Lesson: Factoring by GCF and Trinomials (the more memorable types of factoring)

How do you factor $3x + 3y$?

What type of factoring is this called?

<u>Examples</u> Show how to use the GCF to factor the following:

a. $4x + 12x^2 - 8x^3$

b. $5x^2y + 10x^7y^3 - 15x^5y^2$

Examples Show how to use the GCF to factor the following:

a. $4x^2 + 12xy - 3y^2$

b. $15x^3y^4 - 25x^6y^2 + 20x^7y^8$

What is a trinomial?

Show how to multiply: $(x + 2)(x + 7)$

Show how to factor $x^2 + 3x + 2$ using the *trial and error* method. Write down some reasoning explaining how this method works.

<u>Example</u> Factor the following using trial and error.

a. $x^2 + 5x + 6$

b. $x^2 - 7x + 12$

c. $x^2 - 25$

Show how we can factor $2x^2 + 5x + 3$ using *trial and error.*

<u>Examples</u> Factor the following.

a. $5x^2 + 7x + 2$

b. $3x^2 + 13x - 10$

Lesson: Factoring by Grouping and the Sum and Difference of Cubes

Show how to use grouping to factor: $ax + ay + bx + by$

What are some key hints that you probably need to use the grouping technique?

Write out the steps of how to use grouping for this problem:

<u>Example</u> Show how to finish the factoring for the following:

a. $5(x - 2y) + z^2(x - 2y)$

b. $6x^2(3y - z) - (3y - z)$

Examples Use grouping to factor the following:

a. $4x^3 - 12xy + x^2y - 3y^2$

b. $xy + 4x - y - 4$

For the next example we will use $12x^2 - 15y + 18x - 10xy$

What is the issue when we try to use grouping?

How can we resolve this issue and still use grouping?

Write the sum and difference of cubes formulas below:

Examples Factor the following:

a. $x^3 + 8$

b. $y^3 - 1$

c. $8x^3 + 27y^3$

Lesson: A Few More Tricks for Factoring

List all the types of factoring that you might have to use:

It is always possible that you will have to combine these methods. There are also a few other tricks you can try, which we will discuss in this lesson.

Using **<u>Grouping</u>** with Trinomials

Write out *instructions* on how to use grouping for $4x^2 + 16x + 15$

Below are a few challenging examples of factoring. Write out any notes of how to work these out that might be helpful to you. Remember, you are expected to use these notes as a resource to help you finish your homework – take the time to write down reminders that will be meaningful and helpful to you!

a. $-x^2 - 15x - 36$

What trick can you use here? Show how to use it.

b. $4y^3 - 28y^2 + 48y$

What trick can you use here? Show how to use it.

c. $-6x^2 + 10x - 3$

d. $x^2 + 7xy + 12y^2$

How should you deal with having the multiple variables?

Finally, let's work with a few trickier examples of the difference of squares. Follow the pattern:

a. $x^2 - 16$

b. $x^2 - 25$

c. $x^2 - \frac{16}{25}$

d. $16 - x^2$

e. $25x^2 - 16y^2$

f. $\frac{16}{25}x^2 - \frac{4}{9}y^2$

Rational Expressions
Video 1: The Basics

What are rational expressions?

What do rational expressions look like? Write down some examples:

Simplifying Rational Expressions
Take down some notes on this:

Examples together:

a. $\frac{25x^7y^9}{15x^3y^2z^7}$

Examples on your own:

b. $\frac{21x^7y^9}{14y^8z^3}$

c. $\frac{16x^4y^7z^3}{15x^3y^4z^8}$

Examples together:

a. $\frac{x^2+5x+6}{x^2-9}$

Examples on your own:

b. $\frac{x^2+7x+10}{x^2+10x+25}$

c. $\frac{x^2-x-6}{x^2-9x+14}$

d. $\frac{x^2+10x+16}{x^3+2x^2+x+2}$

Play the video and check your work.

Examples on your own:

e. $\frac{x^3-8}{x^2-9x+14}$

f. $\frac{10x^3+15x^7}{10x^3+40x^2+40x}$

Rational Expressions
Video 2: Other Things to Know

When it comes to fractions, where are you allowed to have a zero? In the numerator (top) or denominator (bottom)?

Therefore, what do we have to worry about with rational expressions?

Examples on your own:

Evaluate the following rational expressions for $x = 2$.

a. $\frac{x^2+6x+8}{x^2+8x+15}$

b. $\frac{x^2+9x+20}{x^2+3x-10}$

Example on your own: (evaluate for x=2)

c. $\frac{x^2-4}{x^2+9x+14}$

To summarize…

When a rational expression has a zero in the numerator, what does this mean?

When a rational expression has a zero in the denominator, what does this mean?

Define the term **domain:**

What are we concerned with for the domain of rational expressions?

A review of interval notation:

Examples:

Write the following in interval notation:

a.

b.

c.

Example together:

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

a. $\frac{x+3}{x-5}$

b. $\frac{2x+1}{x^2+7x+10}$

Examples on your own:

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

c. $\frac{x-3}{x-1}$

d. $\frac{x^2-81}{x^2+5x+6}$

Rational Expressions
Video 3: The Negative One Trick with Simplifying

In its current form, can you simplify: $\frac{x-3}{3-x}$? Why or why not?

Show how you can manipulate this expression to simplify it.

There is no formal name for this trick. Some might refer to it as a *multiplicative property of -1,* but I refer to as "that -1 trick." I know, it's not exactly formal sounding!

Let's do one more example of this. Simplify $\frac{5-x}{3x-15}$

The first thing to get used to it how to rewrite expressions using this trick.

Examples

Rewrite the order of each expression by factoring out -1 first.

a. $4 - x$

b. $3x - 1$

c. $x - 6$

Now let's use this to simplify rational expressions.

Examples Simplify the following.

a. $\frac{6x-12}{2-x}$

<u>Examples</u> Simplify the following.

b. $\frac{4x-20}{30x-6x^2}$

c. $\frac{x^2-4}{2-x}$

d. $\frac{x-6}{36-x^2}$

e. $\frac{5-x}{x^2-7x+10}$

Examples Simplify the following.

f. $\frac{x^2-10x+25}{25-x^2}$

g. $\frac{27-x^3}{x^2-5x+6}$

h. $\frac{x^3-3x^2+5x-15}{6-3x}$

<u>Examples</u> Simplify the following.

i. $\frac{54-9x}{x^2-36}$

j. $\frac{8-x^3}{x^2-9x+14}$

k. $\frac{y^3-x^3}{x^2-y^2}$

Example/Question: Troubleshooting the -1 Trick

Explain the actual DETAIL behind WHY $\frac{4}{5-x} = -\frac{4}{x-5}$

<u>Examples</u> Write an equivalent fraction for the following:

a. $\frac{5}{x-8}$

b. $\frac{x-3}{x+2}$

Lesson: Multiplying Rational Expressions

How do you multiply the fractions $\frac{a}{b} \times \frac{c}{d}$? Include a one-sentence explanation.

Examples Multiply the following.

a. $\frac{4}{5} \times \frac{7}{9}$

b. $\frac{2x^3}{7y^2} \times \frac{5x^5}{9y^7}$

Show how to make cancellations before multiplying $\frac{12}{25} \times \frac{15}{16}$

We can apply this same logic to rational expressions.

<u>Examples</u> Multiply the following and simplify when possible.

a. $\frac{21x^7}{15y^3} \times \frac{5y^8}{42x^4}$

b. $\frac{2x^2+5x+3}{x^2-25} \times \frac{x^3+125}{4x^2+6x}$

Lesson: Dividing Rational Expressions

How do you divide $\frac{a}{b} \div \frac{c}{d}$? Write a one sentence explanation of how to do this too.

Example Divide the following: $\frac{5x}{9} \div \frac{2}{7x^4}$

When can you make cancellations in the process?

Examples Divide.

a. $\frac{12x^7}{45} \div \frac{42x^3}{15}$

b. $\frac{x^2+4x+4}{x+5} \div (x+2)^5$

c. $\frac{x^3+1}{6x^3-6x^2+6x} \div \frac{x^2-1}{8}$

Explain why $\frac{\frac{5x+5y}{24}}{\frac{15x^2+15xy}{12}}$ is both a fraction and a division problem.

<u>Example</u> Divide: $\frac{\frac{16x^2-25}{x^5}}{\frac{36x-45}{9x^7}}$

Lesson: Finding the Least Common Denominator for Addition and Subtraction – A Quick Review for Fractions (with only numbers)

We are going to work on adding and subtracting rational expressions with different denominators. Before we do that, we are going to remind ourselves how to find common denominators for fractions that only have numbers.

The simplest case: what is the least common denominator between the fractions $\frac{1}{2}$ and $\frac{1}{3}$?

Why is that the common denominator?

Now let's look at something slightly more complicated. What is the least common denominator between $\frac{1}{6}$ and $\frac{1}{10}$?

First, take a guess. It's totally fine if this guess is wrong, but just make a guess.

Why is this your guess? What did you do?

There is a specific procedure that you want to use when finding least common denominators in this class. Write down instructions that were given in the video, but write them our in a way that makes sense to you.

Repeat this procedure to find the least common denominator $\frac{1}{15}$ and $\frac{1}{10}$.

Repeat this procedure to find the least common denominator $\frac{1}{12}$ and $\frac{1}{8}$.

Now we will focus on how to write fractions with this desired denominator.

The simplest case: using the least common denominator you found for $\frac{1}{2}$ and $\frac{1}{3}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Using the least common denominator you found for $\frac{1}{6}$ and $\frac{1}{10}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Using the least common denominator you found for $\frac{1}{12}$ and $\frac{1}{8}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Lesson: Finding the Least Common Denominator for Rational Expressions

The simplest case: what is the least common denominator between the fractions $\frac{1}{2x}$ and $\frac{1}{3y^2}$?

Examples Find the least common denominator between the following fractions.

a. $\frac{1}{2x^2}, \frac{1}{3xy^2}$

b. $\frac{1}{2x^2y^5}, \frac{1}{3x^7y^2}$

c. $\frac{1}{2x^3y^5z^2}, \frac{1}{3x^5y^7z^8}$

<u>Examples</u> Find the least common denominator for the following.

a. $\frac{1}{14x^3y^2}, \frac{1}{21x^2y^7}$

b. $\frac{1}{24x^3y}, \frac{1}{12x^2y^5}$

What is the least common denominator between $\frac{1}{x+2}$ and $\frac{1}{x}$?

What is the least common denominator between $\frac{1}{x^2-4}$ and $\frac{1}{x^2+5x+6}$?

<u>Examples</u> Find the least common denominator between the following fractions.

a. $\frac{3}{5x-10}, \frac{1}{7x-14}$

b. $\frac{1}{x^2-12x+35}, \frac{1}{x^2+x-20}$

Examples Find the least common denominator between the following fractions.

a. $\frac{1}{x-3}, \frac{1}{3-x}$

b. $\frac{1}{x^2-4}, \frac{1}{2-x}$

c. $\frac{1}{x^2+9x+20}, \frac{1}{x^2-25}, \frac{1}{x^2-x-20}$

Lesson: Writing Equivalent Rational Expressions with the Same Denominator

Examples Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{2}{15x^2y^5}, \frac{4}{9x^3y^2}$

b. $\frac{5}{x+3}, \frac{4}{x}$

Examples Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{x}{x^2+9x+18}, \frac{2}{x^2+5x+6}$

b. $\frac{x}{5x^2-30x}, \frac{x-2}{2x^2-17x+30}, \frac{2}{2x^2-5x}$

Lesson: Adding and Subtracting Rational Expressions

Show how to add or subtract the following:

a. $\frac{5}{x+2} - \frac{3}{x+2}$

b. $\frac{5}{x-4} + \frac{7}{x}$

c. $\frac{5}{12xy^2} - \frac{7}{8x^3y}$

d. $\frac{x}{x+3} - \frac{2}{x^2-9}$

e. $\frac{x}{x^2-25} - \frac{5}{x^2-3x-40}$

Example/Question: When Are You Allowed to Cancel with Rational Expressions?

What is the wrong way to cancel $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and explain why you can't cancel like that? How can you tell it's wrong?

What is the difference between $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and $\frac{3(x+1)(x-3)}{2(x-3)(x+1)}$?

How do you simplify $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$?

