Draw and explain what makes a transversal line. Draw a big picture, you will use it for the next part of your notes:

Label the diagram and note the relationships between the different angles below:

Example Draw and label the lines from the video. Find the measures of angles 1, 2, 3, and 4.

Example Draw and label the lines from the video. Find the measures of angles 1, 2, 3, and 4.

Lesson: Review of Triangles and Similar Triangles

The sum of angle measures of any triangle is:

Draw the different types of triangles:

<u>**Examples**</u> Draw the triangles in the example, then classify the triangles as acute, right, or obtuse. Also classify as equilateral, isosceles, or scalene.

What makes 2 triangles similar?

What makes 2 triangles congruent?

What is the key relationship between similar and congruent triangles?

<u>What are the 2 conditions for similar triangles?</u>

In other words, triangle ABC is similar to triangle DEF if:

Example Draw the similar triangles in the video, and then find any missing angle measures.

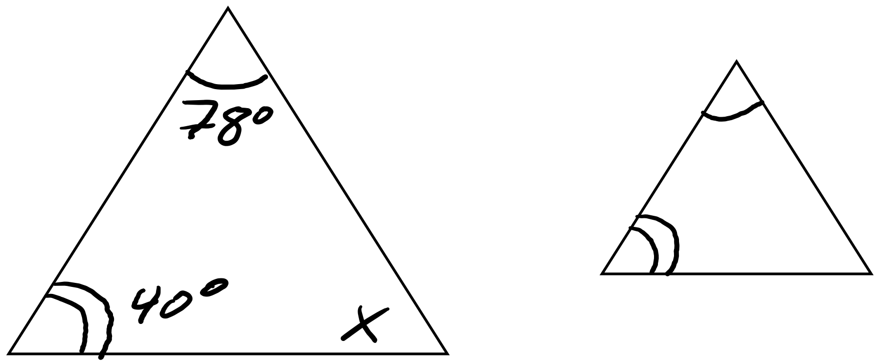
Example Draw the similar triangles in the video, and then find any missing side lengths.

Example Draw the similar triangles in the video, and then find any missing measures.

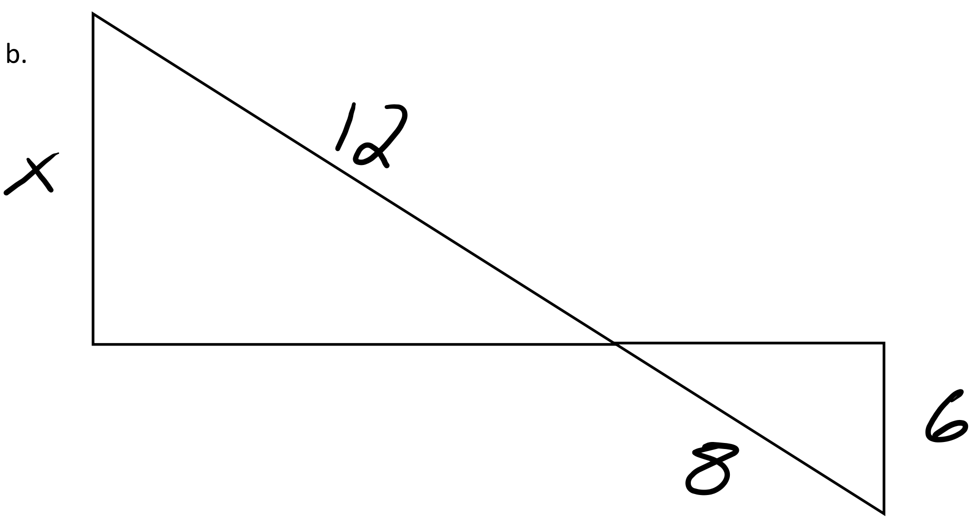
Examples: Similar Triangles

For each example, assume the triangles are similar. Triangles are not drawn to scale. Find any missing angle or side measures.

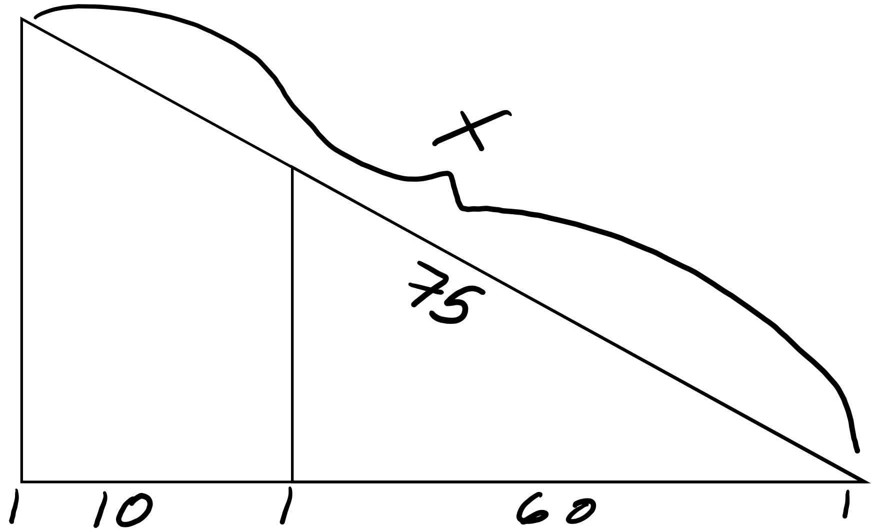
a.



b.



c.



Lesson: The Pythagorean Theorem, The Distance Formula, and Intro to Trig Functions

State the Pythagorean theorem:

State the distance formula:

How does the Pythagorean theorem relate to the distance formula?

Now we will introduce **trigonometric functions!** Explain what makes trig functions unique:

How do trig functions relate to the distance formula? Draw the picture associated with this.

Explain the 6 trig functions and how they relate to what we discussed above.

Sometimes you will see expressions such as $\sin^2 90°$. What does this mean?

Example The terminal side of an angle θ in standard position passed through the point (-12, -5). Find the values of the 6 trig functions of angle θ.

Example Find the 6 trig function values of an angle θ in standard position if the terminal side of θ is defined by

$3x + 5y = 0, x \geq 0$

Example Suppose the point (x, y) is in the indicated quadrant. Determine whether given ratio is positive or negative.

$$III, \frac{y}{r}$$

Examples: Finding Values of Trig Functions

For each given point, sketch the angle of θ in standard position such that θ has the least positive measure and the given point is on the terminal side. The find the values of the 6 trig functions.

a. $(-3, -4)$

b. $(7, 4)$

c. $(-\sqrt{2}, \sqrt{2})$

Examples: Finding the 6 Trig Functions Given an Equation of the Terminal Side

An equation of the terminal side of an angle θ (in standard position) is given with a restriction on x. Sketch the situation with least positive angle θ, and find the values of the 6 trig functions.

a. $-6x - y = 0, x \geq 0.$

b. $x + y = 0, x \geq 0$

c. $x = 0, y \geq 0.$

Lesson: Quadrantal Angles and Trig Functions

What were the quadrantal angles?

If one of the quadrantal angles is involved, how does this impact how we think about the trig functions? Draw a sketch to help illustrate this point. Explain how these are different from non-quadrantal angles, and how we choose points in these cases.

Example Find the indicated value of: $\sin 90°$.

How should we think about something like $\sin^2 90°$?

Example Use trigonometric function values of quadrantal angles to evaluate: $\tan 0° - 6\sin 90°$.

Example Use trigonometric function values of quadrantal angles to evaluate: $\sin^2 180° + \cos^2 180°$.

Examples: Quadrantal Angles and Trig Functions

Find the function values:

a. $\cot 90°$

b. $\sec 180°$

c. $\sin(-270°)$

d. $3\sec 180° - 2\tan 360°$

e. $\sin^2 90° - 3\cos^2 90°$

Lesson: Reciprocal Identities of Trig Functions

First, what is a reciprocal? Give an example.

What are the 6 reciprocal identities?

1.	4.
2.	5.
3.	6.

You should NOT write out something like ________________ or _________________as a shortcut for a reciprocal identities. Why not?

How can you shortcut the reciprocal identity then with notation?

What are some other ways we can relate the reciprocal identities to one another?

Examples Find the function values.

a. $\tan\theta$ given that $\cot\theta = 3$

b. $\sec\theta$ given that $\cos\theta = -\frac{5}{\sqrt{50}}$

c. $\sin\theta$ given that $\csc\theta = 1.25$

Lesson: Signs and Ranges of Trig Functions

Review: what does r represent when we are talking about trig functions?

Since r is a distance, we know ________________________.

What determines if a trig function has a positive or negative value?

Draw a picture to help breakdown and analyze the signs of theta:

Now fill in this table to help you remember the sign of each function:

θ IN QUADRANT	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
I						
II						
III						
IV						

One way to remember the signs is with the phrase All Students Take Calculus. Break down this this is helpful.

Examples Determine the signs of the trig functions of an angle in standard position with the given measure.

a. $23°$

b. $255°$

c. $-30°$

Examples Determine which quadrant of an angle θ that satisfies the given conditions.

a. $\sin\theta > 0, \csc\theta > 0$

b. $\sec\theta > 0, \csc\theta > 0$

Now we will show how to get ranges of the six trig functions.

Draw a picture to demonstrate how angle θ increase in measure from $0° - 90°$.

As the measure of an angle ___________________, _____ ________________ but never exceeds ______. Therefore:

Let's repeat this idea but in quadrant IV. Follow along with the video below and write down the observations again.

What can we conclude from these sketches and observations?

What can we conclude about the values of $\tan\theta$ and $\cot\theta$? And why?

What can we conclude about the values of $\sec\theta$ and $\csc\theta$? Why?

We have now discovered the ranges of all the trig functions. Let's summarize this in the table.

TRIG FUNCTION	RANGE (WRITE IN BOTH SET-BUILDER NOTATION AND INTERVAL NOTATION)
$\boldsymbol{\sin\theta, \cos\theta}$	
$\boldsymbol{\tan\theta, \cot\theta}$	
$\boldsymbol{\sec\theta, \csc\theta}$	

<u>Example</u> Determine if each statement is possible or impossible.

a. $\cot\theta = -0.98$

b. $\cos\theta = -1.25$

c. $\sec\theta = 0.72$

Lesson: Pythagorean and Quotient Identities

Show how we can build the first Pythagorean identity. These are actually pretty darn important, so you really do need to understand this argument!!

Now let's build another one:

And now let's build the last one:

Summarize the Pythagorean identities:

What is only key about these identities?

Now let's show how to build the quotient identities:

By the way, similar reasoning holds for $\cot\theta$.

Summarize the Quotient Identities below:

Example Find the value of $\cos\theta$ and $\tan\theta$, give that $\sin\theta = -\frac{\sqrt{2}}{2}$ and $\cos\theta > 0$.

Example Find $\sin\theta$, given that $\cos\theta = \frac{4}{5}$ and θ is in Quadrant IV.

Example Give all 6 trig function values given that $\cot\theta = \frac{\sqrt{3}}{8}$ and θ is in quadrant I.

Examples: Using Pythagorean and Quotient Identities to Find Trig Function Values

Find all 6 trig function values given the information.

a. $\cos\theta = -\frac{3}{5}$, θ is in quadrant III.

b. $\csc\theta = 2, \theta$ is in quadrant II.

c. $\cos\theta = 1$

Lesson: Right-Triangle Based Definitions of Trigonometric Functions and Cofunctions

Instead of using standard position to describe trig functions, we can alternatively use right triangles.

Draw a right triangle to orient yourself.

What does A represent?

Show how to define the 6 trig functions with this right triangle.

1.	4.
2.	5.
3.	6.

What mnemonic device can help you remember the trig functions?

<u>Example</u> Draw the triangle in the video, and then show how to find the values of sine, cosine, and tangent.

<u>Example</u> Using the same example above, show how to find the values of cosecant, secant, and cotangent.

Draw a new right triangle, and show an alternative angle measure we can find from this picture:

These relationships are called the **cofunction identities,** summarize these below:

Examples Write each function in terms of its confunction

a. $\sin 10°$

b. $\cot 65°$

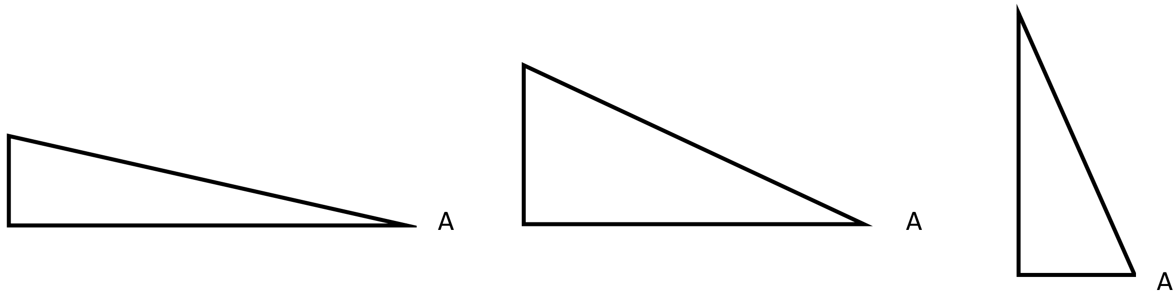
c. $\csc 34°$

Example Find one solution for each equation. Assume all angles involved are acute angles.

a. $\cos\theta = \sin(2\theta - 30°)$

b. $\sec(\theta + 15°) = \csc(2\theta + 25°)$

Lesson: More on Trig Function Values as Angles Change and Special Angles



Explain how angle A changes in these pictures:

<u>Examples</u> Determine whether each statement is true or false.

a. $\tan 25° < \tan 22°$

b. $\cos 25° < \sin 25°$

There are a few angles that occur frequently in trig and have some *nice* properties that we like to call them out. Draw out a triangle where each angle is $60°$ and describe any other properties:

If you bisect this triangle, what happens? Draw a picture to illustrate, and then describe the trig functions associated with it:

Finally, draw an isosceles right triangle and describe any special characteristics with it:

We have now examined some *extremely* common trig measures that will come up all the time in trig. They are so important that we will create a separate reference sheet for them.

Read this! I highly recommend that you remove the last page from this packet, and instead put it at the very front of your notes so that you can easily reference it. If you are moving on to calculus, KEEP THIS PAGE IN GOOD CONDITION FOR THE NEXT FEW YEARS. You will thank yourself later that you have this reference as these angles will come up over and over again, and students frequently forget them. I will occasionally create these handouts and make these suggestions to you as we go along in trig – you will thank yourself (and me) later if you keep these. It is extremely common for students to forget some of KEY parts of trig and then struggle in calculus – I don't want this to happen to you!

Some Function Values of Special (Common) Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$0°$						
$30°$						
$45°$						
$60°$						
$90°$						

Lesson: Reference Angles and Non-Acute Angles

Review: what is a quadrantal angle?

What is associated with every non-quadrantal angle?

Show how we can find reference angles by drawing the reference graphs in the video:

Example Find the reference angle for each angle.

a. $274°$

b. $822°$

Draw how to find the reference angle based on the quadrant (follow along with the video and draw the diagram on your own):

Now let's talk about using $30°, 45°$, and $60°$ as reference angles.

<u>**Example**</u> Find the values of the 6 trig functions for $240°$. Draw a reference!!! (This class is all about pictures!!)

Don't forget the helpful device: All Students Take Calculus 😊

Let's outline the steps for finding trig function values for any non-quadrantal angle:

1.

2.

3.

4.

How can you avoid making an error with finding a trig function value?

<u>Examples</u> Find the exact values:

a. $\sin(-150°)$

b. $\cos(-510°)$

Example Evaluate: $\sin^2 120° + \cos^2 120°$

Example Find all values of θ, if θ is in the interval $[0°, 360°)$ and has the given function value: $\cos\theta = \frac{\sqrt{3}}{2}$

Example Is the following true or false? $\cos 60° = 2\cos 30°$

Examples: Reference Angles and Finding Trig Value Functions

Find the exact values of all 6 trig functions of each angle. Rationalize when applicable.

a. $315°$

b. $-855°$

Evaluate

a. $\cot^2 120° + \tan^2 60° - \cos^2 330°$

b. $\cot^2 135° - \tan^4 45° - \sin^2 150°$

Examples: Finding the Value of Theta

Find all values of θ if θ is in the interval $[0°, 360°)$ and has the given value.

a. $\sec\theta = -\sqrt{2}$

b. $\cot\theta = -\frac{\sqrt{3}}{3}$

c. $\cos\theta = \frac{1}{2}$

Lesson: How to Approximate Trigonometric Function Values

We will discuss using a calculator to do approximations of trig functions.

If you are going to use a calculator, what is key?

<u>Examples</u> Use a calculator to evaluate the following expressions.

a. $\cos 32°15'$

b. $\sec 55.321°$

c. $\sin(-105°)$

d. $\frac{1}{\csc(233.45°)}$

To find the value of θ (certain angle measures), we can use inverse trig functions. What are the three inverse trig functions that a calculator will typically have?

What do these functions means?

Example Show how to use completely proper notation to find the angle θ in the interval $[0°, 90°)$.

a. $\cos\theta = 0.931145$

b. $\cot\theta = 1.7538932$

Examples: Solving for Theta with a Calculator

Find all the values of θ in $[0, 360°)$ where $\tan\theta = 0.9845331$

Lesson: Significant Digits and Solving Triangles

What is an exact number?

We've seen that many values for trig functions are not exact. Consider this: a room is 12ft x 18ft. We are doing a project where we need to know the length of the diagonal. What is it?

What's the issue with reporting this number?

What do we call the number of digits obtained by actual measurement?

Show some examples of significant digits:

Significant Digits for Angles

Angle Measure to Nearest	Number of Significant Digits Used	Write an Example

How do you perform calculations in this case?

What should you keep in mind about the answer in this case?

We can practice significant digits by solving triangles.

Example Solve the right triangle presented in the video.

Example Solve the right triangle presented in the video.

Examples: Significant Digits and Rounding

How many significant digits are there in each?

a. 421	e. 400.01
b. 420	f. 400.00
c. 400	g. $72°31'$
d. 400.	h. $72°30'$

Your answer needs to be 2 significant digits. You calculate the answer 421. What should your final answer be for significant digits?

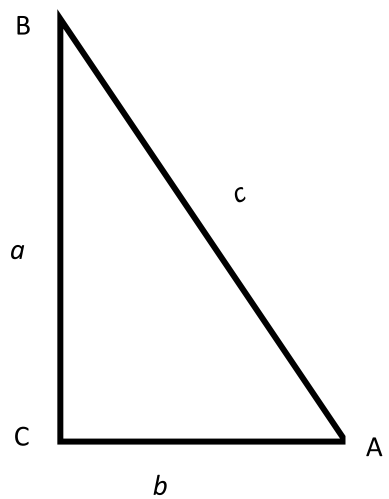
Your answer needs to be 4 significant digits. You calculate the answer 421. What should your final answer be for significant digits?

Your answer needs to be 3 significant digits. You calculate the answer $42°18'$. What should your final answer be for significant digits?

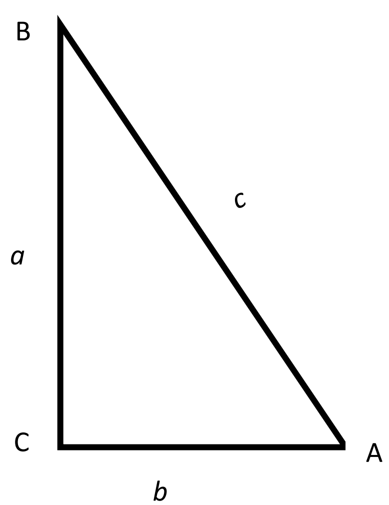
Examples: Solving Triangles

Find all missing lengths and angle measures for the given triangles. Use significant figures.

a. $A = 28.3°, c = 18.5$



b. $b = 85m, c = 112m$



Lesson: Angles of Elevation or Depression

What is an angle of elevation and what is its context?

What is an angle of depression?

What should you be careful when interpreting either?

Example Outline the general steps to solving the application problem: the angle of elevation from a point on the ground 122ft from the base of a flagpole is $25°20'$. Find the height of the flagpole.

Step 1 (write out the general approach and then show how to do it for each step)

Step 2:

Step 3:

Example A 14.00m fire truck ladder is leaning against a building. What is the distance that the ladder goes up the wall if the ladder makes an angle of $42°25'$ with the horizontal?

Example A company safety committee installs a floodlight to illuminate the exit of a building, with the dimensions shown in the video. Draw the situation, and then find the angle of depression to the nearest minute.

Examples: Angles of Elevation or Depression

A pyramid has a square base with side 600 ft long and height 300 ft. Find the angle of elevation made from the sides of a corner to the center of the pyramid. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.

Lesson: Applications of Right Triangles with Bearing

What is bearing?

What are some examples of bearing?

Explain method 1 to express bearing:

What is key with these types of problems?

Example Two radar stations are on an east-west line, 9.2 km apart from one another. The first station detects a plane on a bearing of $63°$. The second station detects the same plane on a bearing of $333°$. Find the distance from the second station to the plane.

Explain method 2 to express bearing:

Show some examples:

<u>Example</u> A ship leaves port and sails on a bearing of N 35° E for 3 hours. Then it turns and sails on a bearing of S 55° E for 2.5 hours. If the ships rate is 22 knots (nautical miles per hour), find the distance that the ship is from the port.

Examples: Applications of Right Triangles with Bearing

Two lighthouses are located on a north-south line. From lighthouse A, the bearing of a ship 3520 meters away is 139°43^′. From lighthouse B, the bearing of a ship is 49°43^′. Find the distance between the lighthouses.

The bearing from A to C is S 52°E. The bearing from A to B is N 84° E. The bearing from B to C is S 38°W. A plane flying at 375 mph takes 1.5 hours to go from A to B. Find the distance from B to C.

Lesson: Applications of Right Triangles with Distance and Height

What is the subtense bar method?

<u>Example</u> Draw a situation between points P and Q, then solve the following:

a. Find d when $\theta = 2°3'15''$ and $b = 3.5000$cm.

b. How much change would there be in the value of d if θ measured $1''$ larger?

We will now examine a few other problems involving distances and heights.

Example Copy the illustration shown in the video of the angle of elevation from a point on the ground to the top of a pyramid. The angle of elevation is $32°15'$. The angle of elevation from a point 120ft farther back is $20°25'$. Find the height of the pyramid.

Example In a certain area, the lowest angle of elevation of the sun in winter is $22°15'$. Find the minimum distance x that a plant needing full sun can be placed from a fence 4.75 ft high.

Example: Modeling Distance with Right Triangles

A highway curve connects 2 straight sections road and may be circular. Use the diagram in the video to illustrate the situation.

a. Find the distance between A and C if r=925ft and $\theta = 34°$.

b. Find an expression in terms of r and θ for the distance from P to Q.

Lesson: Radian Measure

Radian measure is another way to describe angle measure, as opposed to degrees.

What does radian measure allow us to do?

What is the definition of a radian?

Are radians a unit of measure like feet or meters? Circle: Yes No

What is the formula for the distance around a circle?

What does this formula show?

Write a way to relate radians to degrees:

So what is π radians?

Show a conversion rate between radians and degrees:

<u>Examples</u> Convert each degree to radians:

a. $35°$

b. $-175°$

c. $27.8°$

Convert each radian measure to degrees:

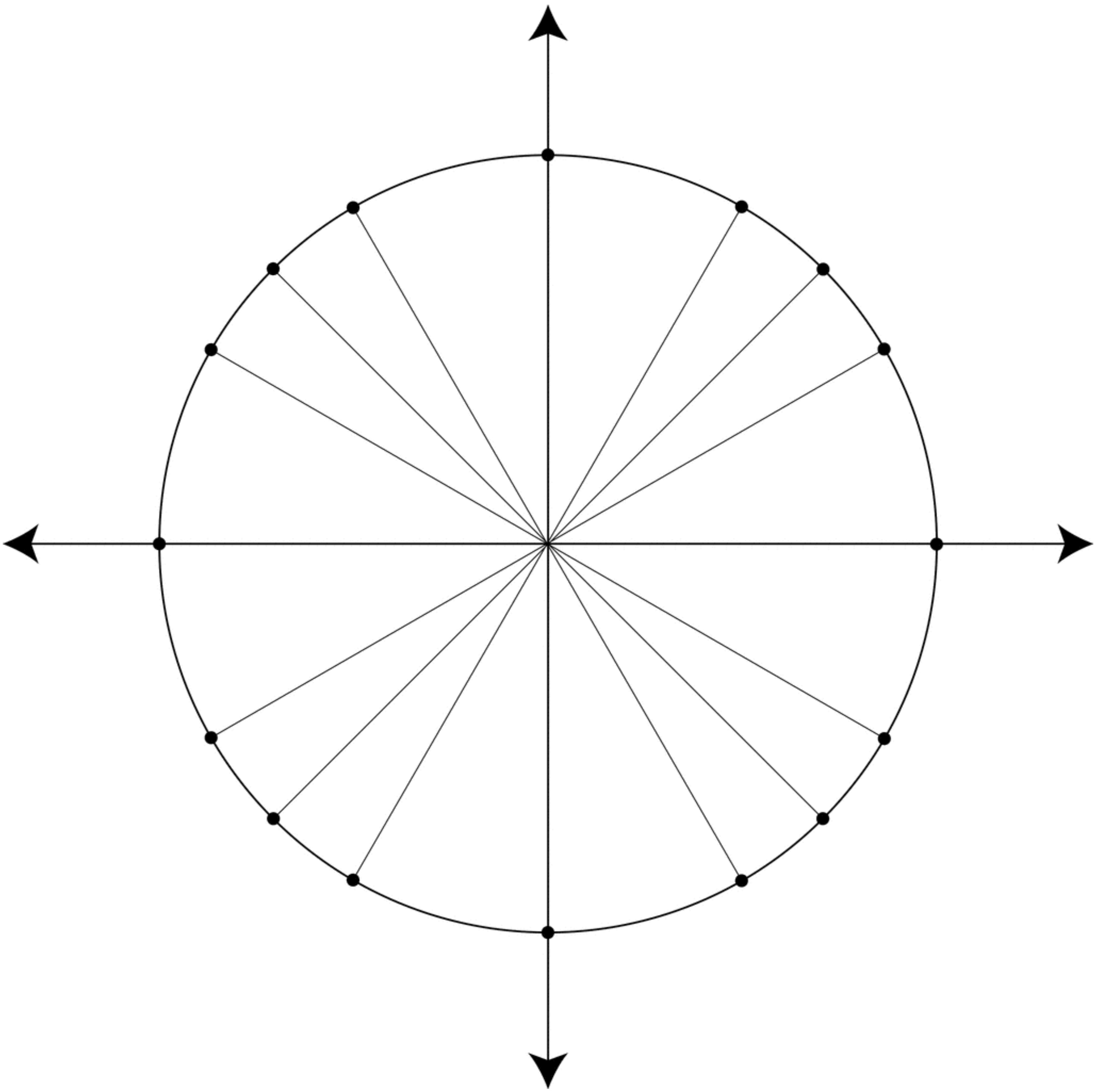
a. $\frac{7}{4}\pi$

b. $-\frac{2\pi}{3}$

c. 7

If no unit of angle measure is specified, what should you assume?

Below we will construct an illustration of all the common degree measures we have discussed, and then write their radian equivalent. Take your time to copy this perfectly!



You will need to know all of those common degrees AND radian measures for the exam!

You may want to reference cheat sheets to help you with the next few problems.

Examples Find the value of each function.

a. $\sin\frac{5\pi}{6}$

b. $\tan\frac{3\pi}{4}$

c. $\csc\frac{5\pi}{3}$

d. $\cos\left(-\frac{2\pi}{3}\right)$

Lesson: Arc Length on a Circle and Some Applications

We can determine the formula for the arc length of a circle from the definition of radians.

Regarding radians, what does θ equal? Show how this relates to arc length.

What is key about the arc length formula?

Examples A circle has radius 22.30 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{5\pi}{6}$ radians

b. $66°$

Examples A circle has radius 15.22 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{3\pi}{8}$

b. $125°$

What is the definition of latitude?

Example The latitude of Panama City, Panama is $9°$ N and the latitude of Pittsburgh, PA is $40°$ N. Find the north-south distance of these 2 cities. The radius of the Earth is 6400 km.

Example A rope is being wound around a drum with a radius of 0.7525 ft. How much rope will be wound around the drum if the drum is rotated through an angle of $42.25°$?

Example Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of $150°$, through how many degrees will the larger gear rotate? Refer to the photo in the video.

Examples: Applications using Arc Length

Charleston, SC and Toronto, Canada, are 1100 km apart and lie on the same north-south line. If the latitude of Charleston is $33°$ N, what is the latitude of Toronto?

Find the radius of the larger wheel in the figure if the small wheel rotates $75.0°$ when the large wheel rotates $45°$.

How many inches will a small weight rise if a pulley is rotated through an angle of $65°15'$? Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 4 inches?

Lesson: Area of a Sector of a Circle and Some Applications

What do we define as a sector of a circle?

What is the formula for the area of a section of a circle with radius r?

Examples Find the area of a sector of a circle having the given radius and central angle θ. Answers should be to the nearest tenth.

a. $r = 11.5$ ft, $\theta = \frac{5\pi}{6}$ radians

b. $r = 13.5$ m, $\theta = 125°$

Example Find the measure (in radians) of a central angle of a sector of a circle of area $25\ in^2$ in a circle of radius 2.5 in.

Example A center-pivot irrigation system provides water to a sector-shaped field as shown in the figure in the video. Find the area of the field if $\theta = 35.0°$ and $r = 155$ yd.

Example A circular sector has an area of $60\ in^2$. The radius of the circle is 6 inches. What is the arc length of the sector?

Lesson: The Unit Circle and Circular Functions

Explain how we create the unit circle:

What is the equation for the unit circle?

Recall the formula for arc length is $s = r\theta$. When the radius of the circle is 1, what does this formula become?

Why does this matter / what is cool about this?

Since r=1 on the unit circle, what can we say about our previously studied trig functions? Just summarize what we discussed in the video as best as you can.

Write out the **circular functions**

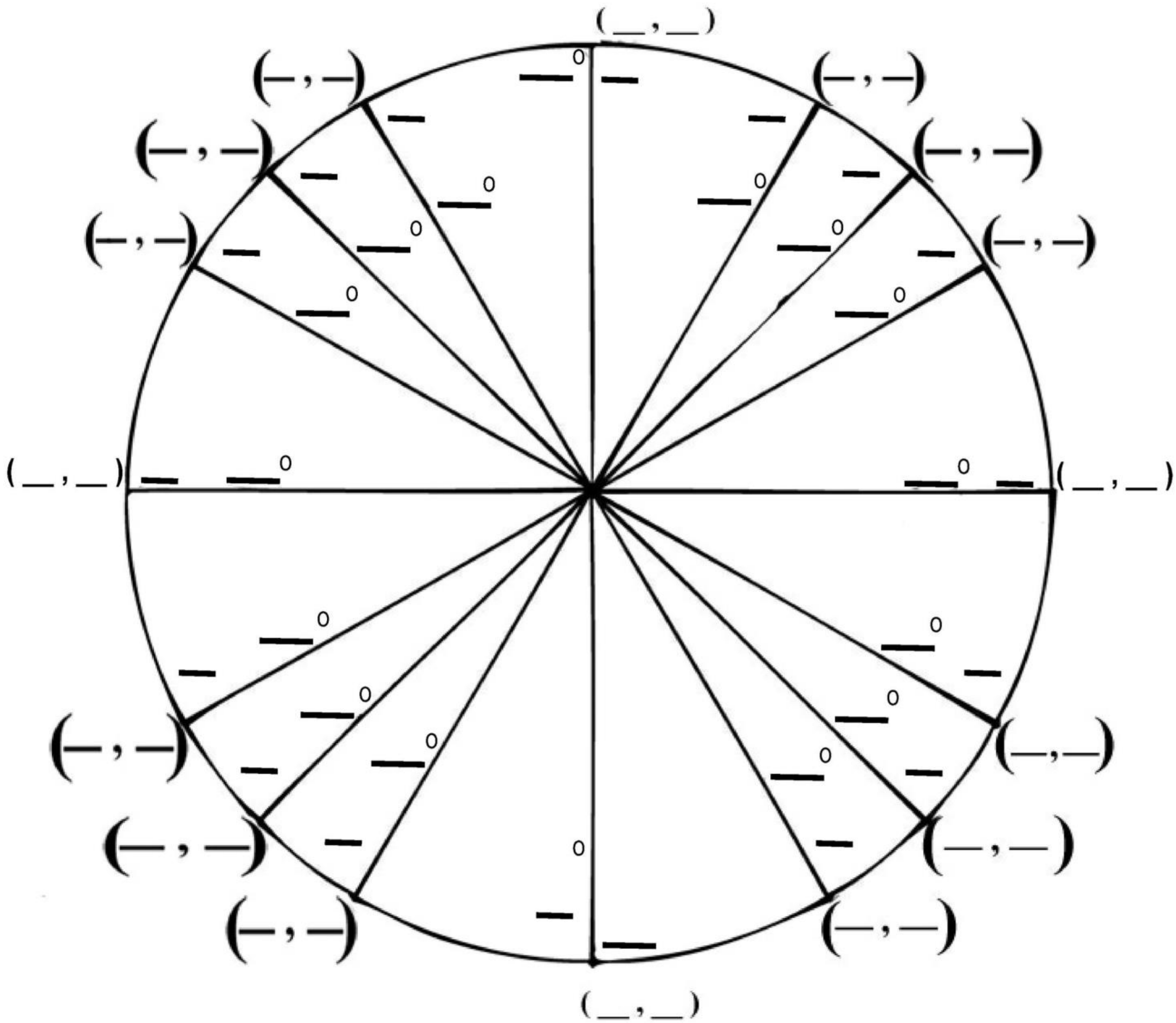
1.	4.
2.	5.
3.	6.