Lesson: Complex Fractions

First let's review how to simplify a complex fraction we've already discussed. Show how to simplify:

$$\frac{\frac{2x}{x+5}}{\frac{2}{x^2-25}}$$

Explain what makes the problem below different from the one we just reviewed above.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

Outline the steps required to simplify. Write out the instructions as they were explained in the video.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

Example Simplify the following

$$\frac{\frac{7}{9}-\frac{1}{3}}{1+\frac{5}{9}}$$

Examples Simplify the following.

a. $\dfrac{\frac{5}{x}+\frac{2}{y}}{\frac{1}{y}-\frac{3}{x}}$

b. $\dfrac{\frac{6}{x+3}-\frac{4}{x-1}}{\frac{2}{x-1}+\frac{5}{x+3}}$

Example Simplify the following.

$$\frac{1-\frac{4}{t+5}}{\frac{4}{t^2-25}+\frac{t}{t-5}}$$

Complex Fractions
Video 2: More Complicated Examples

Complex fractions look like this:

$$\frac{\frac{4x}{15y^2}}{\frac{6x^4}{25y^8}} \quad \text{or} \quad \frac{\frac{1}{3}-\frac{1}{6}}{\frac{3}{2}+\frac{5}{6}} \quad \text{or} \quad \frac{\frac{4}{x^2}-\frac{4}{y^2}}{\frac{1}{xy}+\frac{5}{y}}$$

First we will discuss the basics.

__Examples__

a. $\dfrac{\frac{1}{x^2}-\frac{1}{y}}{\frac{3}{x}-\frac{4}{y^2}}$

b. $\dfrac{\frac{3}{x}-x}{\frac{2}{x}+x}$

Examples

a. $\dfrac{\frac{3}{x-1}-\frac{2}{x+3}}{\frac{5}{x+3}-\frac{2}{x-1}}$

b. $\dfrac{\frac{5}{x-2}-\frac{3}{x+5}}{\frac{1}{x+5}+\frac{2}{x-2}}$

Examples

a. $\dfrac{1-\frac{1}{x+5}}{\frac{x}{x-5}+\frac{4}{x^2-25}}$

b. $\dfrac{1+\frac{12}{x^2-16}}{\frac{x}{x+4}-\frac{6}{x-4}}$

c. $\dfrac{\frac{1}{x+2}+\frac{4}{x^2-4}}{\frac{3}{x-2}+1}$

Lesson: Solving Rational Equations

How do you know that you need to solve a problem versus simplify it?

Explain how to solve the rational equation. Include some instructions in your notes.

$$\frac{2}{x-1}+\frac{1}{x+4}=\frac{4}{x+4}$$

What do we have to watch out for when solving rational equations?

<u>Examples</u> For the following, which solutions would NOT work for these rational equations?

a. $\frac{4}{x+1} - \frac{2}{x+3} = \frac{5}{x+7}$

b. $\frac{4}{x^2-9} + \frac{3}{x-3} = \frac{2}{x+3}$

Examples Solve.

a. $1-\frac{8}{x}=\frac{10}{x}$

b. $\frac{5}{x-3}-\frac{2}{x^2-9}=\frac{5x+13}{x^2-9}$

c. $\frac{x^2}{2} = \frac{x^2-6x}{3}$

d. $\frac{4}{x+1} = \frac{10}{x^2-1} - \frac{5}{x-1}$

Solving Equations with Rational Expression

We will learn how to solve equations that look like this:

$$z = \frac{x+3y}{4w} \quad \text{or} \quad \frac{1}{a} - \frac{2}{b} = \frac{3}{c}$$

__Examples__

a. Solve for y: $z = \frac{3xy}{5w}$

b. Solve for x: $w = \frac{7+2y}{xz}$

c. Solve for z: $y = \frac{4xw}{z-w}$

Examples

a. Solve for x: $\frac{3}{x} + \frac{1}{y} = \frac{2}{z}$

b. Solve for b: $\frac{1}{a} - \frac{1}{b} = \frac{3}{c}$

c. Solve for z: $\frac{5}{x} - \frac{3}{y} = \frac{7}{z}$

Lesson: Three Ways to Use the LCD with Rational Expressions

We have spent quite a bit of learning how to find the LCD between rational expressions and then using it in very different ways. We are going to compare the 3 techniques for how we used the LCD in this video.

First, simplify completely:

$$\frac{9}{x+5}-\frac{4}{x+1}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

Simplify completely:

$$\frac{\frac{9}{x+5}-\frac{4}{x+1}}{\frac{10}{x^2+6x+5}-\frac{3}{x+1}}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

Solve:

$$\frac{9}{x+5} - \frac{4}{x+1} = \frac{10}{x^2+6x+5}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

Example/Question: When Can You Cross Multiply

Explain what a ratio is:

When can you cross-multiply?

Solve: $\frac{8}{x+2} - \frac{12}{x-4} = \frac{2}{x+2}$. Show that you CANNOT cross-multiply in this situation.

Now show why cross-multiplying works with $-\frac{x}{5} = \frac{3}{x+8}$

Rational Expression Applications
Video 1: Proportion Type Problems

What is a proportion?

A bakery can make 300 cookies in 1.5 hours. How long would it take to make 525 cookies?

The ratio of patrons at a café who purchase coffee versus those who purchase tea is 5 to 3. On Monday, the café sold 45 teas. How many coffees were sold?

A painter makes a particular shade of orange by using 3 parts red and 4 parts yellow. The painter has four more gallons of yellow paint than he has of red paint. How many gallons of red paint should he use?

Rational Expression Applications
Video 2: Distance, Rate, and Time Problems

In these types of problems, we need to use the equation:

$$d = rt$$

Where d=distance, r=rate/speed of a vehicle, and t=time

We can also see that we can rearrange this formula in a few ways:

A boat travels 30 miles downstream in the same time that it takes to travel 20 miles upstream. The current of the river is 5mph. How fast would the boat travel in still water?

A boat travels 16 miles downstream in the same time that it travels 12 miles upstream. In still water, the boat can travel 6mph. What is the speed of the current?

A plane travels 550 miles with the wind. In the same amount of time, it travels 375 miles against the wind. If the wind is blowing at a speed of 25 mph, what is the speed of the plane when there is no wind?

Rational Expression Applications
Video 3: Work Problems

What is the setup for work problems?

It takes Maria 3 hours to paint a room, while it takes her younger sister Lupe 5 hours to paint the same room. How long would it take if the girls painted the room together?

It takes Max 30 minutes to shovel the driveway, while it takes his brother Darrell 45 minutes to do the same job. How long would it take for the guys to do it together?

Variation
Video Notes

The term “variation” refers to a particular way of describing how certain things relate to one another.

There are three types of variation we will look at. Write down the definitions for each, you will remember it better!

1. Direct Variation:

2. Inverse Variation

3. Joint Variation

It’s also possible to combine different types of variation.

The first thing we have to do is learn how to interpret and write variation equations.

Examples Write the variation equation:

a. P varies directly with Q

b. J varies inversely with L

c. H varies jointly with A and B

d. Z varies inversely with the square of R

e. M varies directly as the square root of N

f. Y varies jointly with the cube of M and the square root of L

g. M varies directly as L and inversely as N

Now we can take a look at how to solve general variation equations.

Example Write down the detailed steps / directions for this example.

J varies directly with L. If J = 12 when L = 4, find J when L = 7.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Examples Using the steps outlined from the previous example, try the following:

a. M varies inversely as the square of L. If M = 12 when L = 2, find M when L = 4.

b. A varies jointly with B and C. If A = 30 when B = 2 and C = 5, find A when B = 3 and C = 4.

c. L varies directly as M and inversely as N. If L = 15 when M = 5 and N = 2, find L when M = 7 and N = 3.

Example Jada's weekly income varies directly with the amount of hours that she works. Last week, she earned $570 for working 38 hours. This week she worked 36 hours, how much will she earn?

Absolute Value Equations
Video 1: The Basics

Absolute value equations look like this:

$$|x+2| = 7 \quad \text{or} \quad |3x-5|+4=2$$

To understand how to solve these types of equations, first think about: what are the solutions to $|x| = 5$? Explain why.

From here, we can see that we must consider 2 possible solution sets to find all solutions to absolute value equations. Write down the general strategy for solving absolute value equations:

Solve: $|2x+3| = 9$

Examples Solve the following:

a. $|3x - 9| = 6$

b. $|7 - 4x| = 3$

To solve $|5x - 4| + 2 = 13$, what must we do first?

Solve:

Examples Solve:

a. $|3x-6|+4=10$

b. $\left|\frac{1}{2}x+\frac{1}{3}\right|+5=10$

The next video will discuss a special situation and a more complicated equation type.

Absolute Value Equations
Video 2: More Complicated Situations

In this video, we will discuss how to solve equations like:

$$|x+2| = -7 \quad \text{or} \quad |3x-5| = |2x+7|$$

What can we say about the solutions to $|x| = -5$?

Examples Solve the following:

a. $|2x-7| = -8$

b. $|3x+5| + 7 = 2$

Absolute value equations can also look like this: $|3x+1| = |2x-5|$. Explain how to solve this.

Examples Solve the following:

a. $|5x - 10| = |10x + 15|$

b. $|3x - 5| = |5x + 7|$

c. $|4x - 6| = |3x - 8|$

Example/Question: Why Do You Only Make One Side Negative with Solving $|\boldsymbol{ax+c}| = |\boldsymbol{bx+d}|$ type problems?

What is the approach for solving $|x+1| = 5$?

What is key to notice here?

How would you solve $|x+1| = |5x|$?

How would you solve $|x+1| = |5x-3|$?

A review of number lines, interval notation, and set notation

This is a general review of these concepts. We will look at how to represent solutions for problems like:

$$x > 5 \quad \text{or} \quad -3 \leq x < 4 \quad \text{or} \quad x > 2 \text{ or } x < -3$$

Use the example $x < 4$ to describe the 3 types of inequality notation. Write down any special symbols associated with it.

<u>Number line:</u>

The special signs/symbols are:

Graph $x < 4$ on a number line:

<u>Interval Notation:</u>

The special signs/symbols are:

Write $x < 4$ in interval notation:

<u>Set Notation</u>

The format of this is always:

Write $x < 4$ in set notation:

Examples Write the following inequalities on a number line, in interval notation, and in set notation.

a. $x \geq 5$

b. $x < -2.5$

c. $-4 \leq x < 7$

d. $x > 4$ or $x < -5$

Examples Write the following inequalities on a number line, in interval notation, and in set notation.

a. $x \leq -3$

b. $x > 12$

c. $0 < x \leq 8$

d. $x > 3$ or $x < 1$

Lesson: Absolute Value Inequalities

Absolute value inequalities have 2 key aspects to solving them. What are those 2 key aspects?

Explain the setup to solve an absolute value inequality of the form $|P| \leq k$ (or $< k$).

Explain the setup to solve an absolute value inequality of the form $|P| \geq k$ (or $> k$).

Let's try to understand why these are setup this way.

Explain the solutions to $|x| < 3$. Summarize what was explained in the video.

Explain the solutions to $|x| \geq 3$. Summarize what was explained in the video.

Examples Solve the following inequalities. Graph your solution set on a number line and express your solution in interval notation.

a. $|2x - 4| < 6$

b. $|3x + 6| \geq 9$

c. $|x - 5| - 4 \leq 6$

d. $5 \geq |3x - 4|$

e. $7 < 3 + |x - 4|$

f. $6 \leq 4 + |8 - 2x|$

Lesson: Special Absolute Value Inequalities

We will discuss some special situations. Explain the reasoning for each of these situations:

a. $|2x + 3| + 7 < 4$

b. $|3x - 2| > -3$

c. $|5x + 1| \geq 0$

d. $|2x - 4| < 0$

e. $|6x + 2| \leq 0$

The Basics of Roots
Video 1: Square Roots

What are the square roots of 9? Write down what it means to be a square root.

Examples Find the square roots of the following numbers.

a.

b.

What does the symbol $\sqrt{}$ refer to?

How can we refer to the negative square root?

Draw a square root and label its parts:

Examples Evaluate the following:

a. $\sqrt{25}$

b. $\sqrt{81}$

c. $\sqrt{36}$

d. $\sqrt{\frac{1}{36}}$

e. $\sqrt{\frac{16}{49}}$

f. $\sqrt{\frac{9}{25}}$

g. $\sqrt{-36}$

h. $-\sqrt{64}$

Finally, how would you estimate a root like $\sqrt{5}$?

Example Estimate $\sqrt{14}$

The Basics of Roots
Video 2: Higher Roots

How would you evaluate the following? Explain the rationale for how to evaluate these:

$\sqrt[3]{8}$

$\sqrt[4]{81}$

$\sqrt[5]{32}$

The most general symbol for a root is $\sqrt[n]{a}$. Explain what this symbol means and once again, label it.