Explain the symmetry of the unit circle:

If a point (a, b) in on the unit circle, what other points are guaranteed to be on the unit circle?

How can we use symmetry with drawing the unit circle? How is it handy?

Label the unit circle. Note: you will have to have this memorized! These are the standard angle measures that you'd be expected to do without a calculator!



Now, going back to the circular functions we discussed, how do you view the x-coordinates?

How do you view the y-coordinates?

Examples Use the unit circle to find the values of the following:

a. $\sin\frac{\pi}{3}$

b. $\cos\frac{5\pi}{4}$

c. $\sin\frac{11\pi}{6}$

Lesson: Arc Length on a Circle and Some Applications

Using the unit circle, we can also determine the domains of our circular functions.

What is the domain of sine and cosine, and why?

What is the domain of tangent and secant, and why?

What is the domain of cotangent and cosecant, and why?

Lesson: Values of Circular Functions

We have previously discussed the unit circle (you will want to have it handy). One the most important things about the unit circle is that it will quickly tell you the value of the sine and cosine functions.

On the unit circle, what does the y-coordinate represent?

On the unit circle, what does the x-coordinate represent?

When we think about the values of these functions, what do we assume?

What does this mean for your calculator?

<u>Examples</u> Use the unit circle to find the values of:

a. $\sin\frac{\pi}{2}$

b. $\cos\frac{\pi}{2}$

c. $\tan\frac{\pi}{2}$

Examples Use the unit circle to find the values of:

a. $\sin\frac{5\pi}{4}$ b. $\cos\frac{5\pi}{4}$ c. $\tan\frac{5\pi}{4}$

Example Compare methods. Use reference angles and degrees to find the exact value of $\sin\frac{2\pi}{3}$, then confirm your answer with the unit circle.

For approximate values, we still must use a calculator. Remember, when you are not specifically told you are using degrees, then you assume you are using radians.

What must you ensure with your calculator?

Examples Use a calculator to approximate the function values.

a. $\cos(1.72)$

b. $\sin(-3.54)$

c. $\cot(1.438)$

Lesson: Values of Inverse Circular Functions

What do the functions $\sin^{-1} x$, $\cos^{-1} x$, $\cot^{-1} x$, etc. represent?

Your calculator makes a few key assumptions if it uses these functions (the reasons why are explained later):

1. For all x in $[-1, 1]$, using $\sin^{-1} x$ with a calculator will return a value between:

2. For all x in $[-1, 1]$, using $\cos^{-1} x$ with a calculator will return a value between:

3. For all real numbers x, using $\tan^{-1} x$ with a calculator will return a value between:

Sometimes you will be asked to find answers in different quadrants than what your calculator will return!

Example Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\sin s = 0.3311$

Example Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\tan s = 0.2155$

Example Solve for s, given that s is in the interval $\left[\pi, \frac{3\pi}{2}\right]$: $\tan s = \sqrt{3}$

Example Solve for s, given that s is in the interval $[0, 2\pi)$: $\cos^2 s = \frac{1}{2}$

Lesson: The Unit Circle and Circular Functions as Line Segments

The unit circle is really just an extension of how we have previously thought about trig functions as line segments / right triangles.

Draw the illustration in the video, then show how these sides relate to our trig functions.

From your illustration, how do we define the following?

$\cos\theta =$ $\tan\theta =$ $\csc\theta =$

$\sin\theta =$ $\cot\theta =$ $\sec\theta =$

<u>Example</u> Using your illustration from the previous page, suppose the measurement of TVU is $45°$. Find the lengths of the following line segments:

OQ

OU

OV

VR

US

PQ

Lesson: Linear Speed and Angular Speed and Applications

What is the formula for linear speed?

When are some situations where you might use linear speed?

What is the formula for angular speed?

When are some situations where you might use angular speed?

Examples Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

a. $\omega = \frac{\pi}{3}$ radians per second, $t = 6$ seconds

b. $\theta = 3.721$ radians, $t = 18.34$ seconds

Examples Use the formula $v = r\omega$ to find the value of the missing variable.

a. $r = 10\ m$, $\omega = \frac{9\pi}{5}$ radians per second

b. $v = 10$ m per second, $\omega = \frac{3\pi}{2}$ radians per second

Example Suppose a point P is on a circle with a radius of 5 cm, and the ray OP Is rotating with an angular speed of $\frac{\pi}{10}$ radian per second.

a. Find the angle generated by P in 3 seconds.

b. Find the distance traveled by P along the circle in 3 seconds.

c. Find the linear speed of P in cm per second.

The formula $\omega = \frac{\theta}{t}$ can be rewritten as $\theta = \omega t$. We can then use the formula $s = r\theta$ to create a different formula. Show the new formula we can create:

<u>Examples</u> Use the formula above to find the value of the missing variable.

a. $r = 5\ cm, \omega = \frac{2\pi}{3}$ radians per second, $t = 8$ seconds

b. $s = \frac{3\pi}{2}$ km, $r = 4$ km, $t = 3$ seconds

Example The orbit of the earth around the sun is almost circular. Suppose that the orbit has a radius of 93 million miles.

a. We know that a year is 365 days. Find the angle formed by the Earth's movement in one day.

b. Give the angular speed in radians per hour.

C. Find the linear speed of the Earth in miles per hour.

Examples: Applications of Linear Speed and Angular Speed

Example Find the angular speed of the minute hand of a clock.

Example Find the linear speed of the tip of the second hand of a clock, if the hand is 25 mm long.

Example A bicycle has tires with a 12-inch radius. The tires are turning at the rate of 200 revolutions per minute. How fast if the bike traveling in miles per hour?

Example A pulley has a radius of 13.23 cm. It took 15 seconds for 45 cm of belt to go around the pulley.

a. Find the linear speed of the belt in cm per sec.

b. Find the angular speed of the pulley in radians per second.

Example A thread is being pulled off a spool at a rate of 55.4 cm per second. Find the radius of the spool if it makes 155 revolutions per minute.

Lesson: Sine and Cosine as Periodic Functions

What is a periodic function?

What are some examples?

What are 2 known periodic functions?

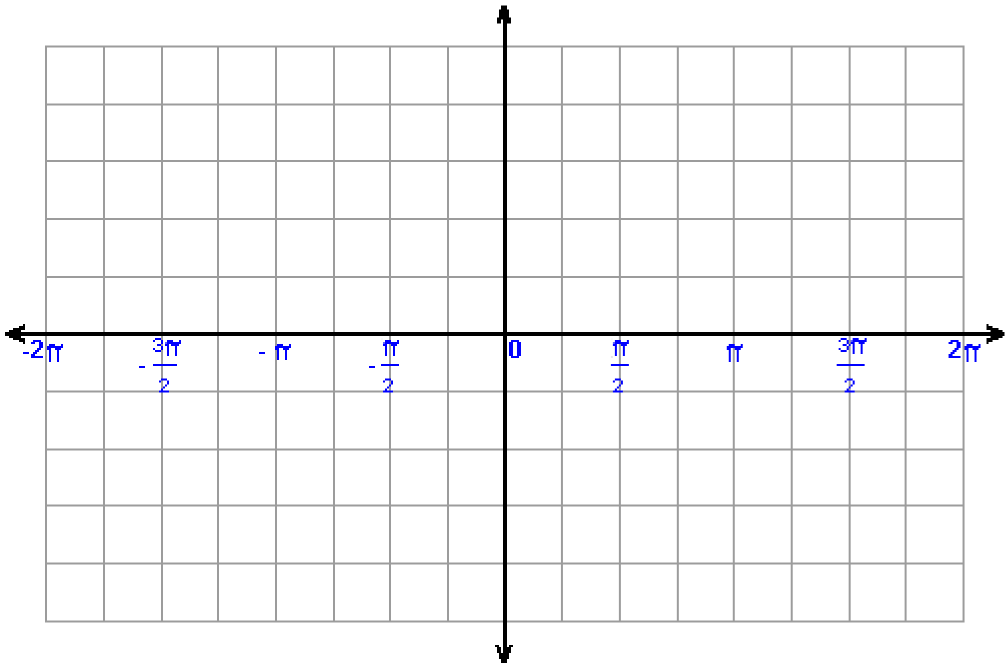
What can we say about the behavior of these functions as we move along the unit circle?

AS s CHANGES FROM	THE VALUE OF $\sin s$	THE VALUE OF $\cos s$
0 TO $\frac{\pi}{2}$		
$\frac{\pi}{2}$ TO π		
π TO $\frac{3\pi}{2}$		
$\frac{3\pi}{2}$ TO 2π		

But let's change s to x going forward so we don't get confused.

Draw the graph of sine, filling in the table and then using the graph paper.

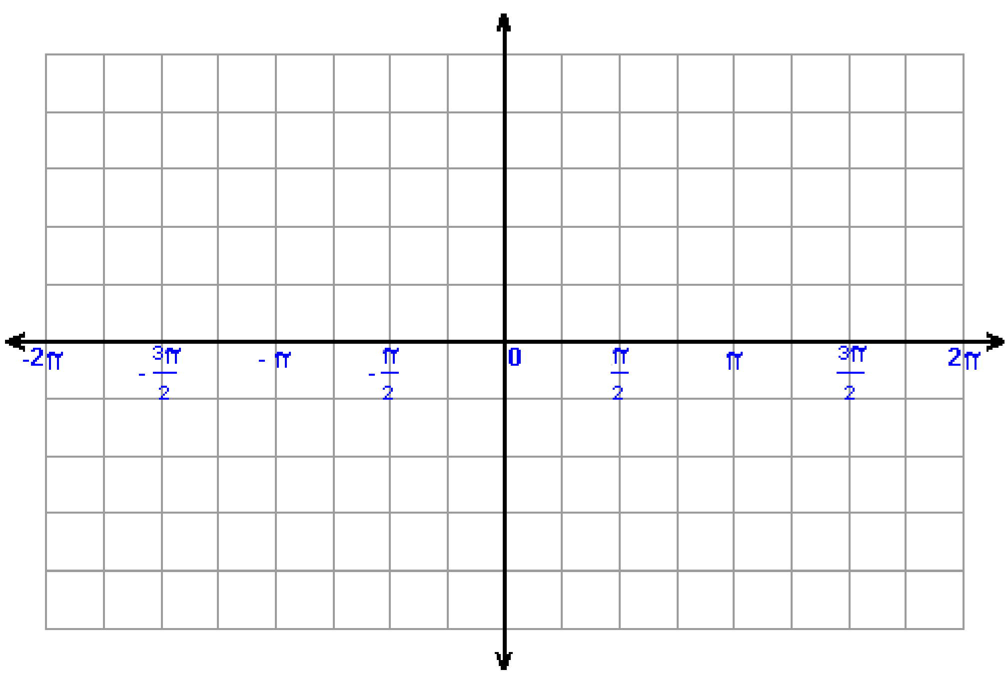
x	$y = \sin x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Math-Aids.Com

Draw the graph of cosine, filling in the table and then using the graph paper.

x	$y = \sin x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Math-Aids.Com

What term do we use to describe the graph of sine?

How do we describe the graph of cosine?

What is the period of both the sine and cosine functions?

Where are the graphs continuous?

What is the domain and range of the graphs?

How can we describe the x-intercepts of sine?

How can we describe the x-intercepts of cosine?

What is the test for even or odd functions?

What does it mean to be an even function?

What does it mean to be an odd function?

How do you know that $y = \sin x$ is odd? How would you show it?

How do you know that $y = \cos x$ is even? How would you show it?

Lesson: Using Amplitude and Period to Graph Sine and Cosine

What is amplitude?

Using the equation $y = a \sin x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

Using the equation $y = a \cos x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

What happens when a is negative? Demonstrate this with some brief sketches so that you can remember.

What is period?

Using the equation $y = \sin bx$, watch the video and write an explanation with sketches to demonstrate period.

Using the equation $y = \cos bx$, watch the video and write an explanation with sketches to demonstrate period.

What formula can you use to determine the period of a function?

Examples Determine the periods of the following functions:

a. $y = \sin 3x$

b. $y = \cos \frac{1}{2} x$

c. $y = \sin 2\pi x$

Summarize amplitude and period when the function is of the form $y = a\sin bx$ and $y = a\cos bx$

Examples Graph the following over 2 period intervals, completing the table with strategic points to help you graph. Give the period and amplitude in each. You will have to sketch your own graph and create your own axes.

$y = \cos\frac{1}{3}x$

Period: **Amplitude:**

x	$y = \cos\frac{1}{3}x$

$y = 2\sin x$

Period: **Amplitude:**

x	$y = 2\ sin\ x$

$y = 3\cos 2\pi x$

Period: **Amplitude:**

x	y

Examples: Using Amplitude and Period to Graph Sine and Cosine

Graph the following over two-period intervals.

a. $y = -\frac{1}{2}\sin x$

b. $y = 3\cos 2x$

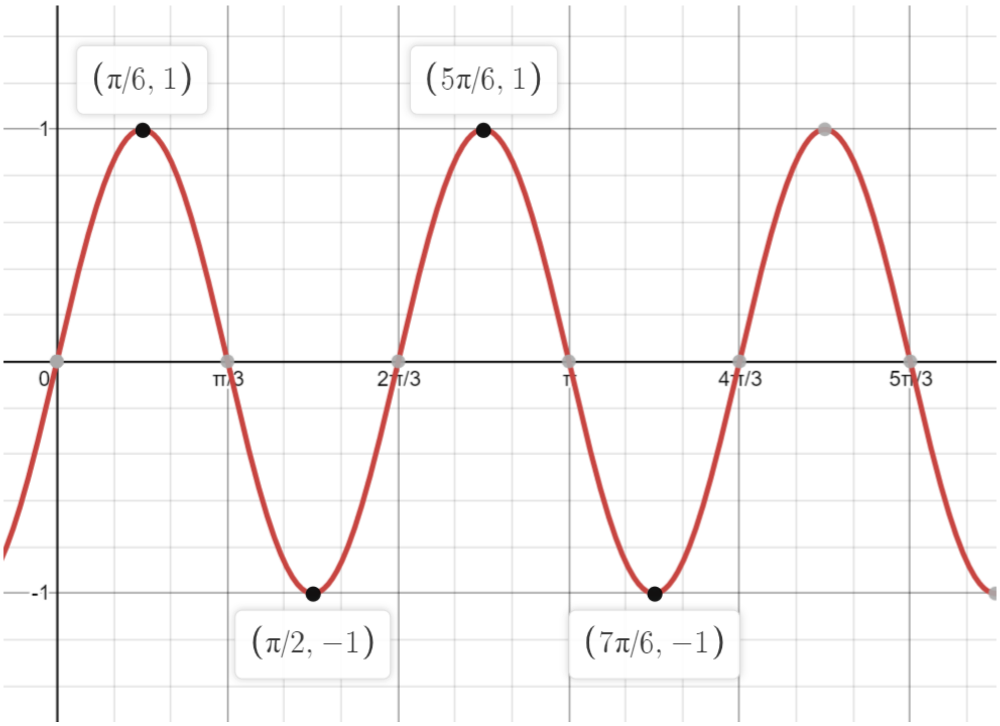
d. $y = -\sin \pi x$

Lesson: Determining the Equations of Sine and Cosine from Graphs

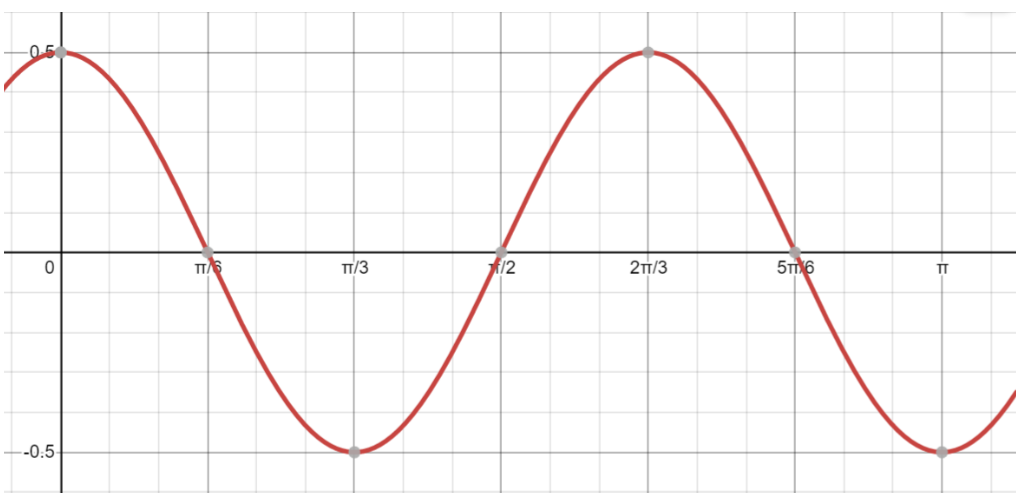
In each graph, determine the equation of the form $y = a\cos bx$ or $y = a\sin bx$, with $b > 0$, for each graph.