Now we can create a list of the most likely roots that we will encounter. You will very likely want to create another list of this on a page you can easily access.

Squares	**Cubes**	**Other**

Examples Evaluate the following. Be careful when you evaluate.

a. $\sqrt[3]{125}$

b. $\sqrt[3]{-125}$

c. $\sqrt[4]{16}$

d. $\sqrt[4]{-16}$

d. $-\sqrt[3]{8}$

e. $-\sqrt[3]{-8}$

f. $\sqrt[5]{-32}$

g. $\sqrt[6]{-64}$

Now let's generalize. For all roots of the form $\sqrt[n]{a}$, what can we say if….

1. The index (n) is an **odd** number?

2. The index (n) is an **even** number and a is a **negative** number?

3. The index (n) is an **even** number and a is a **positive** number?

The Basics of Roots
Video 3: The "Ultimate" Test and A Few Other Notes

To begin, we will first see how well we can put everything we've learned so far about roots together.

Examples Evaluate the following, if possible.

a. $\sqrt[3]{-8}$

b. $\sqrt{-16}$

c. $-\sqrt[4]{16}$

d. $\sqrt[3]{-125}$

e. $-\sqrt{100}$

f. $\sqrt[4]{-81}$

g. $-\sqrt{25}$

h. $-\sqrt[3]{-8}$

i. $-\sqrt[4]{-16}$

j. $-\sqrt[5]{-32}$

And now that you have completed this … it's time for my "ultimate" test … drum roll please:

The "Ultimate" Test for Roots

Evaluate. Note: at this moment in time, we do not know anything about imaginary numbers.

a. $\sqrt{1}$

b. $\sqrt[3]{1}$

c. $\sqrt[5]{1}$

d. $\sqrt[4]{-1}$

e. $\sqrt[3]{-1}$

f. $-\sqrt{1}$

g. $-\sqrt[4]{-1}$

h. $-\sqrt[3]{1}$

i. $-\sqrt{-1}$

j. $-\sqrt[7]{-1}$

k. $-\sqrt[3]{-1}$

l. $\sqrt[8]{-1}$

m. $-\sqrt[10]{-1}$

n. $\sqrt{-1}$

Are you having fun yet???

We have one last thing to discuss: the general rule for evaluating $\sqrt[n]{a^n}$

If n is even ….

If n is odd ….

<u>Examples</u> Evaluate the following:

a. $\sqrt[3]{y^3}$

b. $\sqrt[4]{(x-4)^4}$

c. $\sqrt[7]{(2y-4)^7}$

d. $\sqrt[6]{(3x+2)^6}$

Rational Exponents

Video 1: How to work with rules like $x^{\frac{1}{n}}$ or $x^{\frac{m}{n}}$ or $x^{-\frac{m}{n}}$

In this video, we will learn how to evaluate expressions that look like this:

$8^{\frac{1}{3}}$ or $25^{\frac{3}{2}}$

Define $x^{\frac{1}{n}}$ and show how to use this rule to evaluate $25^{\frac{1}{2}}$

Examples Evaluate the following:

a. $125^{\frac{1}{3}}$

b. $32^{\frac{1}{5}}$

c. $9^{\frac{1}{2}}$

d. $(-9)^{\frac{1}{2}}$

e. $-9^{\frac{1}{2}}$

f. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$

Define $x^{\frac{m}{n}}$ and show how to use this rule to evaluate $25^{\frac{3}{2}}$

Examples Evaluate the following:

a. $27^{\frac{2}{3}}$

b. $4^{\frac{3}{2}}$

c. $16^{\frac{5}{4}}$

d. $-8^{\frac{2}{3}}$

e. $(-8)^{\frac{2}{3}}$

f. $(-16)^{\frac{3}{4}}$

g. $-81^{\frac{3}{4}}$

h. $\left(\frac{125}{27}\right)^{\frac{2}{3}}$

Define $x^{-\frac{m}{n}}$ and show how to use this rule to evaluate $25^{-\frac{3}{2}}$

Examples Evaluate the following:

a. $25^{-\frac{1}{2}}$

b. $27^{-\frac{1}{3}}$

c. $16^{-\frac{3}{4}}$

d. $4^{-\frac{5}{2}}$

e. $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$

f. $(-25)^{-\frac{3}{2}}$

g. $(-27)^{-\frac{2}{3}}$

h. $-16^{-\frac{5}{4}}$

Lesson: Using Exponent Rules with Rational Exponents

First, let's review the main exponent rules:

Next, let's do a quick review on operations with fractions. Complete the following operations:

a. $\frac{2}{3} \times \frac{5}{6}$

b. $\frac{2}{5} \times 10$

c. $\frac{2}{3} + \frac{2}{5}$

Now we will combine all of these ideas into the following examples!

Examples Simplify the following. Your final answer should not include any negative exponents.

a. $x^{\frac{2}{3}} * x^{\frac{1}{5}}$

b. $\left(4x^{\frac{2}{3}}y^{6}\right)^{\frac{3}{2}}$

c. $\dfrac{x^{\frac{2}{3}}y^{-\frac{1}{5}}}{x^{\frac{1}{6}}y^{\frac{2}{5}}}$

d. $\left(\frac{125x^4y^{-\frac{1}{3}}}{x^{-\frac{1}{2}}y^{\frac{2}{5}}}\right)^{-\frac{2}{3}}$

e. $\left(\frac{4x^6y^{-\frac{1}{3}}}{x^{-3}y^{\frac{1}{2}}}\right)^{-\frac{3}{2}}$

Lesson: Simplifying Radicals with Rational Exponents

Convert the following radicals to rational exponents, simplify, and then rewrite in radical form.

a. $\sqrt{7^2}$

b. $\sqrt{9^2}$

c. $\sqrt[3]{x^{15}}$

d. $\sqrt[4]{x^2}$

Rational Exponents
Video 3: Convert a radical expression to exponential form

Examples Convert the following expressions to exponential form, and then simplify if needed.

a. $\sqrt{7}$

b. $\sqrt[3]{8}$

c. $\sqrt{5^2}$

d. $\sqrt[3]{7^6}$

e. $\sqrt[4]{x^{16}}$

f. $\sqrt[6]{x^2}$

Simplifying Radicals
The Basics

We will look at how to simplify radicals like this:

$$\sqrt{50} = 5\sqrt{2} \quad \text{or} \quad \sqrt[3]{250} = 5\sqrt[3]{3}$$

You might want to have the list of squares / cubes / quartics ready for this exercise. If you don't, pause the video and write them all down on a separate sheet of paper so that you can have it handy for this entire lesson.

First we look at **the multiplication property of radicals:**

__**Examples**__ Multiply the following:

a. $\sqrt[3]{2} * \sqrt[3]{3}$

b. $\sqrt{7} * \sqrt{5}$

c. $\sqrt[3]{3} * \sqrt[3]{9}$

d. $\sqrt[3]{6} * \sqrt[3]{9}$

How are examples c and d related?

Being able to multiply radicals together is powerful, because it also allows us to break apart radicals and simplify them. Using the example $\sqrt{12}$, write down how to simplify:

<u>Examples</u> Simplify the following:

a. $\sqrt{50}$

b. $\sqrt{20}$

c. $\sqrt[3]{54}$

d. $\sqrt[4]{32}$

e. $\sqrt[3]{24}$

At this point, we should probably define when a radical is actually simplified.

A simplified radical will ……

<u>Examples</u> Explain why the following radicals are not simplified:

a. $\sqrt{24}$

b. $\frac{5}{\sqrt{7}}$

c. $\sqrt[4]{\frac{3}{5}}$

d. $\sqrt[3]{8x^8y^2}$

Examples Simplify the following:

a. $\sqrt{24}$

b. $\sqrt[3]{32}$

c. $\sqrt[4]{64}$

d. $\sqrt[5]{64}$

e. $\sqrt{250}$

f. $\sqrt[3]{250}$

Lesson: Simplifying Radicals with Variables

We are going to rely on a skill we've already learned to help with simplifying radicals.

<u>Examples</u> Show you can simplify the following utilizing rational exponents.

a. $\sqrt{x^6}$

b. $\sqrt[3]{x^{15}}$

c. $\sqrt{x^6 y^8}$

d. $\sqrt[4]{x^{12} y^{20}}$

How can we simplify $\sqrt{x^5}$? Watch the 2 explanations in the video and summarize the explanation that makes more sense to you.

Example Simplify the following.

a. $\sqrt[3]{x^{13}}$

b. $\sqrt[4]{x^{22}}$

c. $\sqrt{12x^6y^7}$

d. $\sqrt[3]{250x^8y^{16}}$

e. $\sqrt[4]{\frac{32x^7}{y^8}}$

Lesson: Multiplying Radicals

Write a one-sentence explanation of how to multiply $\sqrt[3]{3} * \sqrt[3]{4}$

<u>Examples</u> Multiply the following. Simplify if possible.

a. $\sqrt[3]{6} * \sqrt[3]{4}$

b. $\sqrt[4]{4x^3} * \sqrt[4]{12x^2}$

c. $\sqrt{5x} * \sqrt{10xy}$

Simplifying Radicals
Multiplying and Dividing Radicals with Different Indices

We will look at how to perform operations like:

$$\sqrt{x} * \sqrt[3]{x} \qquad \text{or} \qquad \frac{\sqrt[4]{x}}{\sqrt[3]{x}}$$

Before we begin, you should be comfortable with doing problems like this:

$8^{\frac{1}{3}}$

$25^{\frac{3}{2}}$

Write down the approach for performing this operation: $\sqrt{x} * \sqrt[3]{x}$

Examples Perform the following operations:

a. $\sqrt[3]{x} * \sqrt[4]{x}$

b. $\frac{\sqrt[5]{x^2}}{\sqrt[3]{x}}$

<u>Examples</u> Perform the following operations:

a. $\sqrt[4]{x^3} * \sqrt[5]{x^3}$

b. $\frac{\sqrt{x}}{\sqrt[3]{x^2}}$

c. $\sqrt[3]{x^2} * \sqrt{x}$

Adding and Subtracting Radicals

In this video we will look at how to perform operations like:

$$\sqrt{3} + \sqrt[3]{5} - 5\sqrt{3} + 7\sqrt[3]{3} \qquad \text{or} \qquad \sqrt{24x^4y^3} - 3x^2y\sqrt{6y}$$

Before you begin, you should already be able to simplify radicals like this:

a. $\sqrt{24x^6y^9}$

b. $\sqrt[4]{16x^7y^{11}}$

What are like terms?

Add the like terms in this problem: $3x + 7y - 9x + 7y^2 - 2x - 2y$

What are like radicals?

Add the like radicals in this problem: $5\sqrt{3} - 2\sqrt[3]{3} + 8\sqrt[3]{3} - 2\sqrt{5} + 8\sqrt{3}$

Examples Perform the following operations:

a. $4\sqrt{7} - 5\sqrt[3]{3} + 2\sqrt{3} - 3\sqrt[3]{7} - 9\sqrt{3} + \sqrt[3]{3}$

b. $2x\sqrt{7} - 3\sqrt{7} + 8x\sqrt{7} - 2\sqrt{7}$

c. $3x\sqrt[3]{5} + 2x\sqrt[3]{4} - 3\sqrt[3]{4} + 5x\sqrt[3]{4} - 7x\sqrt[3]{5} + 5\sqrt[3]{4}$

It is possible that you might have to simplify a radical before you can add it. Note how this works with the example:

$$\sqrt{24} + 2\sqrt{54}$$

Examples Perform the following operations:

a. $5\sqrt{8} - \sqrt{18}$

b. $4\sqrt[3]{54} - 7\sqrt[3]{16}$

Examples Perform the following operations:

a. $\sqrt{8x^2y^4} - 9xy^2\sqrt{2}$

b. $4x\sqrt[3]{16x^6y^7} + 9x^3y\sqrt[3]{250y^4}$

c. $5x^2y^3\sqrt[4]{32x^7z^{10}} - 2x^3z^2\sqrt[4]{2x^3y^{12}z^2}$

d. $3x\sqrt[3]{24x^5y^7} + 3xy\sqrt[3]{3x^2y} + y^2\sqrt[3]{375x^8y}$

Lesson: Multiplying Radicals Expressions

First, let's review how to multiply radicals in general. Show how to multiply:

a. $\sqrt[3]{4x^2y} * \sqrt[3]{6x^2y}$

b. $\sqrt{6xy} * \sqrt{10y}$

Now we will apply that logic to larger rational expressions.

Write a one-sentence explanation of how to multiply: $\sqrt{2}(\sqrt{3} + \sqrt{10})$, then multiply.

<u>Example</u> Multiply $2\sqrt[3]{3}(\sqrt[3]{9} + 5\sqrt[3]{6})$

Example Multiply $3x\sqrt{5}\left(\sqrt{10x} - 5x\sqrt{2}\right)$

Write out an explanation of how to multiply: $\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{6} + \sqrt{3}\right)$

Example Multiply: $\left(3\sqrt{5} - \sqrt{2}\right)\left(\sqrt{10} + 4\sqrt{2}\right)$

<u>Example</u> Multiply: $\left(3\sqrt[3]{4}-\sqrt[3]{6}\right)\left(5\sqrt[3]{2}+\sqrt[3]{6}\right)$

How do you multiply $\left(\sqrt{2}+\sqrt{3}\right)^2$

<u>Examples</u> Multiply: $\left(\sqrt[3]{9}+\sqrt[3]{6}\right)^2$

Lesson: Rationalizing a Denominator with One Square Root

What does it mean to rationalize the denominator?

How would you rationalize $\frac{2}{\sqrt{3}}$? Explain *why* we can do this.

Can every fraction with a radical in the denominator be rationalized like this? Circle: Yes No

What types of problems can be rationalized this way?

<u>Examples</u> Rationalize the following.

a. $\frac{20}{\sqrt{8}}$

b. $\sqrt{\frac{40}{60}}$

c. $\sqrt{\frac{81x^7}{2y}}$

d. $\frac{\sqrt{22}}{\sqrt{x^9}}$

Dividing Radicals/Rationalizing the Denominator
Part 2: Higher Roots

In this video we will look at how to simplify expressions like:

$$\frac{5}{\sqrt[3]{3}} \quad \text{or} \quad \frac{\sqrt[4]{2}}{\sqrt[4]{9}}$$

Before you begin, you should already be able to simplify problems like this:

a. $\frac{5}{\sqrt{3}}$

b. $\frac{\sqrt{6}}{\sqrt{18}}$

First, let's observe why higher roots are a different situation. Write down what happens if we use the same methods as above to rationalize the following: $\frac{5}{\sqrt[3]{3}}$

We have to pivot our thinking to "count powers". Note how this works with this examples: $\sqrt[3]{3} * \_\_\_ = 3$

Examples Determine what to fill in the blank to create a true statement:

a. $\sqrt[3]{25} * ? = 5$

b. $\sqrt[4]{2} * ? = 2$

c. $\sqrt[4]{27} * ? = 3$

d. $\sqrt[5]{4} * ? = 2$

e. $\sqrt[3]{7} * ? = 7$

f. $\sqrt[4]{25} * ? = 5$

g. $\sqrt[5]{3} * ? = 3$

h. $\sqrt[3]{36} * ? = 6$

Now you should have a sense of what number you want this to equal. This is a very strange exercise, but using the pattern above, determine what to multiply by to get to the desired root:

a. $\sqrt[3]{4} * ? =$

b. $\sqrt[4]{5} * ? =$

c. $\sqrt[5]{9} * ? =$

d. $\sqrt[4]{36} * ? =$

Now we should have a decent sense of how to rationalize and what the answer should equal. Explain how to rationalize the following:

a. $\frac{5}{\sqrt[3]{2}}$

b. $\frac{7}{\sqrt[4]{5}}$

<u>Examples</u> Rationalize the following denominators:

a. $\frac{6}{\sqrt[3]{5}}$

b. $\frac{2}{\sqrt[4]{9}}$

c. $\frac{2}{\sqrt[5]{125}}$

d. $\frac{7}{\sqrt[4]{8}}$

Examples Rationalize the following denominators:

a. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

b. $\frac{2\sqrt[3]{3}}{\sqrt[3]{4}}$

c. $\frac{5\sqrt[4]{2}}{\sqrt[4]{27}}$

d. $\frac{6\sqrt[3]{5}}{\sqrt[3]{3}}$

e. $\frac{3}{\sqrt[5]{x^3}}$

f. $\frac{\sqrt[5]{25}}{\sqrt[5]{16}}$

Lesson: Rationalizing a Denominator with Two Square Roots

How do you find the conjugate of $4+\sqrt{2}$? Write one sentence to explain what you need to do.

<u>Examples</u> What is the conjugate of the following?

a. $3-\sqrt{5}$

b. $-4-\sqrt{2}$

Show how to rationalize: $\frac{5}{\sqrt{2}-\sqrt{3}}$

Examples Rationalize the following.

a. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}}$

b. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{5}}$

c. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Solving Equations with Radicals
Part 1: One Square Root in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{3x+1} = 2 \qquad \text{or} \qquad \sqrt{4x-2} = 5 - x$$

Use the example $\sqrt{3x+1} = 4$ to show the general procedure for solving these types of equations:

Why must we check the solutions?

Example Solve $\sqrt{3x+1} = 8$

Examples Solve

a. $\sqrt{5x-4}+2=8$

b. $\sqrt{1+3x}+5=2$

Explain what is different about solving $\sqrt{5x+4}=x+2$

Examples Solve the following.

a. $\sqrt{3x-6}=x-2$

b. $\sqrt{2x+4}=x-2$

c. $\sqrt{5x+1}+5=3x$

Solving Equations with Radicals
Part 2: Two Square Roots in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{4x-2}=3\sqrt{2x+6} \qquad \text{or} \qquad \sqrt{3x-1}+2=\sqrt{4x-2}$$

Let’s begin by explaining how to solve: $\sqrt{5x+2}=\sqrt{3x+6}$

Examples Solve:

a. $\sqrt{3-2x}=\sqrt{2x+7}$

b. $\sqrt{5+3x}=\sqrt{3+2x}$

c. $4\sqrt{3-2x}=2\sqrt{1+3x}$

<u>Example</u> Solve: $6\sqrt{x} = 4\sqrt{2x+1}$

Now write out the detailed explanation of how to solve $\sqrt{4x+1} - 2 = \sqrt{2x+1}$

Examples Solve:
a. $\sqrt{3x+6} = 2 + \sqrt{2x-4}$

b. $\sqrt{3x+1} = 2 + \sqrt{2x-2}$

Examples Solve:

a. $\sqrt{2x+4}+\sqrt{1-2x}=3$

b. $\sqrt{2x+4}=2+\sqrt{2x+3}$

Solving Equations with Radicals
Part 3: Two Cube Roots of Higher in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt[3]{3x+1} = 2 \qquad \text{or} \qquad \sqrt[4]{2x-1} = 5$$

Explain how to solve: $\sqrt[3]{3x+1} = 2$

Explain how to solve: $\sqrt[4]{3x+4} = 2$

When is it more likely a solution will not work?

Examples Solve the following:

a. $\sqrt[3]{4x+7}=3$

b. $\sqrt[3]{3-5x}=2$

c. $\sqrt[4]{3-2x}=3$

d. $\sqrt[4]{4x-1}=-7$

How to Evaluate Square Roots that Have Imaginary Number Answers and Dealing with Negatives in Square Roots

In this video, we will look at how to evaluate things like this:

$\sqrt{-4}$ or $\sqrt{-12}$ or $\sqrt{-3} * \sqrt{-6}$

What is an imaginary number?

What is a complex number?

How do you evaluate something like $\sqrt{-9}$?

Examples Compute the following:

a. $\sqrt{-25}$

b. $\sqrt{-81}$

c. $\sqrt{-12}$

d. $\sqrt{-7}$

e. $\sqrt{-\frac{25}{16}}$

f. $\sqrt[3]{-8}$

g. $\sqrt[4]{-16}$

h. $\sqrt{-32}$

How do you evaluate problems like $\sqrt{-3} * \sqrt{-2}$?

Examples Evaluate the following:

a. $\sqrt{-6} * \sqrt{-5}$

b. $\sqrt{-6} * \sqrt{-2}$

c. $\sqrt{-10} * \sqrt{-15}$

d. $\frac{\sqrt{-6}}{\sqrt{-2}}$

e. $\frac{\sqrt{-50}}{\sqrt{-2}}$

f. $\frac{\sqrt{-7}}{\sqrt{-2}}$

Operations with Imaginary Numbers

In this video, we will look at how to evaluate things like this:

$2i(3i-5)$ or $(4i+1)-(5i+3)$ or $\frac{4}{3+2i}$

Adding and Subtracting with Imaginary Numbers:

Use the example $(2i+3)+(5i-6)$ to show how to add or subtract with imaginary numbers.

<u>Examples</u> Simplify:

a. $(3i-2)-(4i-9)$

b. $(5i+3)+(2i-1)$

What can we say about i^2?

With that in mind, explain what is unique about multiplying: $2i(3i + 1)$

Examples Simplify:
a. $4(2 - 3i)$

b. $6i(3i + 2)$

c. $5i(2 - 8i)$

d. $(5i + 1)(2i - 3)$

e. $(4i - 3)(6i + 7)$

Examples Simplify:
a. $(5i+2)^2$

b. $(3i-5)^2$

Why do you need to rationalize $\frac{3}{2i+1}$? Explain how to do this.

Examples Rationalize the following:
a. $\frac{4}{2i+6}$

b. $\frac{7}{i+3}$

<u>Examples</u> Rationalize the following:

a. $\frac{3i+1}{5i-2}$

b. $\frac{7i+3}{2i+1}$

c. $\frac{i-3}{5i+3}$

Evaluating Higher Powers of Imaginary Numbers

In this video, we will look at how to evaluate things like this:

i^{21} or i^{54}

First we need to investigate the powers of i and the patterns with it. Write this down below:

Now that we can see a pattern, explain how to evaluate the following:

i^{15}

i^{22}

Examples Evaluate the following:

a. i^{36}

b. i^{43}

c. i^{52}

d. i^{33}

e. i^{25}

Solving Quadratic Equations with Factoring

In this video, we will look at how to solve equations by factoring, which looks like this:

$$x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0$$

Quadratic equations are equations of the form:

Where a, b, and c are real numbers and $a \neq 0$.

How would you solve the quadratic equation $x^2 + 7x + 6 = 0$ by factoring?

<u>Examples</u> Solve the following by factoring:

a. $x^2 - x - 12 = 0$

b. $x^2 + 9 = -6x$

c. $x^2 = 25$

Examples Solve the following by factoring:

a. $x^2 + 7x = 0$

b. $\frac{1}{10}x^2 + \frac{1}{2}x + \frac{3}{5} = 0$

c. $2x^3 + 28x^2 + 90x = 0$

Solving Quadratic Equations using the Square Root Property

In this video, we will look at how to solve equations by using the square root property, which looks like this:

$$(x+2)^2 = 9 \quad \rightarrow \quad x+2 = \pm 3$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

What are the solutions to $x^2 = 25$?

In general, how do you solve equations of the form $Q^2 = k$, where Q is an expression and k is a constant? In other words, what is the **square root property**?

<u>Examples</u> Solve the following equations using the square root property:
a. $(x+3)^2 = 25$

b. $(x+3)^2 = 12$

Examples Solve the following equations using the square root property:

a. $(5x-2)^2+5=29$

b. $(6x+7)^2+4=-41$

Solving Quadratic Equations by Completing the Square

In this video, we will look at how to solve equations by factoring, which looks like this:

$$x^2 + 6x = 7 \qquad b = 6,\ \ \frac{1}{2}b = 3,\ \ \left(\frac{1}{2}b\right)^2 = 9$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

Before we begin, you should be familiar with the square root property. Solve $(2x + 1)^2 = 49$ using the square root property.

First let's talk about how to complete the square. Show how to complete the square on $x^2 + 6x$ below:

Examples Complete the square on the following:

a. $x^2 + 8x$

b. $x^2 - 10x$

c. $x^2 + 3x$

d. $x^2 - \frac{1}{2}x$

e. $x^2 + \frac{2}{3}x$

Show how to solve $2x^2 + 12x - 16 = 0$ by completing the square:

Examples Solve the following by completing the square:
a. $x^2 + 4x - 6 = 0$

b. $x^2 + 5 = 2x$

<u>Examples</u> Solve the following by completing the square:

a. $3x^2 + 15x = -6$

b. $\frac{1}{2}x^2 + 3x - 2 = 0$

Examples: The Distance Formula

Find the distance between the following sets of points.

a. (6, 2) and (2, 11)

b. (3, -1) and (-4, -2)

Solving Quadratic Equations by the Quadratic Formula

Complete the square on: $ax^2 + bx + c = 0$

State the quadratic formula:

Examples Use the quadratic formula to solve the following:

a. $x^2 + 3x + 2 = 0$

b. $2x^2 + 3x = 5$

c. $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{5}{6} = 0$

Lesson: The Discriminant

Restate the Quadratic Formula, and then label the discriminant.