Desmos.com was used to create these graphs.

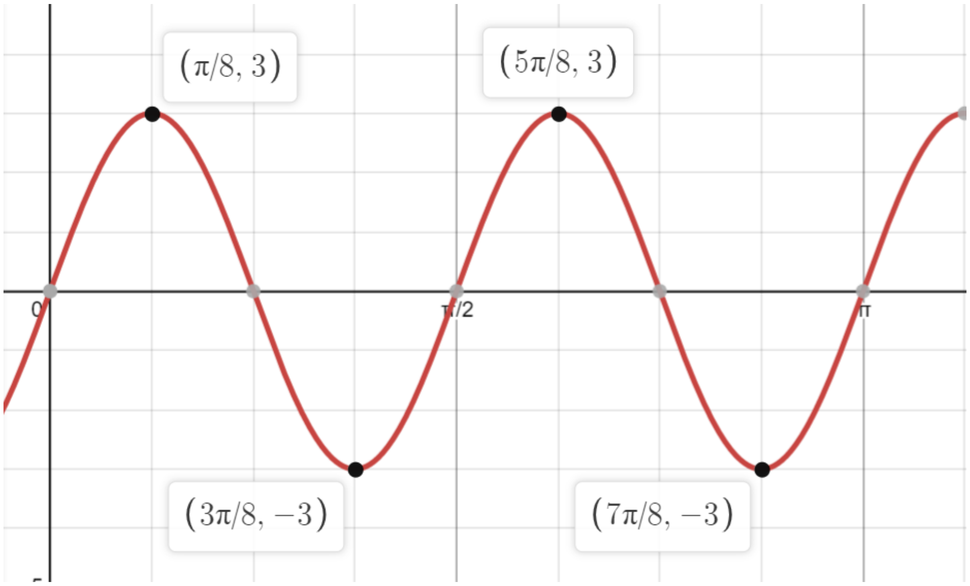
a.



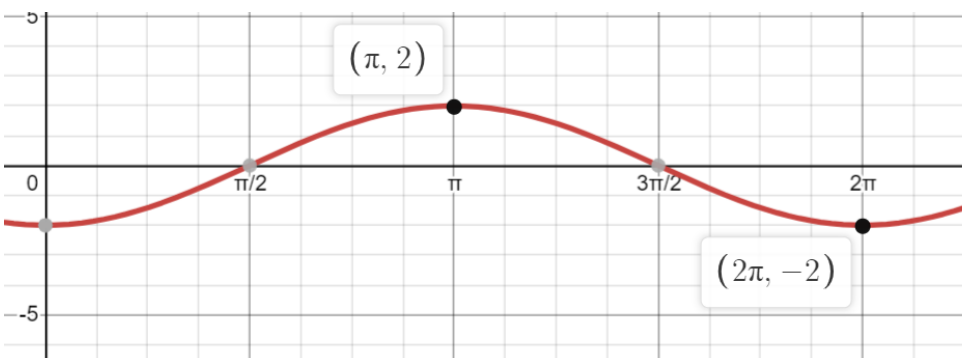
b.



c.



d.



Lesson: Translations of the Sine and Cosine Graphs

How do you describe a horizontal translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What is phase shift?

What is an argument?

How do you describe a vertical translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What happens when a function is of the form: $f(x) = \mathrm{a}\cos(bx + bd)$ with the phase shift?

Examples Identify the amplitude, period, phase shift, argument, and vertical translation.

a. $y = \sin\left(x + \frac{\pi}{4}\right)$

b. $y = 2 - \sin x$

c. $y = 3 + \sin(x + 2)$

d. $y = -4 + \frac{1}{2}\cos x$

e. $y = 2 + \sin(2x - 2\pi)$

f. $y = 1 - 2\cos\left(3x + \frac{\pi}{2}\right)$

g. $y = 2 - 4\cos(3x + 1)$

h. $y = 3\pi - 2\sin(\pi x - 3)$

Examples Show how to find the start and end points of an interval for us to graph.

a. $y = \sin\left(x + \frac{\pi}{2}\right)$

b. $y = 1 - 2\sin\left(3x - \frac{\pi}{2}\right)$

c. $y = 3\pi - 2\sin(\pi x - 3)$

Example Using the graph $y = \sin\left(x + \frac{\pi}{4}\right)$, and *clearly write out the steps* required to graph any sine or cosine graph. Identify the amplitude, period, phase shift, and vertical translation.

Example Graph $y = 1 + \cos x$. Identify the amplitude, period, phase shift, and vertical translation.

Example Graph $y = -1 + \frac{1}{2}(\cos 2x - \pi)$. Identify the amplitude, period, phase shift, and vertical translation.

Examples: Translations with Sine and Cosine

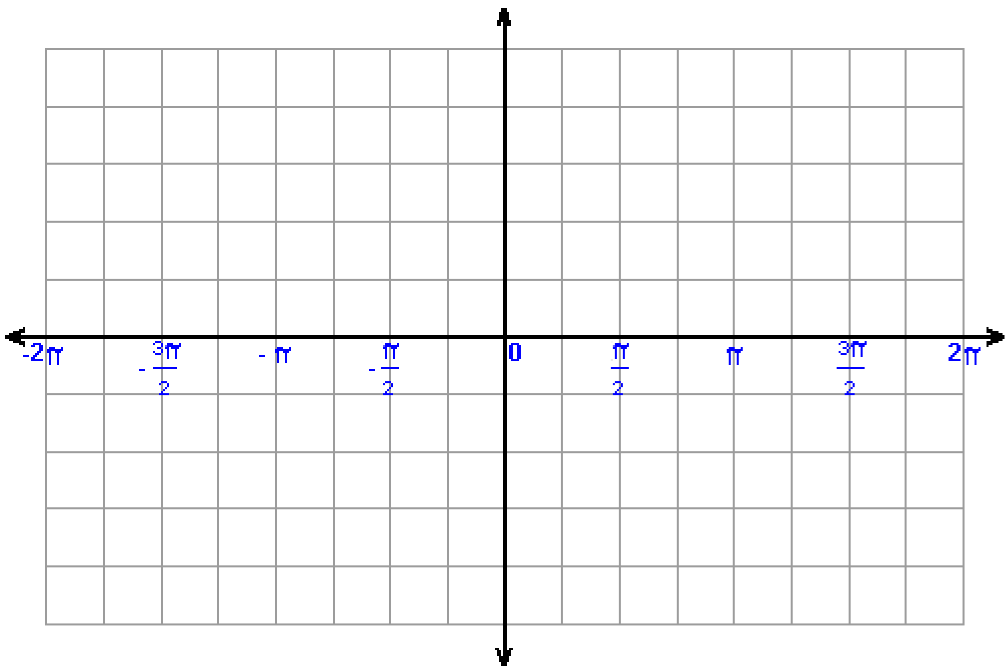
Graph the following: $y = \frac{5}{2}\sin\left(4\left(x + \frac{\pi}{4}\right)\right)$

Lesson: The Graphs of Tangent and Cotangent

What is an asymptote? Draw a sketch to help explain.

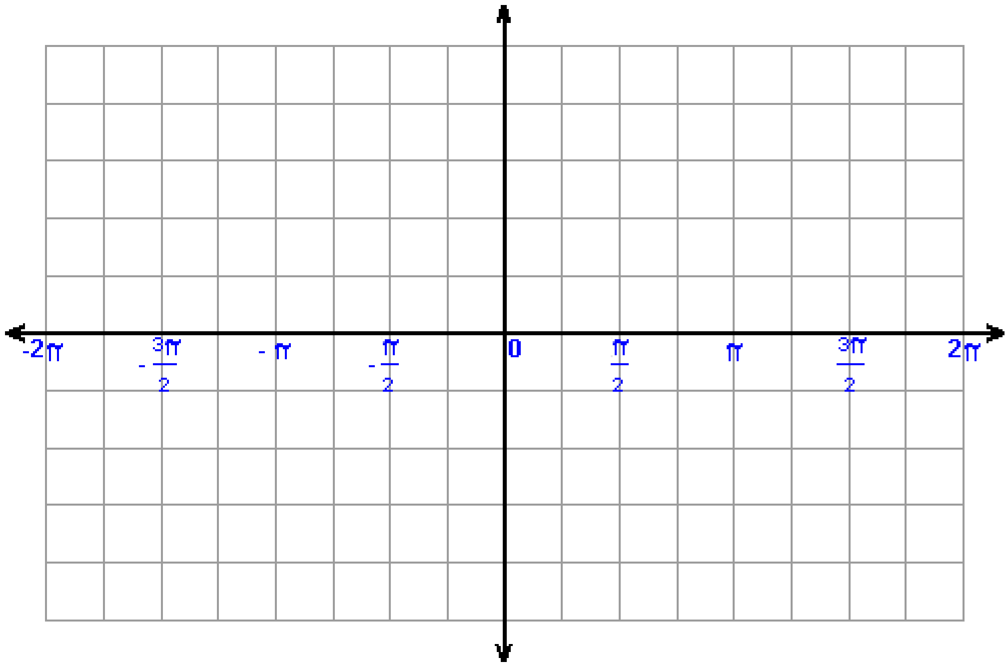
Draw the graph of tangent, filling in the table and then using the graph paper.

x	$y = \tan x$
$-\frac{\pi}{2}$	
$-\frac{\pi}{4}$	
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{2}$	



Draw the graph of cotangent, filling in the table and then using the graph paper.

x	$y = \cot x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{2}$	
π	
$\frac{5\pi}{4}$	



How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

What is the domain and range of the graphs?

How can we describe the x-intercepts of tangent?

How can we describe the x-intercepts of cotangent?

Are either function even or odd?

How do you interpret amplitude with these functions and why?

What is the key for graphing? (Be specific with the equations you need to leverage)

Example Graph over a one-period interval: $y = \tan 2x$

Example Graph over a two-period interval: $y = -\frac{1}{2}\cot 4x$

Lesson: Translations with The Graphs of Tangent and Cotangent

Tangent and cotangent can experience the same types of translations that sine and cosine did (which we discussed in a previous lesson).

<u>Examples</u> Describe the period, phase shift, and vertical translations for each function.

a. $y = 3\tan 4x$

b. $y = -\cot(x + \pi)$

c. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

d. $y = 3 + \cot(x - 4)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

<u>Examples</u> Show how to find 2 adjacent asymptotes for graphing each of the following:

a. $y = 3\tan 4x$

b. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

Example Graph $y = 3 + \cot\left(x + \frac{\pi}{4}\right)$

Example Graph $y = -1 + 2\tan(4x + \pi)$

Examples: Translations with The Graphs of Tangent and Cotangent

Graph over a two-period interval.

a. $y = \tan\left(3x + \frac{\pi}{4}\right)$

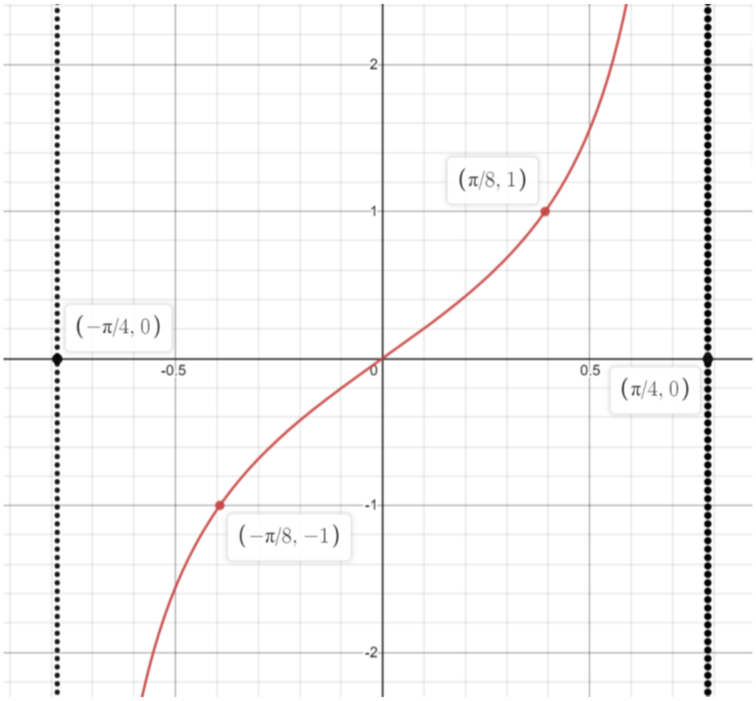
b. $y = -2 + 3\cot(4x - \pi)$

c. $y = 1 - \tan(2\pi x - \pi)$

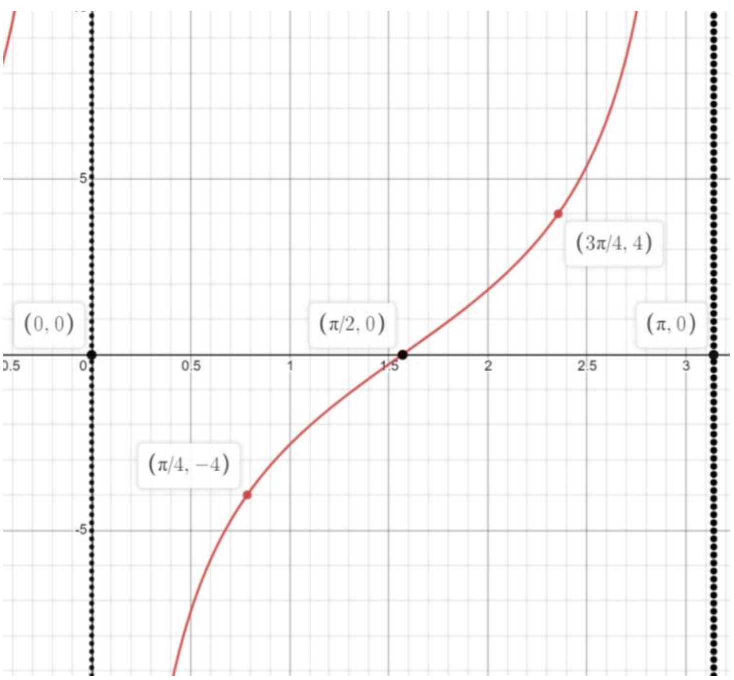
Lesson: Determining the Equations of Tangent and Cotangent from Graphs

Graphs from desmos.com of course!