Value of the Discriminant	Interpretation

Examples Find the values of the discriminant for the following equations. Then solve the equations (using the most efficient method) to determine if this makes sense.

a. $x^2 - 4x + 4 = 0$

b. $x^2 + 6x + 5 = 0$

c. $3x^2 + 2x + 5 = 0$

Choosing the Best Method to Solve a Quadratic

There are 4 ways we can solve quadratics:

1.	3.
2.	4.

Using each of these methods only once, determine the best way to solve these quadratics:
a. $x^2 + 6x = 2$
b. $x^2 + 3 = 2x$
c. $(2x + 1)^2 = 49$
d. $3x^2 + 2 = 3x$

Examples Using each of these methods only once, determine the best way to solve these quadratics:
a. $(3x - 2)^2 = -24$
b. $2x^2 + 5x - 1 = 0$
c. $x^2 - 12 = x$
d. $x^2 - 2x + 5 = 0$

Solving Equations that End Up Having Quadratics

In this lesson, we will review some techniques from previous lessons.

Solve $\frac{2}{x} - \frac{1}{x+1} = \frac{2}{3}$

Solve $\sqrt{2x+8} = x + 4$

Examples Solve the following:

a. $\frac{4}{x-2}+\frac{x}{x-3}=\frac{2}{x-3}$

b. $\sqrt{3x+5}=x-3$

Solving Equations that Are Similar to Quadratics

In this lesson, we will look at how to solve equations that look like this:

$$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 12 = 0 \qquad \text{or} \qquad (2x+3)^2 + 5(2x+3) + 6 = 0$$

Before we begin, what do the following equations have in common if you **look at their exponents**?

a. $x^2 + 5x + 6 = 0$

b. $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} + 6 = 0$

c. $x + 5x^{\frac{1}{2}} + 6 = 0$

d. $x^4 + 5x^2 + 6 = 0$

Explain how to solve $x^4 - 13x^2 + 36 = 0$

Examples Solve the following:

a. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

b. $x - 2x^{\frac{1}{2}} - 8 = 0$

d. $x^{-2} + 8x^{-1} - 9 = 0$

Examples Solve the following:
a. $(x+3)^2-4(x+3)-5=0$

b. $(2x+1)^2-4(2x+1)-21=0$

Solving Formulas with Squares and Square Roots

In this lesson, we will look at how to solve for formulas that look like this:

$$y = 3x^2 \qquad \text{or} \qquad M = \sqrt{\frac{AB}{C}} \qquad \text{or} \qquad kx^2 + cx - n = 0$$

Solving Formulas with Squares

Solve the following for the indicated letters:

a. for x: $y = \frac{4x^2}{z}$

b. for k: $b = \frac{3}{k^2}$

Solving Formulas with Square Roots

Solve the following for the indicated letters:

a. for b: $a = \sqrt{\frac{b}{c}}$

b. for z: $x = \sqrt{\frac{3y}{z}}$

Solving Formulas in other forms

Solve the following for the indicated letters:

a. for x: $kx^2 + cx - d = 0$

b. for b: $ab^2 - 5b + m = 0$

c. for k: $\frac{1}{2}k^2 - 5nk + m = 0$

Solving Applications of Quadratic Equations

An object is thrown upwards off a cliff that is 48 feet above the ground. The height of the object t seconds after the object is released is given by:

$$h = -16t^2 + 32t + 48$$

How long does it take the object to hit the ground?

How long does it take the object to reach a height of 20 feet?

A 12 ft by 16 ft pond is going to be surround by a border of grass of uniform width all the way around. If the area of the enclosure must be 480 feet squared, what is the width of the grass border?

Lesson: An Intro to Functions

Define a relation:

Define a domain:

Define a range:

Example In the set of ordered pairs $\{(2,3), (4,-1), (7,2)\}$, define the domain and range of the relation.

Define a function:

What is the visual explanation that explains what a function is or is not?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, is this a function?

Example Are the following functions?

a. $\{(4,-1),(3,2),(7,-1)\}$

b. $\{(2,4),(3,5),(2,-1)\}$

What is the notation for a function? How do we describe/use this notation?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, what is $f(4)$?

Example Suppose $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find the value of the following:

a. $f(2)$

b. $g(1)$

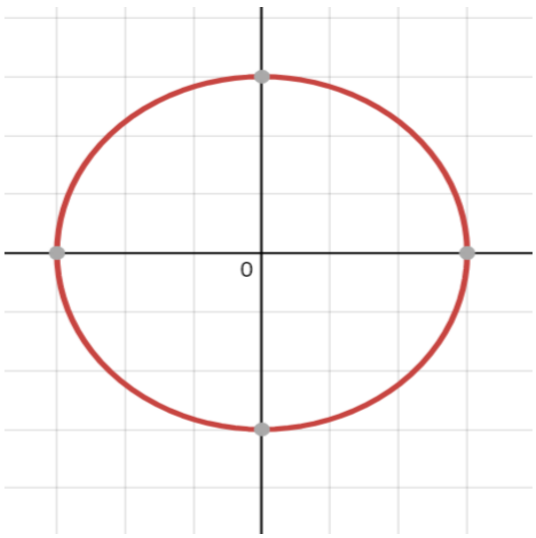
c. $f(a)$

d. $g(h + k)$

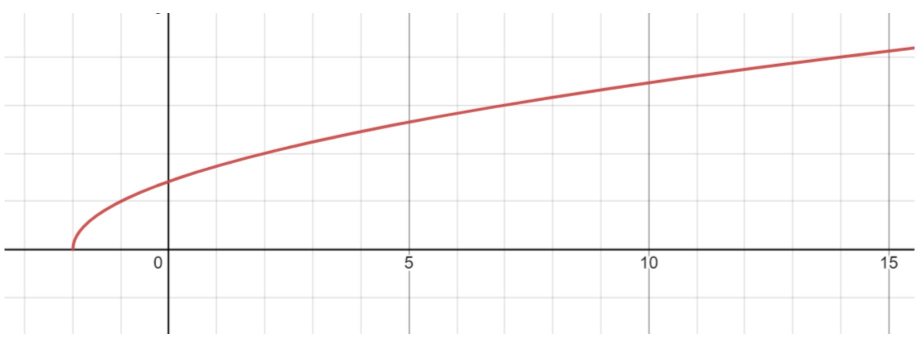
What is the vertical line test?

<u>Example</u> Do the following graphs represent a function?

a.



b.



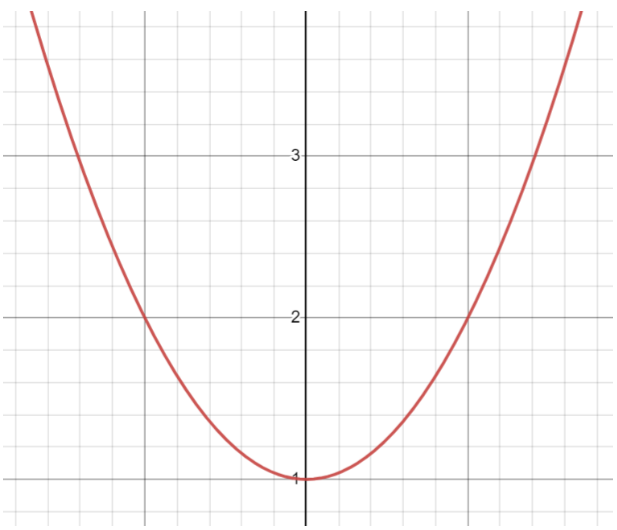
Lesson: Determining the Domain and Range of Functions In General

How do you determine the domain and range of a set of points like $\{(2,4),(5,8),(4,6)\}$?

What is the domain and range of that set of points?

In general, it is easiest to determine the domain and range when we have the graph in front of us.

For the following graph, write out how you should determine the domain and range.



Write a sentence on how you should find the domain:

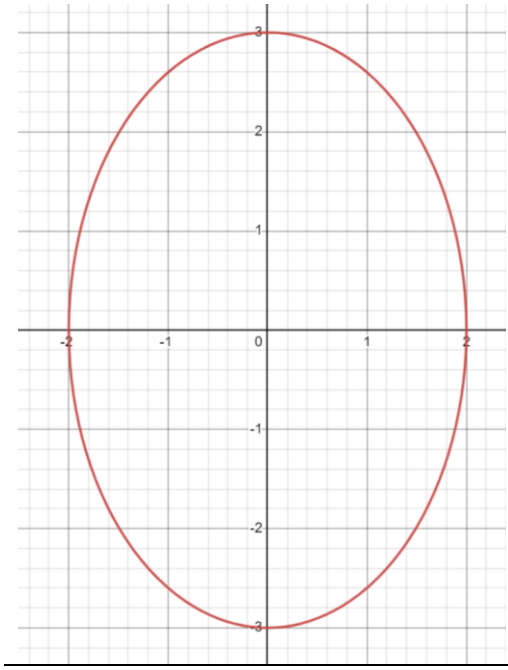
State the domain:

Write a sentence on how you should find the range:

State the range:

Examples Determine if the following graphs represent functions, and then state the domain and range.

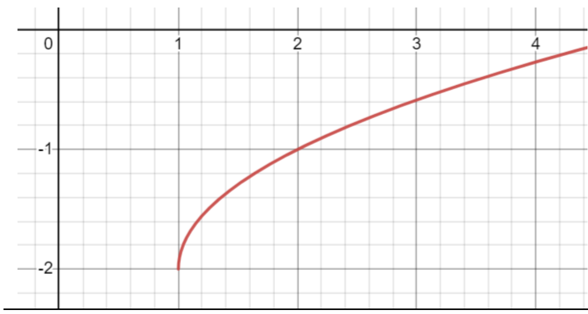
a.



Domain:

Range:

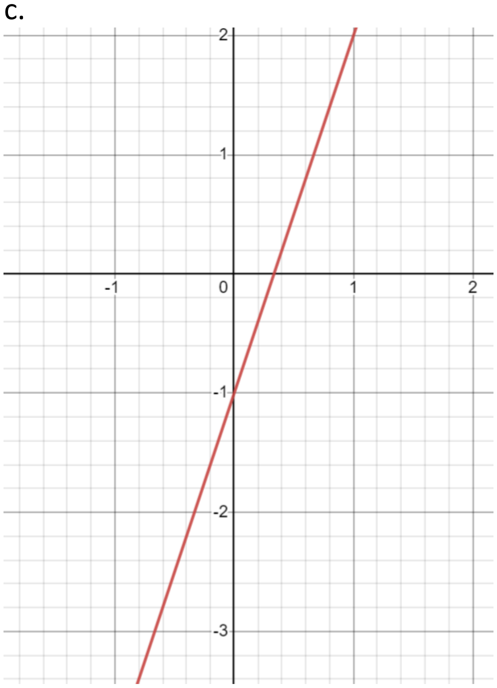
b.



Domain:

Range:

c.



Domain:

Range

Lesson: The Domain of Some Specific Types of Functions

What is a polynomial? Write some examples.

What is the domain of any polynomial function?

What is a rational function?

What is the domain of any rational function?

Examples Determine the domain of the following.

a. $f(x) = 3x - 1$

b. $g(x) = \frac{4}{x+2}$

c. $h(x) = 3x^3 - 2x^2 + 4x - 1$

d. $f(x) = \frac{2x+1}{x^2-9}$

What is a square root function?

What is the domain of a square root function?

Examples Find the domain of the following:

a. $f(x) = \sqrt{x-2}$

b. $g(x) = \sqrt{4x-1}$

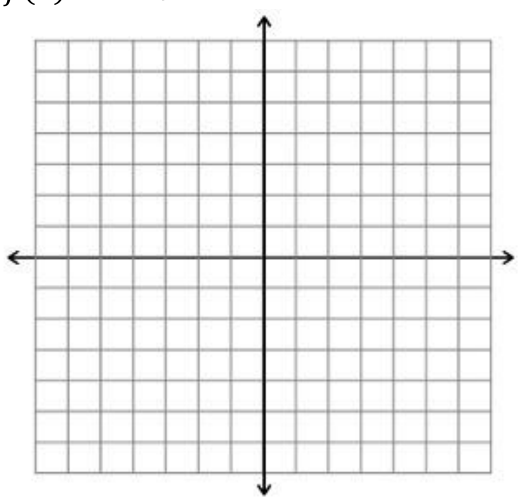
c. $h(x) = \sqrt{4-x}$

Lesson: How to Graph a Function

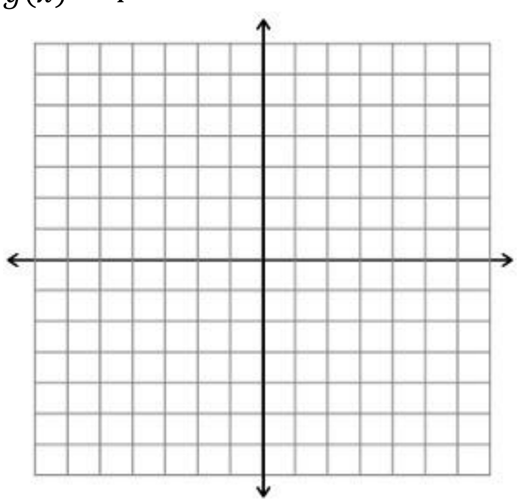
How do you graph a function?

<u>Examples</u> Graph the following functions.

a. $f(x) = 3x + 1$



b. $g(x) = 4$



Lesson: A Few Basic Graphs

There are a few graphs that are helpful to immediately know as we move forward. Let’s break these down.

The Parabola: $f(x) = x^2$

Create table of relevant points and then sketch your own graph.

Absolute Value: $f(x) = |x|$

Create table of relevant points and then sketch your own graph.

X cubed: $f(x) = x^3$

Create table of relevant points and then sketch your own graph.

Square Root: $f(x) = \sqrt{x}$

Create table of relevant points and then sketch your own graph.

Cube Root: $f(x) = \sqrt[3]{x}$

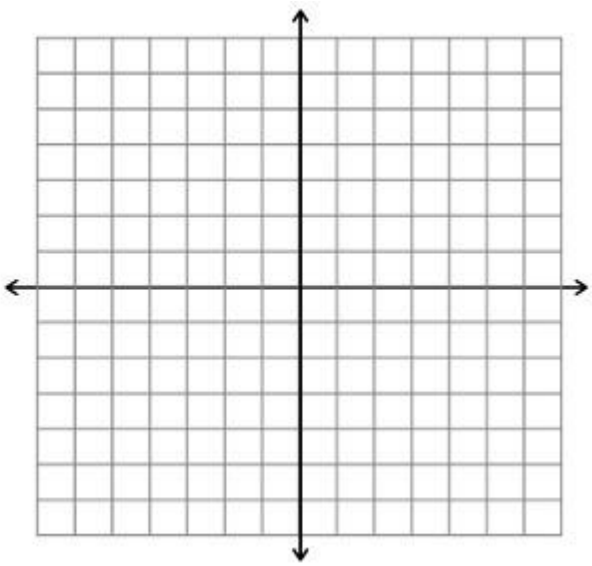
Create table of relevant points and then sketch your own graph.

Lesson: An Intro to Translations and Reflections of Graphs

We are going to develop the intuition for this concept before we actually graph anything.

Vertical Translations

Draw the base graph of $f(x) = x^2$ in one color and then use some other colors to show how the translations work. Label the graphs.

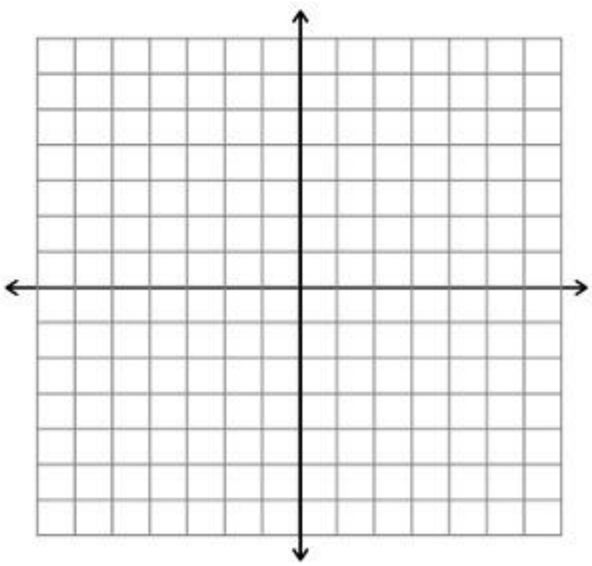


Explain the general way that vertical shifts work:

<u>Example</u> How does $f(x) = \sqrt{x} + 6$ shift?

Horizontal Translations

Draw the base graph of $f(x) = |x|$ in one color and then use some other colors to show how the translations work. Label the graphs.

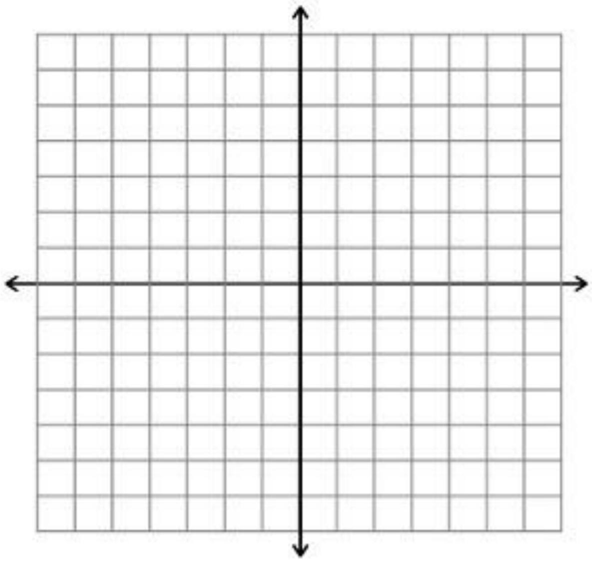


Explain the general way that horizontal shifts work:

Example How does $f(x) = \sqrt[3]{x+3}$ shift?

Reflections Over the X-Axis

Draw the base graph of $f(x) = \sqrt{x}$ and then graph the reflection in another color. Label the graph of the reflection. Label the graphs.



Explain the general way that reflections over the x-axis work:

<u>Example</u> How does $f(x) = -x^2$ look?

Example Describe the translations and reflections of the following.

a. $f(x) = -x^2 + 3$

b. $g(x) = \sqrt{x+2} - 1$

c. $h(x) = (x-5)^3 + 2$

d. $f(x) = -|x+2| + 5$

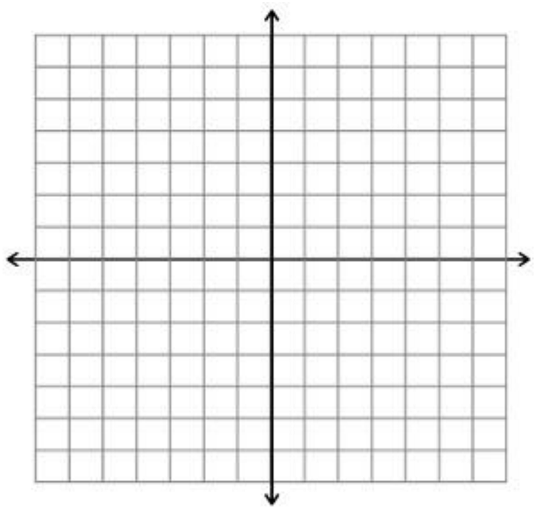
e. $g(x) = -\sqrt[3]{x-1} - 4$

Lesson: Graphing with Translations and Reflections

<u>Examples</u> For each of the following, describe the translations and reflections. Then graph with 2-3 points.

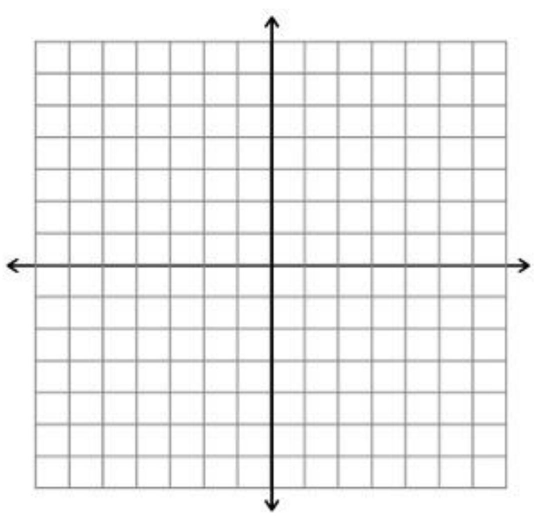
a. $f(x) = x^2 + 1$

Describe any translations or reflections:

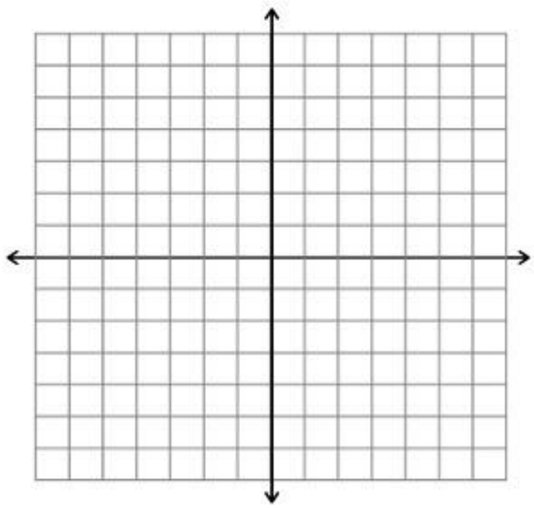


b. $g(x) = \sqrt{x - 2}$

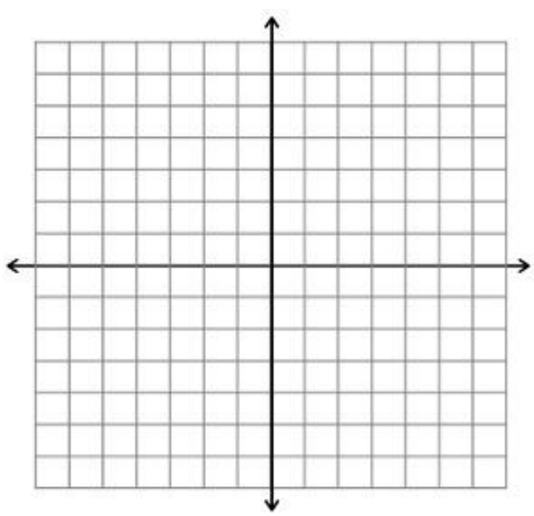
Describe any translations or reflections:



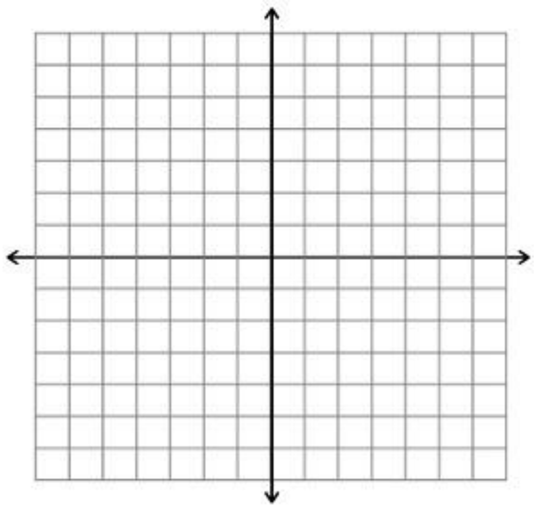
c. $f(x) = (x+2)^3 - 1$
Describe any translations or reflections:



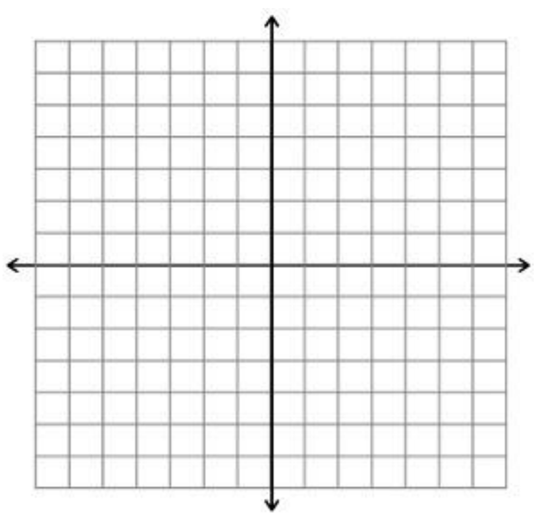
d. $h(x) = |x-3| + 2$
Describe any translations or reflections:



e. $f(x) = -|x - 3| - 2$
Describe any translations or reflections:



f. $f(x) = -\sqrt{x + 3} + 1$
Describe any translations or reflections:



Lesson: Writing Equations of Functions with Translations and Reflections

Example Write the equation of a function $g(x)$ where the following transformations are performed on the graph of $f(x)$

a. $f(x) = \sqrt[3]{x}$ is shifted 3 units down and 2 units right

b. $f(x) = x^2$ is shifted 2.5 units up and π units left

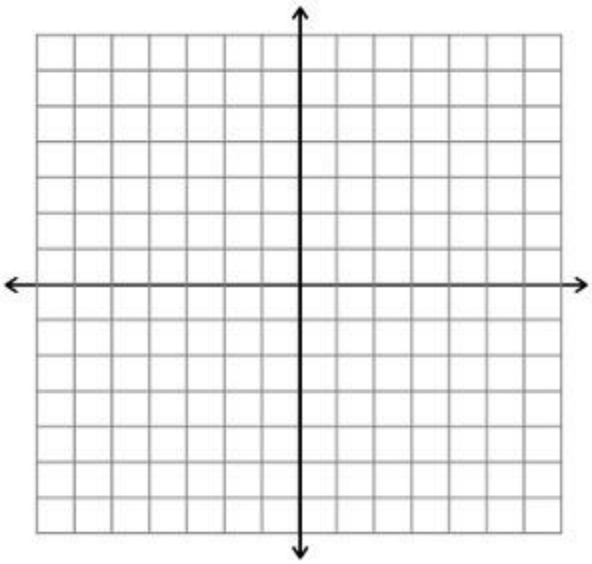
c. $f(x) = e^x$ is reflected about the x-axis, shifted 4 units down and 1 unit left

Lesson: Understanding the Graphs of Quadratic Functions

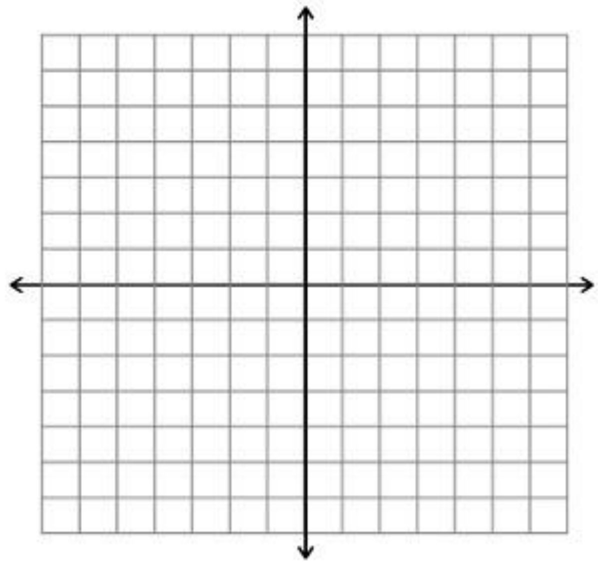
Quadratic functions are functions of the form ________________________________, where $a \neq 0$. But in previous lessons we worked on graphing quadratic functions, ie parabolas in another way. Let's review that first.

<u>Examples</u> Graph the following:

a. $f(x) = (x+2)^2 - 3$



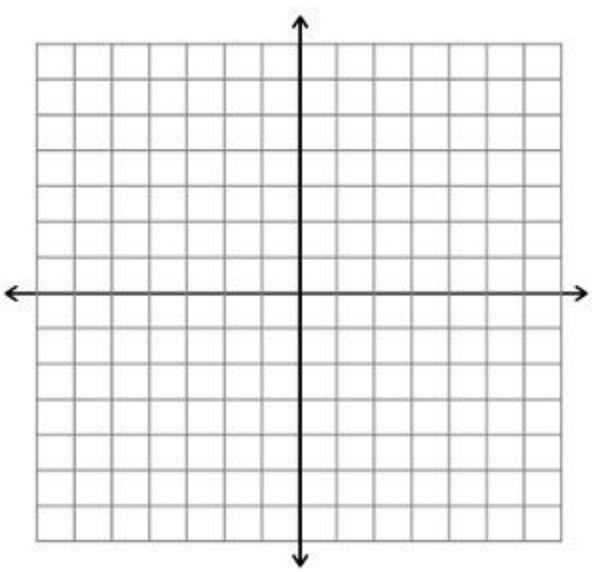
b. $g(x) = -(x-3)^2 + 1$



To begin, we will add on one more way to manipulate the graph of a parabola. Explain how to graph the follow:

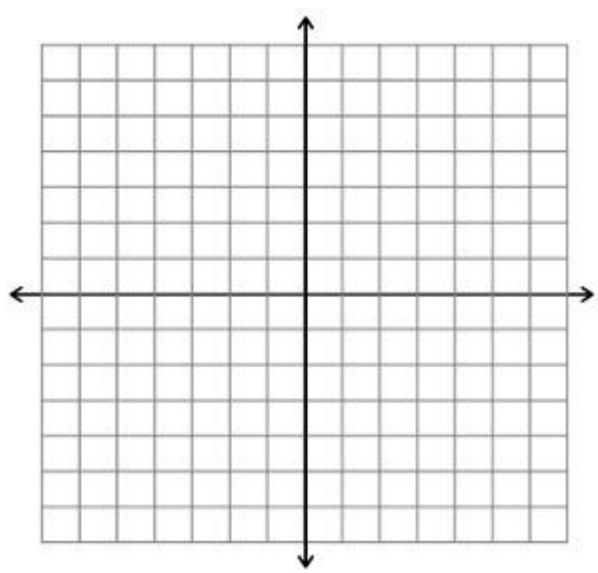
a. $f(x) = 3x^2$

How do you interpret the 3?



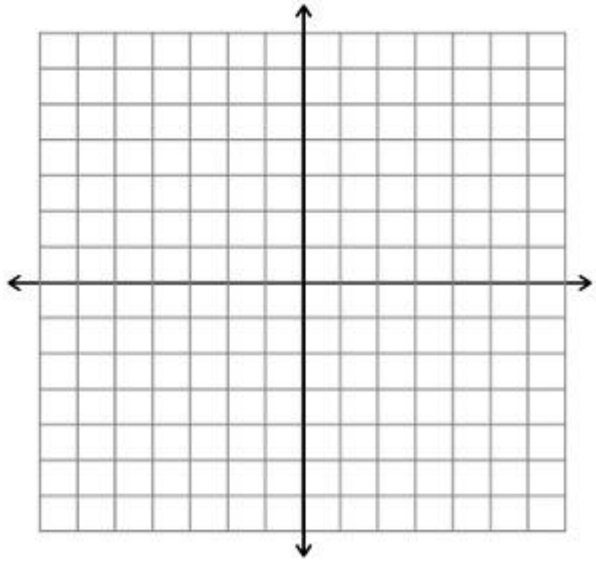
b. $g(x) = -\frac{1}{2}(x-2)^2 + 1$

How do you interpret the $-\frac{1}{2}$?

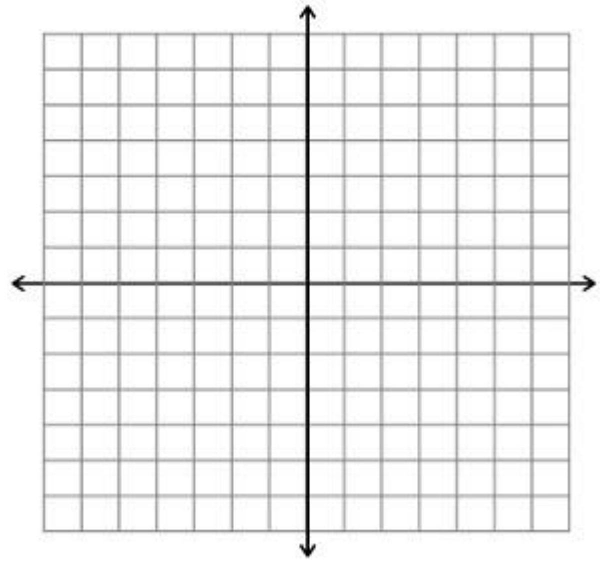


Examples Graph the following:

a. $f(x) = -2(x+1)^2 + 3$



b. $g(x) = \frac{1}{3}(x-2)^2 - 2$

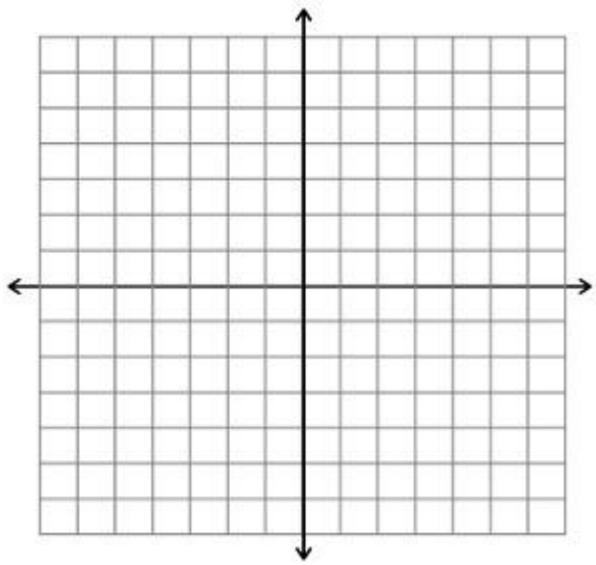


Using all of the examples we have discussed, what is another equation form for the parabola?

Explain some of the characteristics of a parabola that we get from this format:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-3)^2 - 4$



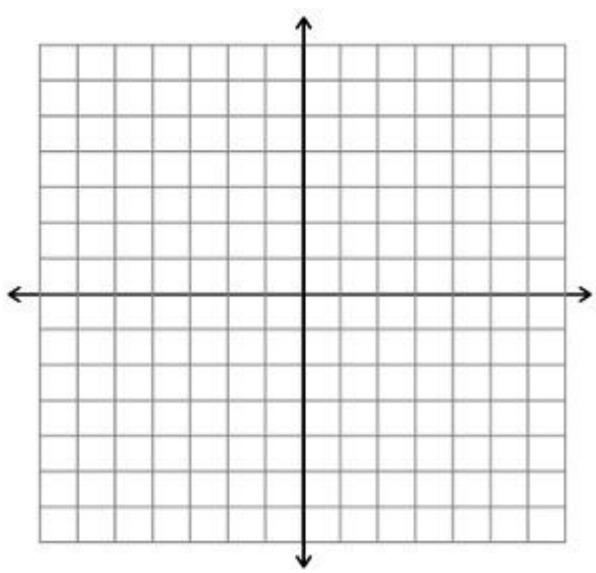
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2 + 8$



Vertex:

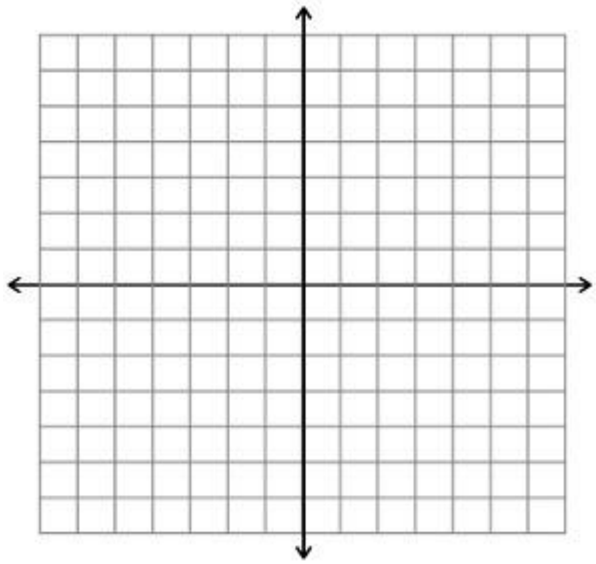
Axis of Symmetry:

X-intercepts:

Y-intercepts:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{4}(x+2)^2 - 3$