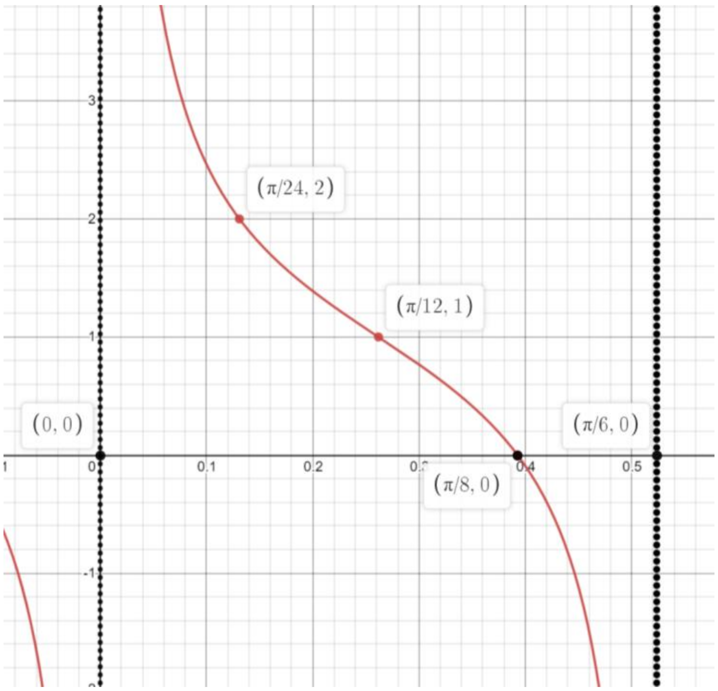
a.



b.



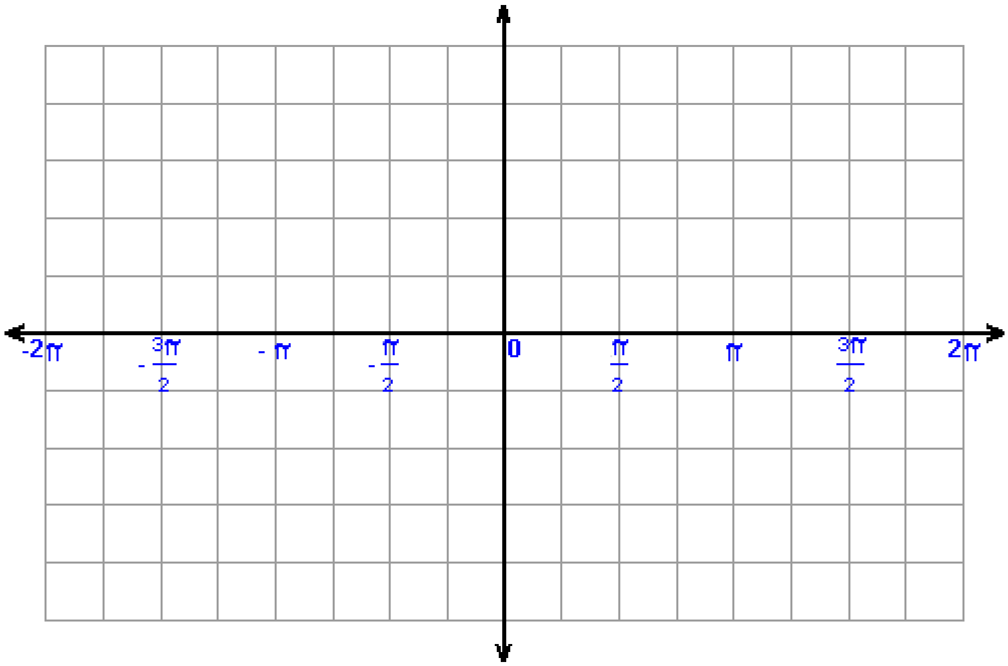
c.



Lesson: Translations of The Graphs of Secant and Cosecant

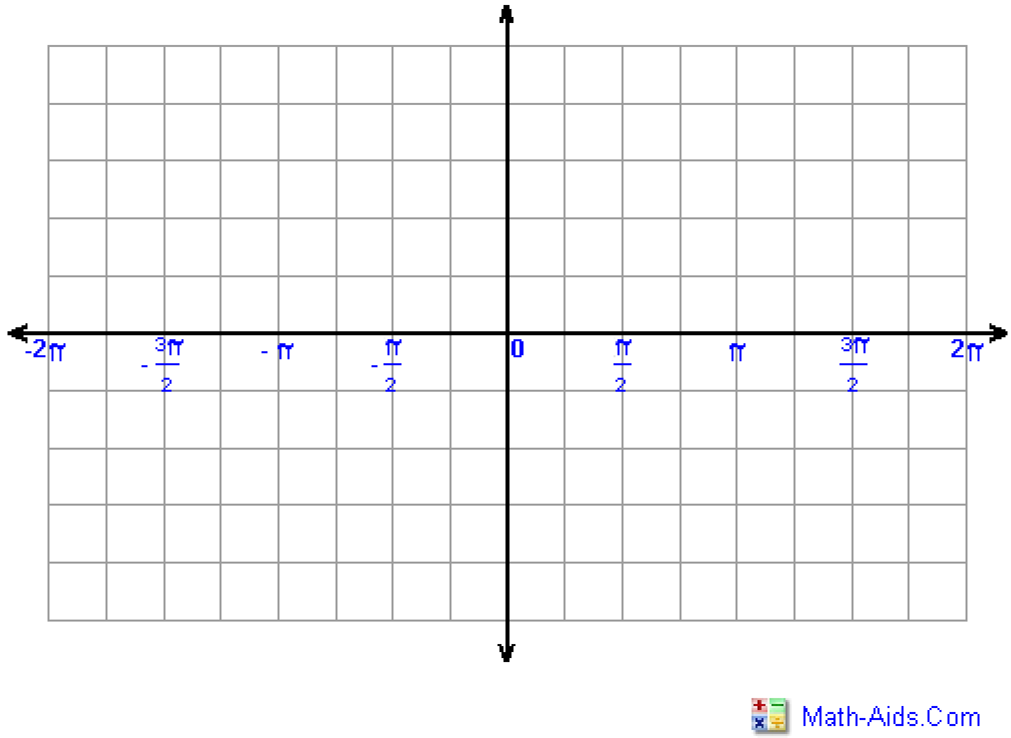
Draw the graph of secant, filling in the table and then using the graph paper.

x	$y = \sec x$
$-\frac{\pi}{2}$	
$-\frac{\pi}{4}$	
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	
$\frac{3\pi}{2}$	



Draw the graph of cosecant, filling in the table and then using the graph paper.

x	$y = \sec x$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{2\pi}{3}$	
π	
$\frac{3\pi}{2}$	
2π	



How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

What is the domain and range of the graphs?

How can we describe the x-intercepts of both graphs?

Are either function even or odd?

How do you interpret amplitude with these functions?

What is the key for graphing? (Be specific with the equations you need to leverage)

Examples Graph $y = 2\csc\frac{1}{2}x$

Examples Graph $y = 1 + \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$

Examples: Translations of The Graphs of Secant and Cosecant

Graph the following over a one-period interval.

a. $y = -2\sec\left(x + \frac{3\pi}{4}\right)$

b. $y = \frac{1}{2}\csc(2x - \pi)$

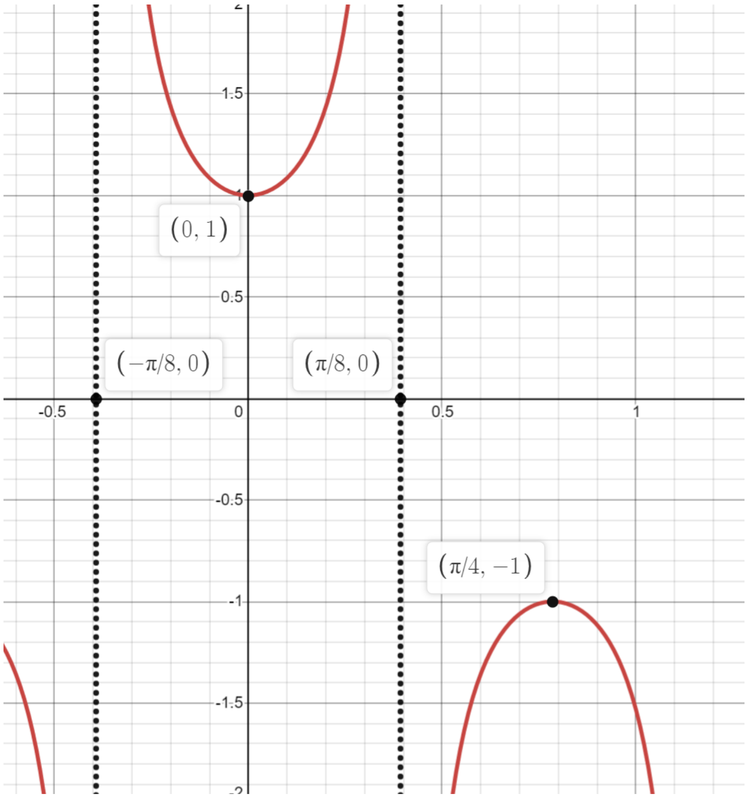
c. $y = 2 + \csc\left(\frac{1}{2}x - \pi\right)$

d. $y = 2 - 3\sec\left(2\pi x - \frac{\pi}{2}\right)$

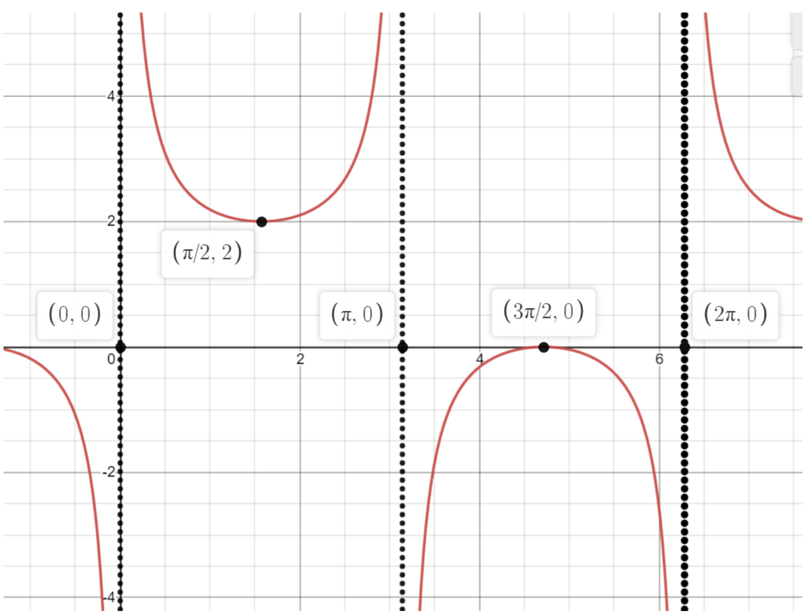
Lesson: Equations of Secant and Cosecant from Graphs

Determine the equation of secant or cosecant from the graphs.

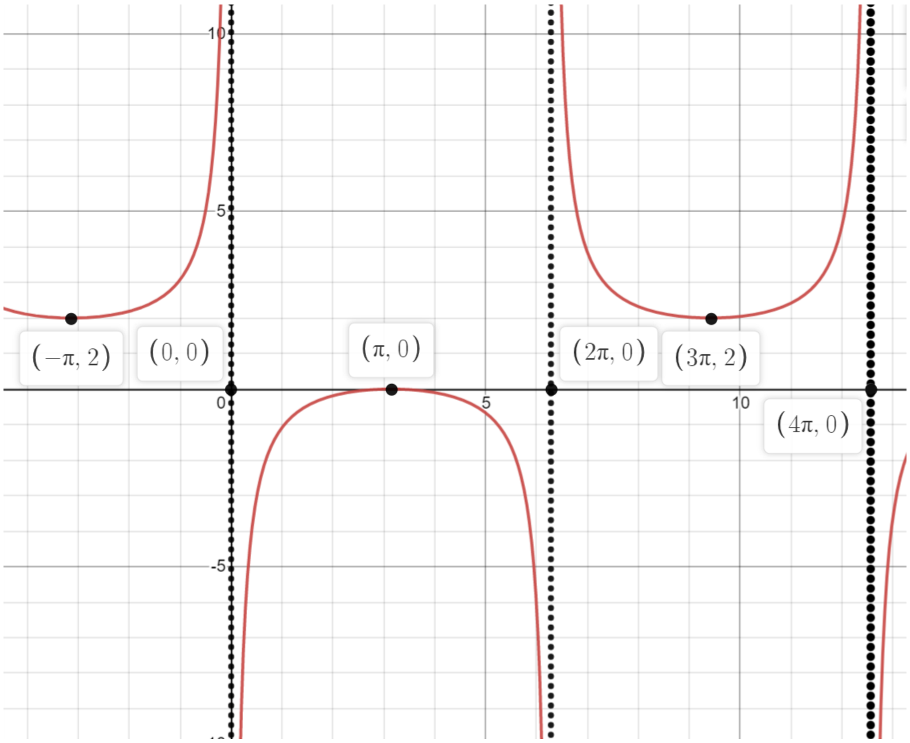
a.



b.



c.



Lesson: Fundamental Identities of Trig

Review the test for symmetry (and the results):

We know that the sine function is odd, show what this means:

We know that the cosine function is even, show what this means:

These are called **identities.**

Using these identities, what other identity can we determine about the tangent function:

These are called the **even-odd identities.**

We have already seen a handful of identities, and we call these the fundamental identities. Summarize them below.

Reciprocal Identities

Quotient Identities

Pythagorean Identities

Even-Odd Identities

Example If $\cos\theta = \frac{3}{4}$ and θ is in quadrant I, find $\sin\theta$.

Example If $\tan\theta = -\frac{1}{4}$ and θ is in quadrant IV, find the values of all the other 5 trig functions.

We use the identities to work on problems called transformations.

Example Write $\tan x$ in terms of $\sec x$.

Example Write the following in terms of sine and cosine, and then simplify until no quotients appear and all functions are of θ only.

a. $\csc\theta\cos\theta\tan\theta$

b. $\cos\ \theta(\cos\theta - \sec\theta)$

Examples: Using Fundamental Trig Identities to Find Values of Other Trig Functions

Find the five remaining trig functions given the following information.

a. $\csc\theta = -\frac{5}{2}$, θ in quadrant III.

b. $\sec\theta = \frac{7}{2}$, $\tan\theta < 0$

Examples: Writing Trig Functions in Terms of Other Trig Functions (Fundamental Identities)

a. Write $\cot x$ in terms of $\sin x$

b. Write $\csc x$ in terms of $\cos x$

Examples: Using Fundamental Identities to Write Expressions in Terms of Sine and Cosine

Write each expression in terms of sine and cosine, then simplify so that no quotients appear and all functions are in θ only.

a. $\cos\theta\csc\theta$

b. $\frac{(1+\cot\theta)}{\cot\theta}$

c. $\frac{\sec^2\theta-1}{csc^2\,\theta-1}$

d. $\frac{\csc\theta}{\cot(-\theta)}$

Lesson: Verifying Trigonometric Identities

As you progress through higher mathematics, sometimes you will need to manipulate equations in very creative ways. One way to learn this level of creativity is by verifying trigonometric identities. We will go through a few techniques of how to do this.

<u>Strategy 1: Rewrite the More Complicated Side / Work from One Side</u>

a. Verify that $\frac{1-\sin^2 x}{\cos x} = \cos x$. Show *why.*

b. Verify that $\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$

Strategy 2: Express the Functions in Terms of All Sine and Cosine

a. $\frac{\tan x}{\sec x} = \sin x$

b. $\sec x\,(\sin x + \cos x) = 1 + \tan x$

Strategy 3: Factor or Complete Indicated Algebraic Operations

Examples Factor.

a. $x^2 - 4$

b. $\sin^2 x - 4$

c. $(\tan x + \cot x)^2 - (\tan x - \cot x)^2$

d. $2\sin^2 x + 3\sin x + 1$

Examples Verify the following identities.

a. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

b. $\frac{1-\cos x}{1+\cos x} = (\cot x - \csc x)^2$

Strategy 4: Manipulate Both Sides

a. $\frac{\cos x}{\sec x}+\frac{\sin x}{\csc x}=\sec^2 x-\tan^2 x$

b. $\tan^2 x\ sin^2x=tan^2x+cos^2x-1$

Strategy 5: Use the Conjugate

What is the conjugate of the following:

a. $x - 4$

b. $\sqrt{2} - \sqrt{3}$

c. $\sin x + 1$

Verify the following identities: $\frac{1-\cos x}{1+\cos x} = \csc^2 x - 2\csc x \cot x + \cot^2 x$

Examples: Verifying Trigonometric Identities

a. $\frac{\cos x}{\cot x \sin x} = 1$

b. $\frac{1}{\sec x - \tan x} = \sec x + \tan x$

c. $\frac{1+\sin t}{1-\sin t} - \frac{1-\sin t}{1+\sin t} = 4\tan t \sec t$

d. $(\cos\theta - \sin\theta)(\sec\theta + \csc\theta) = \cot\theta - \tan\theta$

Lesson: Proofs of the Sum and Difference Identities for Cosine

Write out the proof for $\cos(A - B) = \cos A \cos B + \sin A \sin B$. Do your best to understand the proof and then explain it in a way that you will understand.

Write out the identity for $\cos(A - B)$ below (yes I know I stated it above, but YOU should write it so you have a better chance of remembering it):

Write out the proof for $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Write out the identity below (yes, write it out so you don't forget!):

Lesson: Using Sum and Difference Identities for Cosine

Examples Show how to use the cosine of a sum or difference to find the exact value of the expressions:

a. $\cos 105°$

b. $\cos\frac{\pi}{12}$

c. $\cos 40° \cos 50° - \sin 40° \sin 50°$

Show how the cofunction identities are an extension of the cosine of a difference:

Write out all the **cofunction identities**

Examples Find one value of θ that satisfies the following:

a. $\sec\theta = \csc 72°$

b. $\tan\theta = \cot(-44°)$

c. $\cos\theta = \sin\frac{5\pi}{6}$

Example Write $\cos(180° - \theta)$ as a trigonometric function of θ alone.

Example Suppose $\sin s = \frac{3}{5}$ and $\sin t = -\frac{12}{13}$, where s is in quadrant I and t is in quadrant III. Find $\cos(s + t)$ and $\cos(s - t)$.

Example Verify that $\sec(\pi + x) = -\sec x$

Lesson: Sum and Difference Identities for Cosine

Write out the proof for $\sin(A+B)$

Write out the proof for $\sin(A-B)$

Write out the identities for the sum and difference of sine below:

Write out the proof for $\tan(A+B)$

Write all identities for the sum and difference of tangent below:

Examples Find the exact value of each expression.

a. $\sin(-15°)$

b. $\tan\left(-\frac{5\pi}{12}\right)$

c. $\sin\left(\frac{5\pi}{9}\right)\cos\left(\frac{\pi}{18}\right)+\cos\left(\frac{5\pi}{9}\right)\sin\left(\frac{\pi}{18}\right)$

Examples Write each function as an expression involving functions of θ alone.

a. $\cos(30° + \theta)$

b. $\sin\,(45° + \theta)$

c. $\tan\left(\frac{\pi}{4} + x\right)$

Examples Suppose A and B are angles in standard position such that $\sin A = -\frac{3}{5}\frac{3\pi}{2} < A < 2\pi$ and

$\cos B = -\frac{7}{25}$, $\pi < B < \frac{3\pi}{2}$. Find the following.

a. $\sin(A + B)$

b. $\tan(A - B)$

c. the quadrant of A-B

<u>Examples</u> Verify the identities.

a. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

b. $\frac{\sin(x-y)}{\sin(x+y)}=\frac{\tan x-\tan y}{\tan x+\tan y}$

Examples: Sine of a Sum and Tangent of a Sum

Let $\cos s = \frac{5}{12}, \sin t = \frac{4}{5}$ where s and t are in quadrant I. Find:

$\sin(s + t)$

$\tan\,(s + t)$

The quadrant of $(s + t)$

Lesson: Double-Angle Identities

Show how we derive the first identity for $\cos 2A$:

Show how we derive the second identity for $\cos 2A$:

Show how we derive the third identity for $\cos 2A$:

Show how we derive the identity for $\sin 2A$:

Show how we derive the identity for $\tan 2A$:

Summarize the Double-Angle Identities Below:

Example Give $\sin\theta = \frac{5}{8}$ and $\cos\theta < 0$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

Example Find the values of the six trig functions give $\cos 2\theta = -\frac{11}{13}$ and $180° < \theta < 270°$.

Example Verify the following identities:

a. $(\sin x + \cos x)^2 = \sin 2x + 1$

b. $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$

Examples Simplify the expressions.

a. $\sin^2 15° - \cos^2 15°$

b. $\sin^2 \frac{3\pi}{5} - \cos^2 \frac{3\pi}{5}$

Example Write $\cos 3x$ in terms of $\cos x$

Lesson: Product-to-Sum Identities

Show what happens when we add the identities $\cos(A+B)$ to $\cos(A-B)$

Now show what happens when we subtract them:

Examples Write the following as the sum or difference of 2 trig functions.

a. $2\sin 48° \cos 104°$

b. $8\sin 9x \sin 5x$

<u>Examples</u> Write the expression as a product of trigonometric functions.

a. $\cos 6x + \cos 9x$

b. $\sin 106° - \sin 96°$

Lesson: Half-Angle Identities

Show how we derive the identity for $\sin\frac{A}{2}$:

Show how we derive the identity for $\cos\frac{A}{2}$:

Show the first way we can derive an identity for $\tan\frac{A}{2}$:

Show the second way we can derive an identity for $\tan\frac{A}{2}$:

Show the third way we can derive an identity for $\tan\frac{A}{2}$:

Summarize the half-angle identities below:

<u>Example</u> Show how to use the half-angle identity to evaluate: $\sin 12.5°$

<u>Example</u> Find the exact value of $\cos 165°$

<u>Example</u> Find the exact value of $\tan 75°$ using the identity $\tan A = \frac{\sin A}{1+\cos A}$

<u>Example</u> Given $\cos s = \frac{2}{7}$ with $0 < s < \frac{\pi}{2}$, find $\sin\frac{s}{2}$, $\cos\frac{s}{2}$, and $\tan\frac{s}{2}$

Example Simplify each expression.

a. $\sqrt{\frac{1-\cos 40°}{2}}$

b. $\frac{1-\cos 64.72°}{\sin(64.72°)}$

c. $\pm\sqrt{\frac{1+\cos 10x}{2}}$

Example Verify the identities.

a. $\cot^2 \frac{x}{2} = \frac{(1+\cos x)^2}{\sin^2 x}$

b. $\cos x = \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$

Lesson: Summarizing All of the Identities

Using all the trig identities that we've discussed, verify the following:

a. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

b. $1 - \tan^2 \frac{x}{2} = \frac{2\cos x}{1+\cos x}$

c. $\tan\left(\frac{a}{2}+\frac{\pi}{4}\right)=\sec a+\tan a$

d. $\sin 7x+\sin 5x+\sin 3x+\sin x=4\sin 4x\cos 2x\cos x$

e. $\tan 4x = \frac{2\tan 2x}{2-\sec^2 2x}$

Lesson: Inverse Functions (a review)

What does it mean to be an "inverse function" ?

<u>Example</u> If a function includes the points $\{(2,3), (5,4), (-7,2), (-1,5)\}$, then what points will the inverse have?

What is the horizontal line test?

<u>Example</u> Draw the graphs from the video, then determine if they have inverses:

How do you find the inverse of a function $f(x)$?

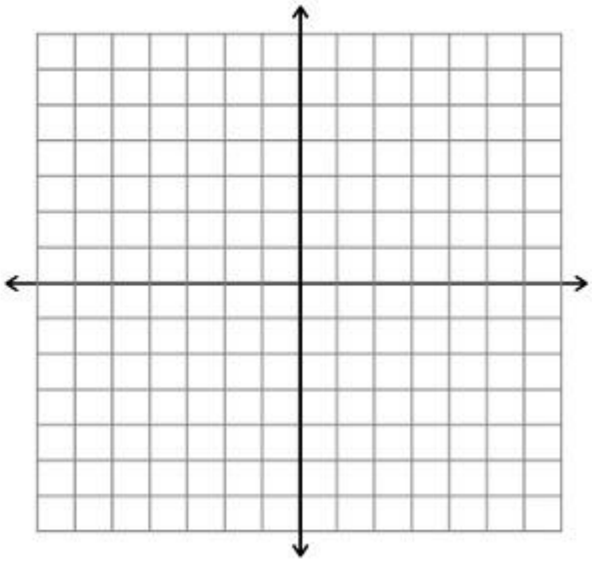
Example Find the inverse of the following:

a. $f(x) = 2x + 1$

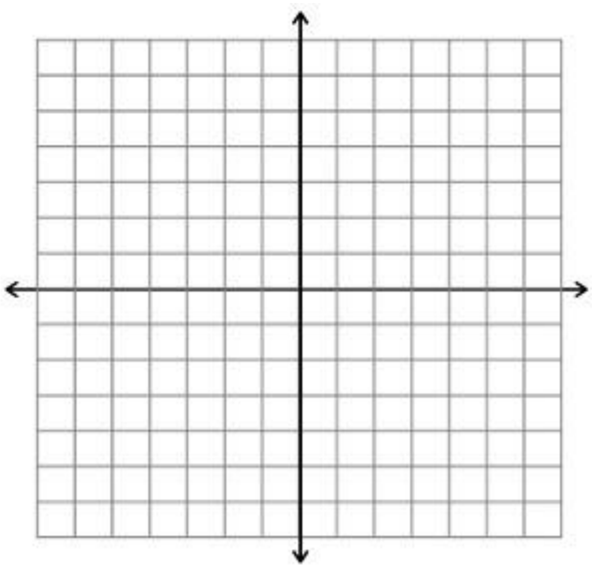
b. $f(x) = \sqrt{x}$

Examples Graph the following functions with their inverses:

a. $f(x) = 2x + 1$



b. $f(x) = \sqrt{x}$



From these graphs, we can see that inverses of functions are really just what?

What can we say about the domain and range inverses of functions?

Lesson: Inverse Trig Functions

Does the sine function have an inverse? Why or why not?

How can we manipulate the graph of the sine function to determine an inverse?

What are two ways that we denote the inverse sine function?

How do we “formally” define the inverse sine function?

Show how to graph the inverse sine function. Draw both the table and graph discussed from the video:

Example Find the value of each of the following:

a. $y = \arcsin \frac{\sqrt{3}}{2}$

b. $f(x) = \sin^{-1}\left(-\frac{\pi}{4}\right)$

c. $y = \sin^{-1} -\sqrt{2}$

Does the cosine function have an inverse? How should we think about this?

How do we denote the inverse of cosine?

How do we define the inverse cosine function?

Show how to graph the inverse cosine function. Draw both the table and graph discussed from the video:

<u>Examples</u> Find the value of each of the following:

a. $y = \arccos 1$

b. $f(x) = \cos^{-1}\left(\frac{1}{2}\right)$

How do we denote the inverse of tangent?

How do we define the inverse tangent function?

Show how to graph the inverse tangent function. Draw both the table and graph discussed from the video:

Define the other inverse circular functions:

Summarize the domains, ranges, and quadrants of the inverse trig functions below (this is important, you'll need to study this for the next exam):

Function	Domain	Interval	Quadrants

<u>Examples</u> Find the degree measure of θ:

a. $\theta = \arctan\sqrt{3}$

b. $\theta = \csc^{-1}(-\sqrt{2})$

Lesson: Finding Values of Inverse Trig Functions

Here we will just examine a wealth of examples of how to find different values of inverse trig functions.

Examples Use a calculator to determine the appropriate values.

a. Find y in radians if $y = sec^{-1}(-4)$

b. Find θ in degrees if $\theta = \text{arccot}(-0.25431)$

Before you evaluate those, you will have to watch some explanation of how / why we evaluate these the way that we do in the calculator. Write out the explanation of how/why we evaluate $\csc^{-1}(2)$ in a calculator below:

Cheat Sheet – How to Evaluate Inverse Trig Functions That Don't Have a Button on a Calculator

Cheat Sheet (for Comparison) – How to Evaluate Regular Trig Functions That Don't Have a Button on a Calculator

<u>This might get confusing so maybe rip this sheet out and put it someplace where you can easily reference it??</u>

Examples Use a calculator to find each value.

a. $\sin(\cos^{-1} 0.25)$

b. $\cot\,(\cos^{-1}(0.56331233)$

Examples Evaluate each expression without using a calculator:

a. $\cos\left(sin^{-1}\left(\frac{2}{3}\right)\right)$

b. $\sec\,(\sec^{-1} 2)$

c. $\sin\left(tan^{-1}\frac{4}{3} - cos^{-1}\frac{12}{13}\right)$

Examples Write each expression as an algebraic expression in terms of u.

a. $\cot\left(\sec^{-1} u\right)$, $u > 0$

b. $\sec\left(\operatorname{arccot}\left(\frac{\sqrt{4-u^2}}{u}\right)\right.$

Lesson: Solving Trig Equations, part 1 – linear methods

Recall how linear methods work first by solving: $2x + 4 = 8$

Now compare this approach with the equation: $3\tan\theta - \sqrt{3} = 0$, over the interval $[0°, 360°)$

How does this answer change if we want to solve for *all* solutions?

<u>Example</u> Solve: $2\sin x + 3 = 4$ over $[0, 2\pi)$

Lesson: Solving Trig Equations, part 2 – quadratic methods

Using quadratic methods to solve trig equations relies on using the zero-factor property. Show how the zero-factor property works by solving $(x+5)(x-3)=0$.

Compare this approach by solving $\cos\theta\sin\theta=0=-\cos\theta$, over $[0°, 360°)$.

<u>Example</u> Solve $(\csc x+2)\left(\csc x-\sqrt{2}\right)=0$ for all solutions.

Recall how to use quadratic methods by solving $x^2 + 5x + 6 = 0$.

Compare this approach by solving $\cos^2 x + 2\cos x + 1 = 0$ over $[0°, 360°)$.

<u>Example</u> Solve $2\cos^2 x + \cos x - 1 = 0$ for all possible solutions.

Example Solve $2\cos^2 x - \sqrt{3}\cos x = 0$ over $[0, 2\pi)$

Example Solve $\tan x \csc x - \sqrt{3}\csc x = 0$ for all solutions.

Example Solve $\csc^2 x - 2\cot x = 0$ over $[0°, 360°)$

Example Solve $\tan x - \sqrt{3} = \sqrt{3}\cot x - 1$ for all possible solutions.

Lesson: Solving Trig Equations, part 3 – equations with half-angles

<u>Example</u> Solve $2\cos\frac{x}{2} - \sqrt{2} = 0$ over the interval $[0, 2\pi)$

How would the answers change if you needed to find all solutions?

Example Solve $2\sqrt{3}\sin\frac{x}{2} = 3$ for all possible solutions.

Example Solve $\sin^2\frac{x}{2} - 2 = 0$ for all possible solutions.

Lesson: Solving Trig Equations, part 4 – equations with multiple angles

Example Solve $\cos 2x = \sin x$ over $[0, 2\pi)$

Example Solve $2\cos^2 x - 2\sin^2 x + 1 = 0$ over $[0°, 360°)$

How would this change for all solutions?

Example Solve $\cot 2x - \csc 2x = 1$ over $[0, 2\pi)$

Example Solve $\tan 2x - \sec 2x = 2$ over $[0, 2\pi)$

<u>Example</u> Solve $\cos x - 1 = \cos 2x$ for all solutions.

<u>Example</u> Solve $\sin 2x = 2\cos^2 x$ for all solutions.

Lesson: Equations Involving Inverse Trig Functions

Example Solve $y = 4\tan 3x$, where x is restricted to $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$.

Example Solve $y = \tan\,(2x - 1)$ for x in $\left(\frac{1}{2} - \frac{\pi}{4}, \frac{1}{2} + \frac{\pi}{4}\right)$

Example Solve $3\tan^{-1}x = \pi$

Example Solve $6\arccos\left(x - \frac{\pi}{3}\right) = \pi$

Example Solve $\cot^{-1}x = \tan^{-1}\frac{4}{3}$

Example Solve $\arccos x + 2\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Example Solve $\sin^{-1} x + \tan^{-1} x = 0$

Lesson: Oblique Triangles and the Law of Sines

First, we should review some facts about congruent triangles. When are 2 triangles congruent?

Congruence Axioms

Side-Angle-Side (SAS)

Angle-Side-Angle (ASA)

Side-Side-Side (SSS)

When does Side-Angle-Angle (SAA) hold?

When a triangle is <u>not</u> a right triangle, what is it it called?

When can a triangle be solved?

What data is required for solving oblique triangles?

Case 1:

Case 2:

Case 3:

Case 4:

Is there an Angle-Angle-Angle congruency?

When Case 1 or Case 2 is involved, what is required?

When Case 3 or Case 4 is involved, what is required?

This Law of Cosines will be discussed in another video.

Show how we derive the Law of Sines:

State the Law of Sines:

What is an alternative form of the law of sines?

<u>Example</u> Solve Triangle ABC if $A = 28.5°, C = 102.3°, c = 25.1$ in.

Recall the formula for the area of a triangle:

Using SAS, how else can you find the area of a triangle?

Example Reference the sketch in the video. Find the area of the triangle.

Example Find the area of a triangle DEF if $D = 58°10', e = 31.2in, F = 73°30'$.

Lesson: Applications of the Law of Sines

<u>Example</u> An engineer wishes to measure the distance across a river. Reference the sketch in the video.
If $C = 117.4°, A = 30.0°, b = 75.8$ ft

Review the two methods of bearing:

Example The bearing of a lighthouse from a ship was found to be $N\ 52°$ W. The ship sailed 12.3 miles south, and the new bearing was $N\ 23°W$. Find the distance, to the nearest mile, between the ship and the lighthouse at each location.

Example A balloonist is directly above a straight road, 2 miles long that joins the 2 villages. The closer town to him is an angle of depression of $45°$, and the farther town is at angle of depression of $41°$. How high above the ground is the balloon?

Lesson: The Ambiguous Case of the Law of Sines

Describe the ambiguous case of the law of sines as describe in the video. There's going to be quite a bit of information presented – watch the entire explanation of why we need the law of sines and do your best to describe the various cases to sum up. Summarize the explanation so that it makes sense to YOU.

What were some tips for applying the law of sines?

Example Solve triangle ABC if $a = 16.2cm, c = 14.1\ cm$, and $C = 72°30'$.

Example Solve triangle DEF if $D = 62°8', a = 34.5in, b = 38.1in$.

Explain the number of triangles satisfying the ambiguous case (SSA):

<u>Examples</u> Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

a. $B = 80°, C = 35°, b = 27$

b. $A = 60°, B = 70°, c = 5$

Examples Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

c. $a = 15, b = 24, B = 33°$

d. $A = 25°, b = 8, a = 11$

<u>Examples</u> Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

e. $A = 61.4°, b = 35.5, b = 39.1$

f. $a = 17.1, c = 13.1, C = 76°$

Example Without using the law of sines, explain why no triangle ABC exists satisfying $B = 95°, b = 40cm, c = 46cm.$

Lesson: The Law of Cosines

What is the property of triangle side length restriction?

Show how to derive the law of cosines:

State the Law of Cosines:

Where is one place where we’ve seen this?

Example Two boats leave a harbor at the same time, traveling on courses that make an angle of $81°30'$ between them. When the slower boat has traveled 65.2 miles, the faster boat has traveled 84.3 miles. At that time, what is the distance between the boats?

Example Solve triangle ABC if $B = 72.4°, a = 31.4ft, c = 44.3ft$.

Example Solve triangle DEF if $d = 24.2in, b = 42.6in, c = 59.1in$.

Example The simplest type of roof truss is a triangle. Refer to the sketch in the video. Find angle C to the nearest degree for the truss shown.

Explain the 4 Cases of Solving Oblique Triangles

Example: The Law of Cosines with SSS Triangles

Find the angles of the triangles given the following sides:

a. $a = 3ft, b = 5ft, c = 6ft$

b. $a = 324m, b = 421m, c = 298m$

Lesson: Heron's Formula for the Area of a Triangle

Explain how to use Heron's formula (as explained in the video):

<u>Example</u> Find the area of triangle ABC where $a = 12m, b = 16m, c = 25m$

Example Find the area of the Bermuda Triangle if the sides of the triangle have approximate lengths 800mi, 875mi, and 1250mi.

Lesson: Vectors

What is a scalar?

What are vector quantities?

What is a vector?

Draw some vectors showing the differences with magnitude and directions.

Explain some of the notation we use regarding these concepts:

When are two vectors equal?

Is the sum of two vectors still a vector? (circle) Yes No

How do you find the sum of two vectors?

Vector addition is commutative. What does this mean?

For every vector, there is an opposite vector. Explain what this means:

The sum of **v** and **-v** results in what?

What happens when you multiply a vector and a scalar?

Explain the geometric properties of parallelograms:

Examples For each pair of vectors and the angle θ between them, sketch the result.

a. $|\boldsymbol{u}| = 12, |\boldsymbol{v}| = 22, \theta = 25°$

b. $|\boldsymbol{u}| = 20, |\boldsymbol{v}| = 40, \theta = 30°$

Examples Use the parallelogram in the video to find the magnitude of the resultant force for the 2 forces shown in the figure in the video.

Lesson: Vectors and Applications of Magnitude

Example A boat is being towed by two other boats, with the first boat having a force of 840lbs and the second have 925lb. The angle between these forces is 26°. Find the direction and magnitude of the equilibrant.

Example Two forces of 150 lbs and 275 lbs act at a point. The resultant force between them is 350 lbs. Find the angle between the forces.

Lesson: Vectors and Applications of Navigation

<u>Example</u> A ship leaves port on a bearing of $27°$ and travels 62.4miles. The ship then turns due east and travels 85.2 miles. How far is the ship from the port? What is its bearing from port?

Lesson: Vectors and Applications of Incline

Example Find the force required to keep 2700-lb car parked on a hill that makes a $13°$ angle with the horizontal.

Example A force of 18.00 lb is required to hold a 74.0 lb crate on a ramp. What angle does the ramp make with the horizontal?

Lesson: Vectors (Algebraic Interpretation)

Vectors are defined algebraically in the rectangular coordinate system with what notation?

Draw how we interpret this vector below.

In a previous video, we discussed how to define vectors geometrically. How is this definition different from our algebraic interpretation?

Show how to define a direction angle.

How do we calculator the magnitude of a vector?

How do we calculation the direction angle of the same vector $\boldsymbol{u} = \langle a, b \rangle$?

<u>Example</u> Calculate the magnitude and direction for $\boldsymbol{u} = \langle 5, 2 \rangle$.

Draw how we can view the horizontal and vertical components of vectors.

Example A vector has magnitude 36.0 and direction angle $35°$. Find the horizontal and vertical components.

Example Copy the sketch in the video. Write the vectors in the form $\langle a, b \rangle$.

What are the vector operations?

<u>Example</u> Let $\boldsymbol{u} = \langle -3, 4\rangle, \boldsymbol{v} = \langle 5, 7\rangle$. Find and illustrate each of the following.

a. $\boldsymbol{u} + \boldsymbol{v}$

b. $-3\boldsymbol{v} + 2\boldsymbol{u}$

What is the i,j form of vectors?

Example Write $\langle 3, 5 \rangle$ in i,j form

Example Write $2\boldsymbol{i} - 4\boldsymbol{j}$ in bracket vector form.

Lesson: The Dot Product

What is the dot product?

What is another name for the dot product?

Which is true: the dot product results in a vector the dot product results in a real number

<u>Example</u> Find the dot products.

a. $\langle 3, 5\rangle \cdot \langle 2, -1\rangle$

b. $\langle 2, -3\rangle \cdot \langle -4, -5\rangle$

What are the properties of the dot product?

<u>Examples</u> Let $\boldsymbol{u} = \langle 1, 3 \rangle, \boldsymbol{v} = \langle 2, 4 \rangle, \boldsymbol{w} = \langle -1, 2 \rangle$. Evaluate the following.

a. $\boldsymbol{u} \cdot \boldsymbol{v} - \boldsymbol{u} \cdot \boldsymbol{w}$

b. $(3\boldsymbol{u}) \cdot \boldsymbol{w}$

What are the possible outcomes of the dot product?

What is the geometric interpretation of the dot product? Draw a sketch.

Example Find the angle between the vectors.

a. $\boldsymbol{u} = \langle 1, 3 \rangle, \boldsymbol{v} = \langle 2, 4 \rangle$

b. $\boldsymbol{u} = \langle 1, -2 \rangle, \boldsymbol{v} = \langle -1, -3 \rangle$

Explain how to interpret the dot product in the following scenarios …

The Dot Product is …	The Angle between the Vectors is …
Positive	
Negative	
Zero	

When the dot product is 0, what does this mean?

What's the better word for perpendicular? (fancier, makes you sound smart, you should definitely use it to impress people at parties):

Example Determine if the pair of vectors is orthogonal: $\boldsymbol{u} = \langle 1, 2 \rangle, \boldsymbol{v} = \langle -6, 3 \rangle$

Lesson: A Review of Imaginary Numbers

Please note: Knowing imaginary numbers if one of the “prerequisite” topics for this course. Ie – you should already know this (though I also realize you probably need a review!). If you have *never* seen imaginary numbers before, then I strongly suggest that you watch my series on imaginary numbers (because, yes, I have one). So if you felt like this went too fast and you want more details, please take responsibility for your education and go back to review this!

How do we represent an imaginary number?

What is i^2?

What is a complex number?

When are 2 complex numbers equal?

What does $\sqrt{-a}$ mean?

Examples Write using the definition of $\sqrt{-a}$

a. $\sqrt{-12}$

b. $\sqrt{-25}$

Examples Complete the following operations.

a. $\sqrt{-5} * \sqrt{-3}$

b. $\frac{\sqrt{-12}}{\sqrt{-18}}$

c. $\frac{\sqrt{-50}}{\sqrt{9}}$

To add or subtract complex numbers, we can add/subtract the real parts and the imaginary parts similar to working with like terms.

Examples Find the sum or difference.

a. $(2-6i)+(8+2i)$

b. $(5-7i)-(-4+2i)$

When multiplying complex numbers, recall that $i * i = i^2 = -1$!

Examples Multiply.

a. $(2 + 4i)(3 - 5i)$

b. $(4 - 5i)(3 - 2i)$

c. $(3 + 2i)(3 - 2i)$

Recall, the conjugate is found by using the opposite side for the second term!

Examples What is the conjugate of the following?

a. $4 + 2i$

b. $-3 - 4i$

c. i

To find the quotient of 2 complex numbers, we multiply by the conjugate of the denominator.

<u>Examples</u> Find each quotient. Write your answer in *standard form,* ie $a + bi$ form

a. $\frac{3-4i}{2+i}$

b. $\frac{5}{i}$

How do the powers of i cycle through?

<u>Examples</u> Show how to simplify the following powers of i. Write your work in great detail so that you remember how to do this!

a. i^{17}	c. i^{-14}
b. i^{28}	d. i^{-5}

<u>Examples</u> The following quadratic equations have imaginary solutions. Show how to solve them.

a. $x^2 = -25$ using the **square root property**

b. $x^2 + 75 = 0$ using the **square root property**

c. $2x^2 - 3x = -6$ using the **quadratic formula**

d. $x^4 - 16$ using **factoring** and also the **square root property**

Lesson: Trigonometric (Polar) From of Complex Numbers

Complex numbers cannot be ordered like real numbers can. What is one way that we can organize complex numbers?

Show how we can graph complex numbers:

Complex numbers correspond to vectors. For a complex number $x + yi$, show how this corresponds to a vector (sketch), then label the sketch.

<u>Examples</u> Add $3 + 2i$ and $2 - 5i$, then graph both numbers and their resultant.

Define the relationships between x, y, r, θ below (with a sketch)

What is the trigonometric form (or polar form) of a complex number?

How do we refer to the number r?

How do we refer to θ?

Example Write $3(\cos 300° + i \sin 300°)$ in rectangular form.

How do you convert from rectangular from to trigonometric form?

Examples Write in trigonometric form.

a. $-\sqrt{2} + i$

b. -12

<u>Examples</u> Write the complex number in its alternative form.

a. $8(\cos 200° + i \sin 200°)$

b. $-8 + 5i$

Lesson: The Product and Quotient Theorems

First, let's review how to multiply $\left(2 + i\sqrt{3}\right)(-2\sqrt{3} + 2i)$

Let's rewrite both of those complex numbers in their trigonometric form:

Now multiply their trigonometric forms together:

What do you notice about the result?

This problem demonstrates the product theorem.

State the Product Theorem:

Example Find the product of $3(\cos 45° + i\sin 45°)$ and $5(\cos 120° + i\sin 120°)$. Then write the answer in rectangular form.

(PART A) Show how to find the quotient of $(2\sqrt{6} - 2i\sqrt{2})$ and $\left(\sqrt{2} - i\sqrt{6}\right)$

Now rewrite these complex numbers in their trigonometric form.

How can the "answer" to PART A be rewritten in trigonometric form?

What do you notice about this result?

This demonstrates the quotient theorem.

State the Quotient Theorem.

<u>Example</u> Find the quotient $\dfrac{35\ cis\ 45°}{7\ cis\ (-90°)}$. Write the result in rectangular form.

Example Use a calculator to find the following. Put your answer in rectangular form.

a. $(7.2\ cis\ 38.5°)(3.5\ cis\ 49.8°)$

b. $\dfrac{15.42\ (\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})}{5.21\ (\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})}$

Lesson: De Moivre's Theorem and Powers and Roots of Complex Numbers

Find the product: $[r\,(\cos\theta + i\sin\theta)]^2$

This is one example of De Moivre's theorem.

State De Moivre's Theorem:

<u>Example</u> Find $\left(\sqrt{3} + i\sqrt{5}\right)^8$ and write the answer in rectangular form.

We can extend this theorem to find all nth roots of complex numbers.

Explain the concept of finding the nth root of a complex number.

State the nth Root Theorem

Example Find the three cube roots of -8

Example Find all complex number solutions to $x^4 + 1 = 0$.

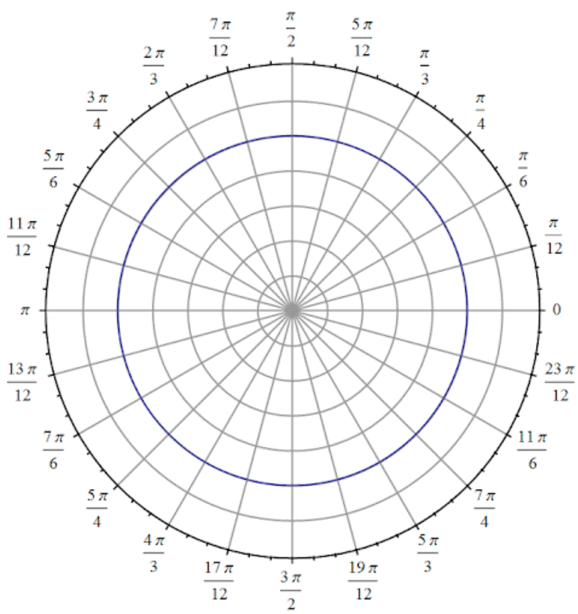
Lesson: Polar Equations and Graphs

There are other coordinate systems besides the rectangular coordinate system.

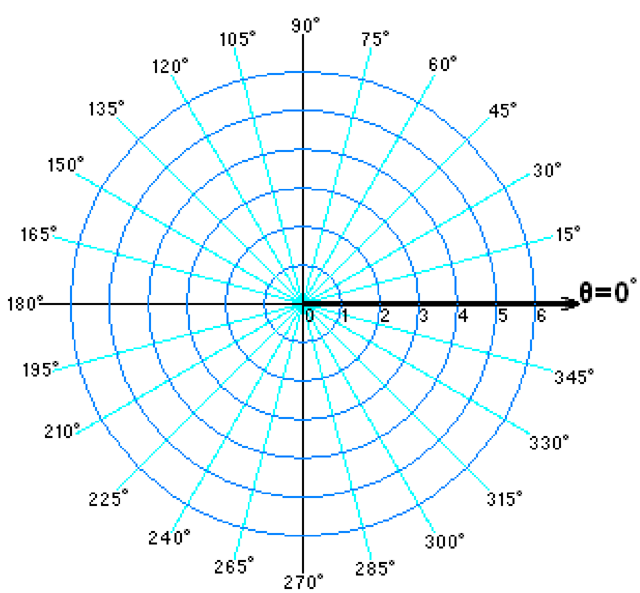
We will discuss the polar coordinate system!

What is a pole? What is the polar axis? Draw a sketch.

Here is what the polar coordinate system looks like:



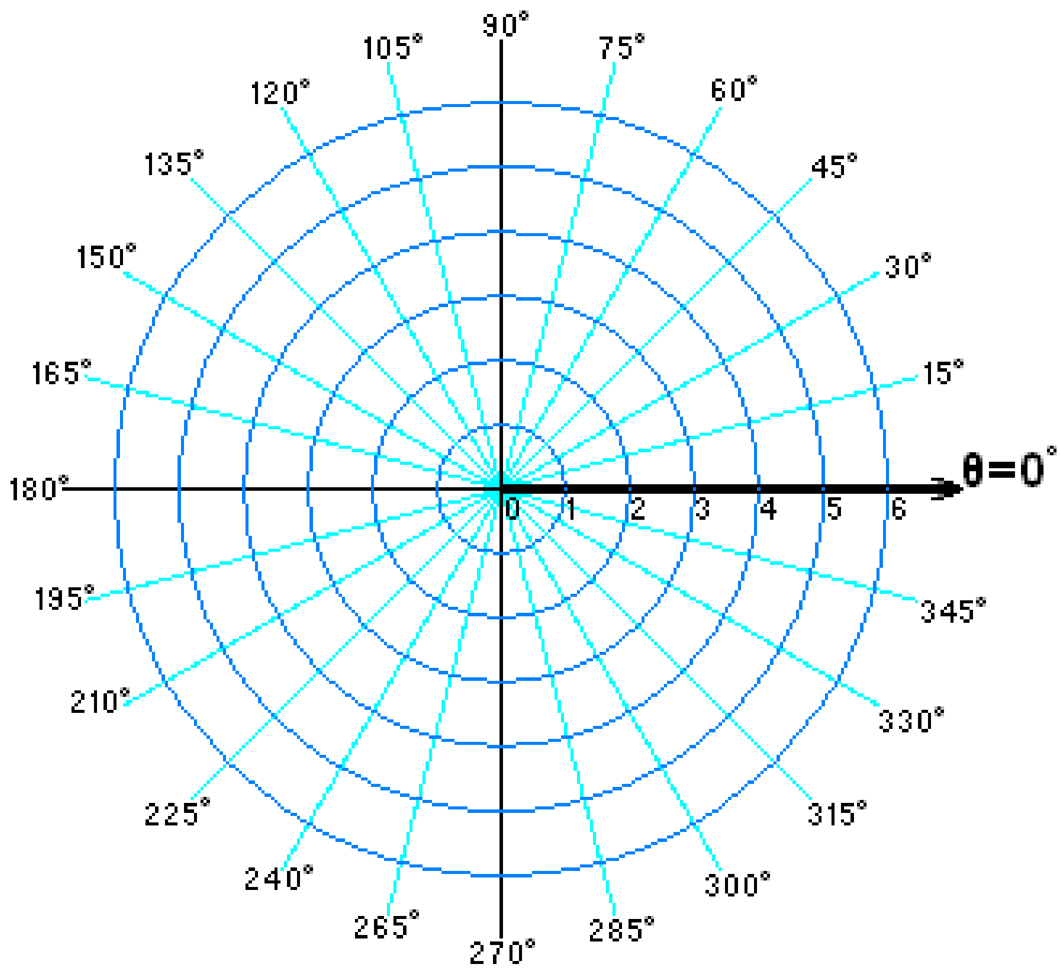
It can also be in degrees:



How do we convert between rectangular and polar coordinates?

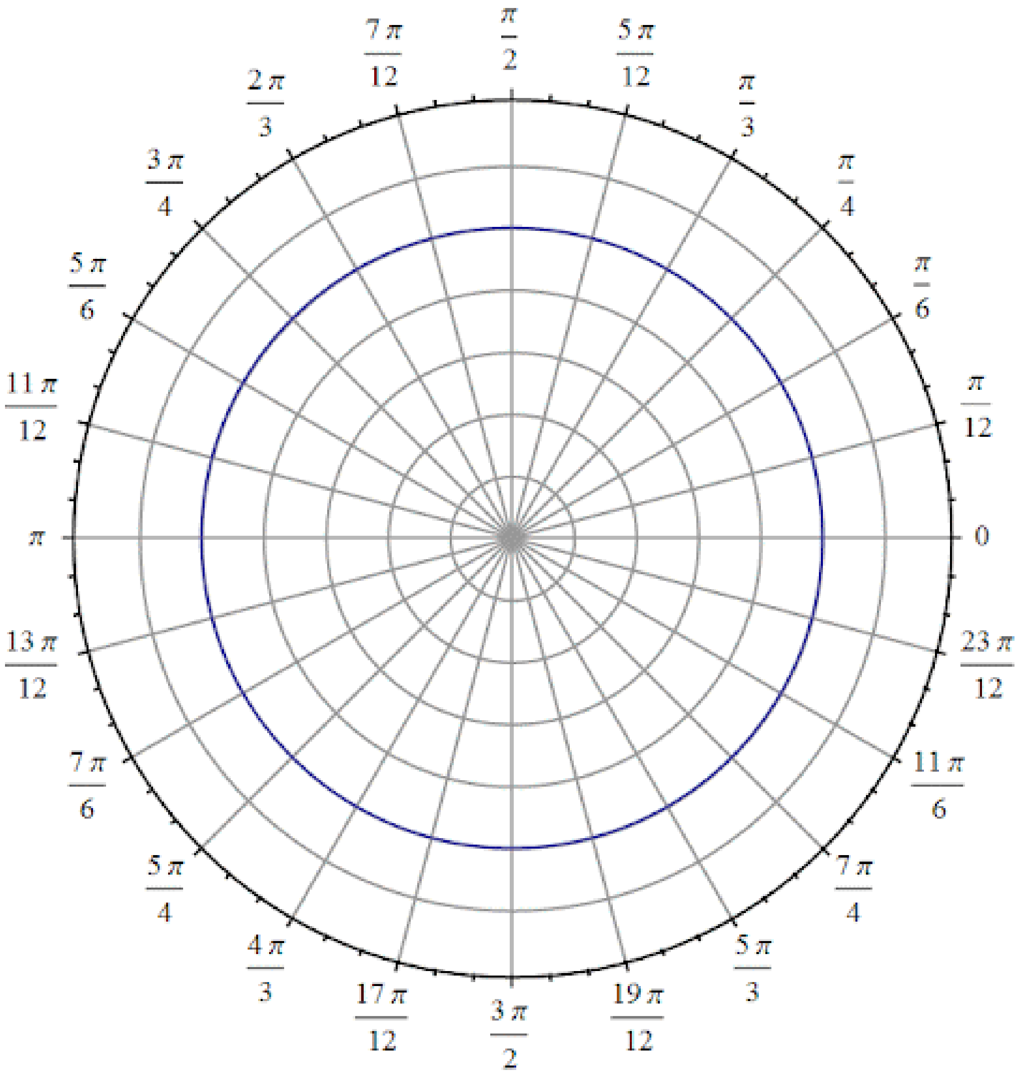
Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

a. $P(3, 45°)$



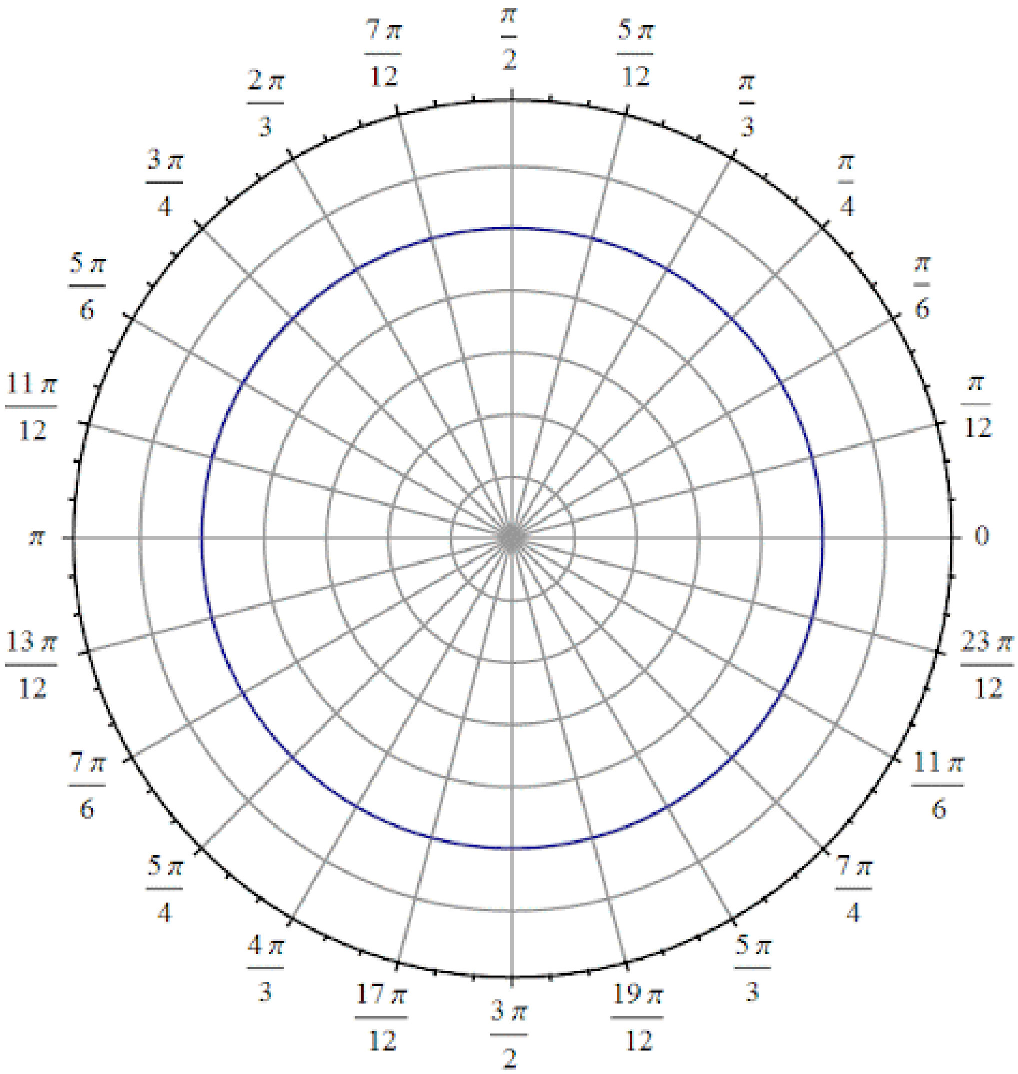
Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

b. $P\left(-4, \frac{\pi}{3}\right)$



Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

c. $R\left(3, -\frac{\pi}{2}\right)$



While a given point in the plane can only have 1 pair of rectangular coordinates, how many pairs of polar coordinates can it have?

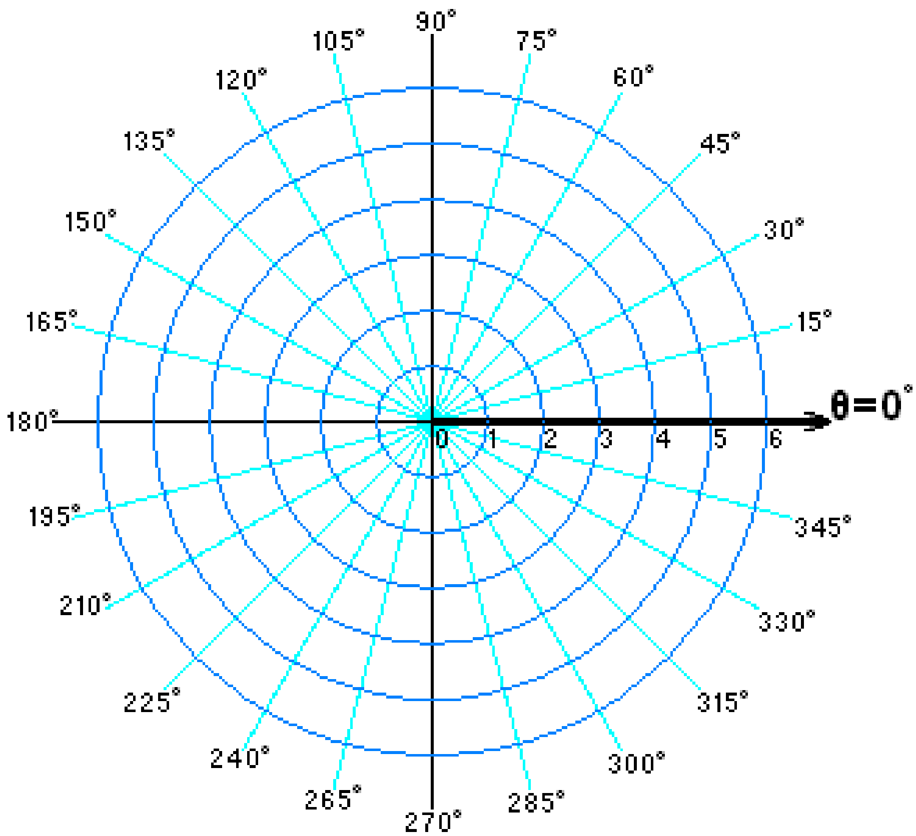
Examples Determine the following:

a. Three other pairs of coordinates for the point $P(3, -110°)$.

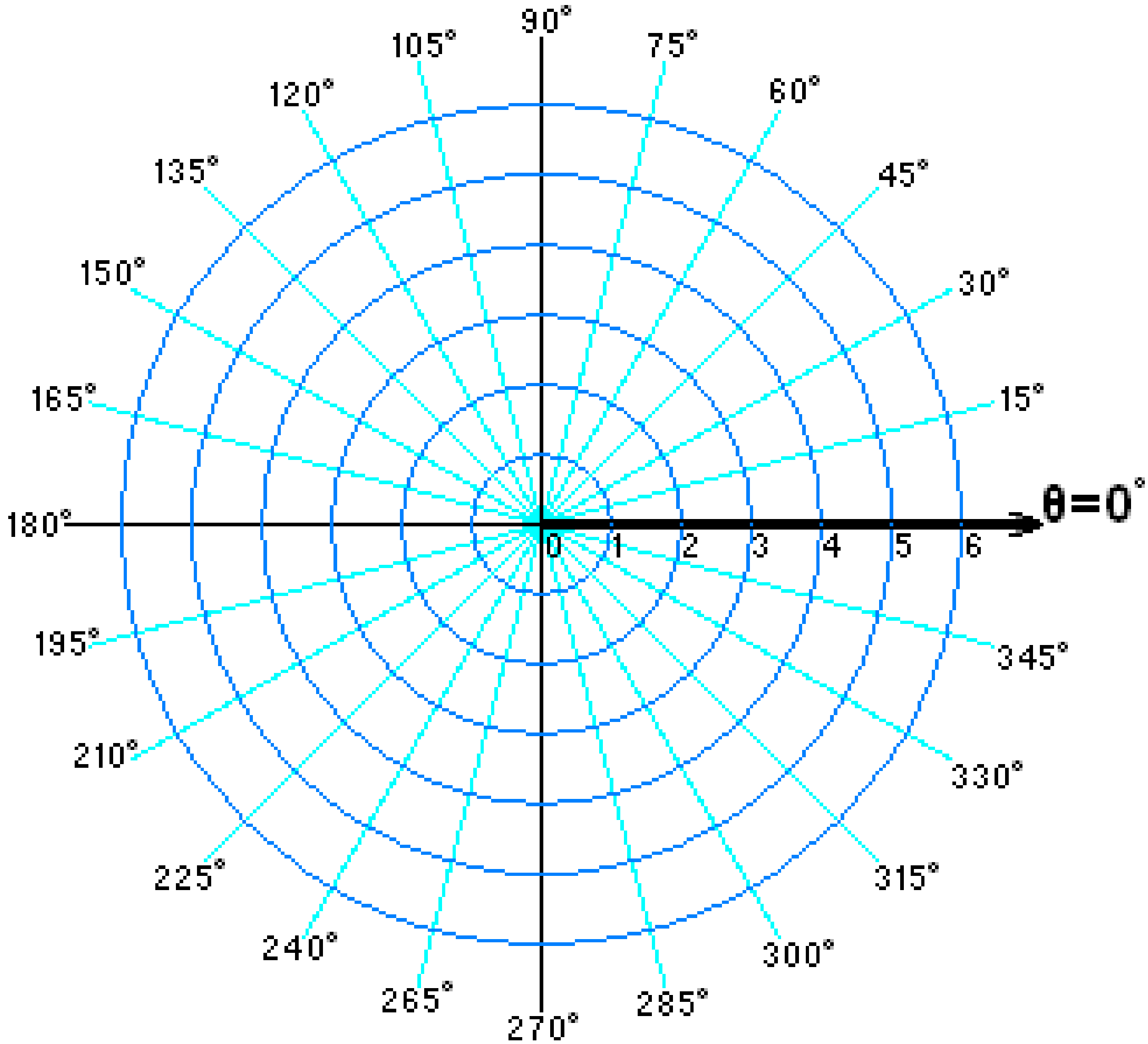
b. Two pairs of polar coordinates for the point with rectangular coordinates $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

What makes an equation a “polar equation” ?

Example Convert the equation to a polar equation, and then graph it: $y = x - 1$



Example Convert the equation to a polar equation, and then graph it: $x^2 + y^2 = 25$



Example Graph $r = 1 - \sin\theta$