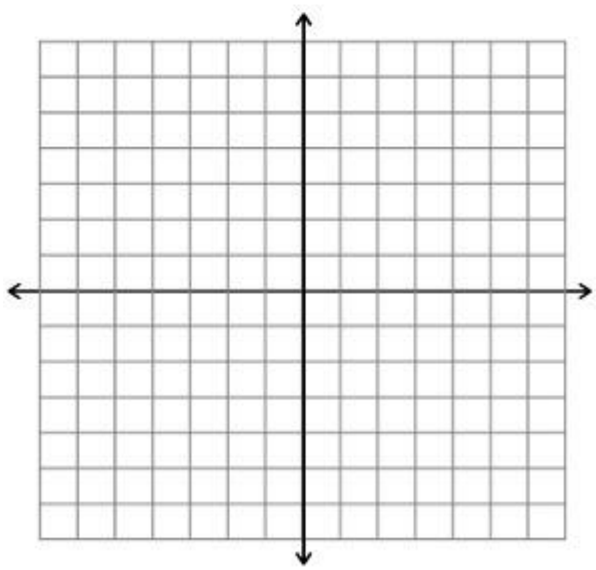
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -\frac{1}{3}(x-1)^2 + 3$



Vertex:

Axis of Symmetry:

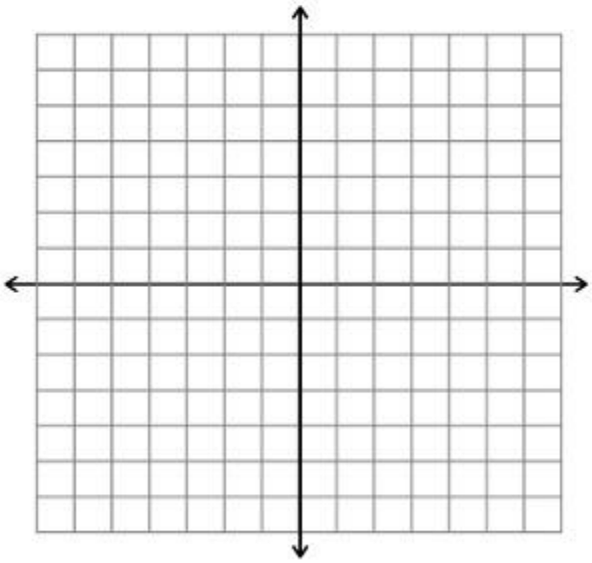
X-intercepts:

Y-intercepts:

Examples: Understanding How Many X-Intercepts There Are

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{2}(x+1)^2 - 8$



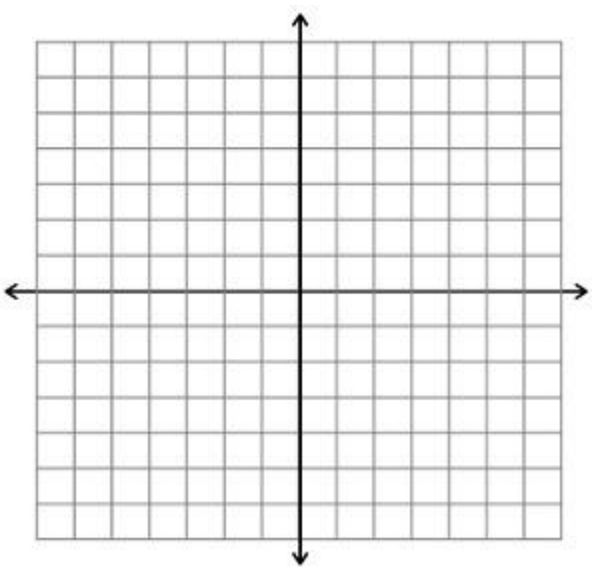
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2$



Vertex:

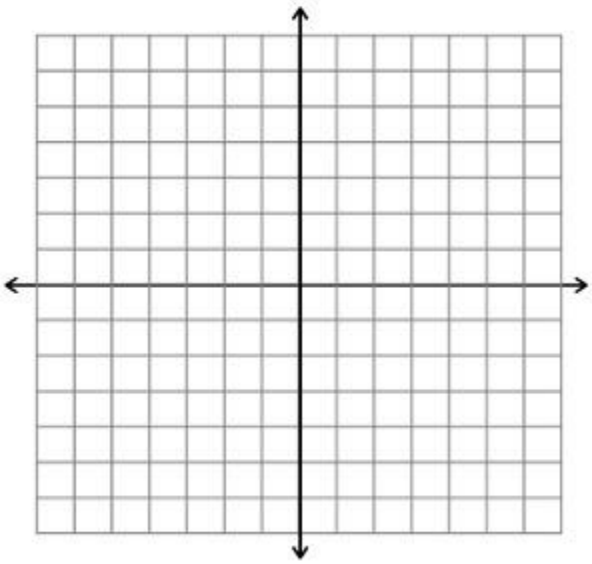
Axis of Symmetry:

X-intercepts:

Y-intercepts:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x - 2)^2 + 1$



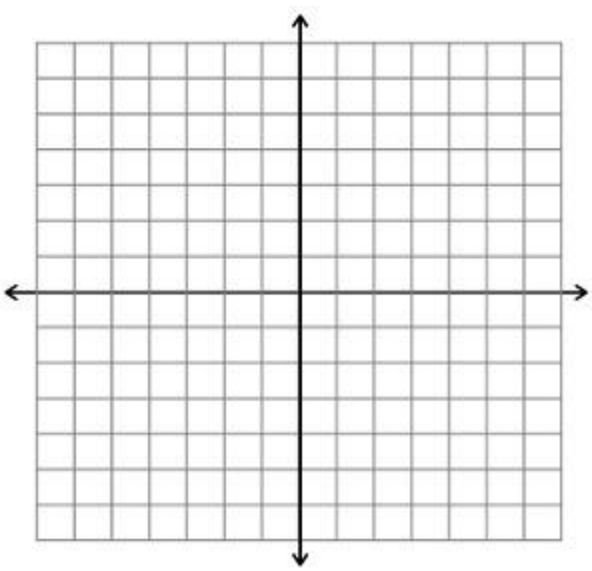
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -(x + 3)^2 - 1$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

Lesson: Finding the Vertex of Quadratic Equations in the form $\boldsymbol{f(x) = ax^2 + bx + c}$

What is the vertex formula?

Show how to use the vertex formula for $f(x) = 3x^2 + 6x - 2$

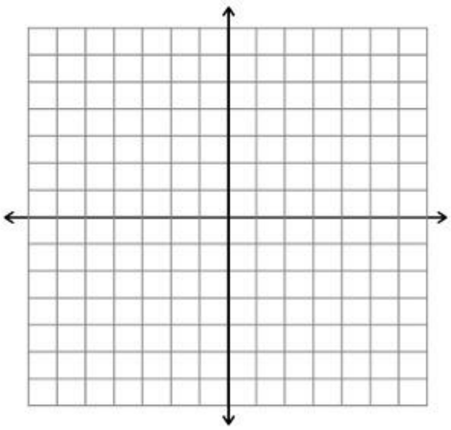
<u>Examples</u> Find the vertex of the following:

a. $f(x) = -x^2 + 4x - 2$

b. $f(x) = -\frac{1}{2}x^2 + 3x - 1$

Lesson: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form

Before you begin this lesson, you should already feel comfortable graphing an equation like $f(x) = 2(x-1)^2 + 3$. Graph this below:



Now show all the steps to put $f(x) = 2x^2 - 8x + 2$ into $f(x) = a(x-h)^2 + k$ form:

Examples Put the following quadratic functions into $f(x) = a(x - h)^2 + k$ form:

a. $f(x) = x^2 + 4x - 2$

b. $g(x) = x^2 + 6x + 10$

c. $h(x) = -x^2 + 2x - 4$

d. $y = 3x^2 + 6x - 12$

Examples: Putting Quadratics into the $\boldsymbol{f(x) = a(x-h)^2 + k}$ Form

<u>Examples</u> Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = \frac{1}{2}x^2 + x - 3$

b. $g(x) = -\frac{1}{3}x^2 + 4x - 2$

c. $y = x^2 + 3x + 2$

Lesson: Determining Whether a Quadratic Function will have a Minimum or a Maximum

Using the form $f(x) = ax^2 + bx + c$, explain what determines whether the parabola opens up or down:

<u>Examples</u> Determine whether the following quadratics open upwards or downwards, and then determine their vertex:

a. $f(x) = -2x^2 + 4x - 2$ upwards downwards

b. $g(x) = -\frac{1}{2}x^2 - 2x + 2$ upwards downwards

c. $h(x) = x^2 + 6x + 1$ upwards downwards

Lesson: Applications of Quadratic Functions 1

A ball is thrown upwards from a cliff 24 feet about the ground. The height h of the ball, in feet, t seconds after it is thrown is given by:

$$h(t) = -16t^2 + 32t + 24$$

How long does it take the ball to reach its maximum height?

What is the maximum height attained by the ball?

Lesson: Determining if Functions are One-to-One

Before we begin, you should already be familiar with the concept of functions and the vertical line test. First let's do some review.

Review Determine whether the following relations are functions, then determine the domain and range.

a. $\{(2,3),(5,7),(2,4)\}$

b.

Now we will discuss a specific type of function: a one-to-one function. Define this below:

How can we visualize this?

Example Determine if any of the following are i) functions and ii) one-to-one functions

a. $\{(2,4),(-1,7),(3,7)\}$

b. $\{(3,6),(-1,2),(-3,-4)\}$

Similar to how we have a test to determine if a graph represents a function, we have another test to determine if a function is one-to-one.

Describe the horizontal line test below:

Examples Determine whether the following graphs represent i) functions and ii) one-to-one functions (draw your own):

a.

b.

c.

Lesson: Finding the Inverse of Functions

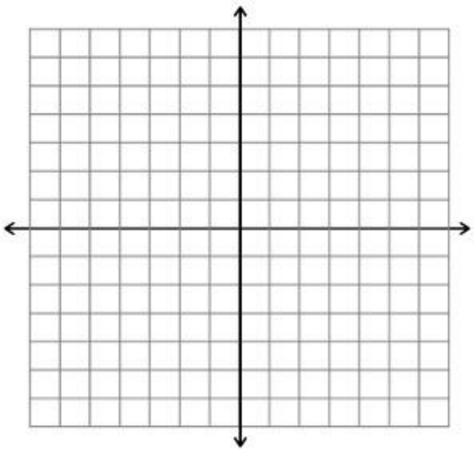
One-to-one functions have inverses. Describe what the inverse of a function is:

<u>Example</u> Find the inverse of the following function: $\{(1, 4), (7, 8), (3, -2)\}$

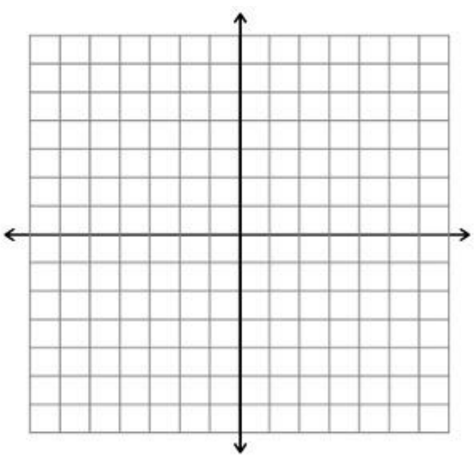
Describe the steps to find the inverse of a function:

<u>Examples</u> Find the inverse of the following one-to-one functions. Then plot the original function on the graph with its inverse.

a. $f(x) = 3x + 1$



b. $g(x) = -\frac{1}{2}x + 4$



What is the relationship of a graph with its inverse?

Examples Find the inverse of the following one-to-one functions.

a. $h(x) = \sqrt[3]{x}$

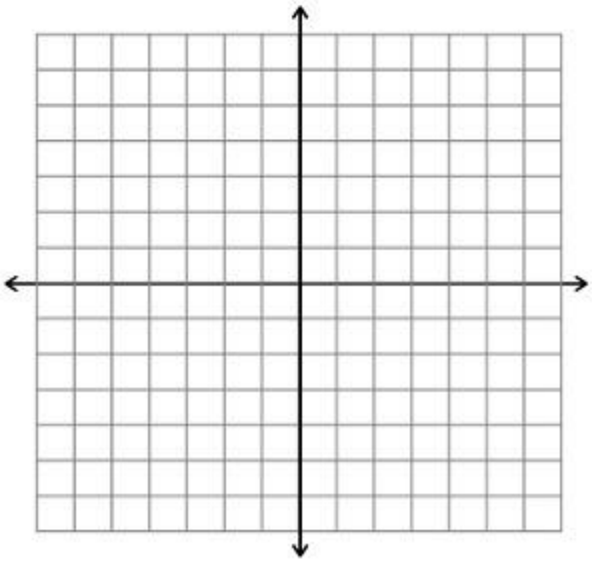
b. $f(x) = x^2, x \geq 0$

Lesson: Exponential Functions

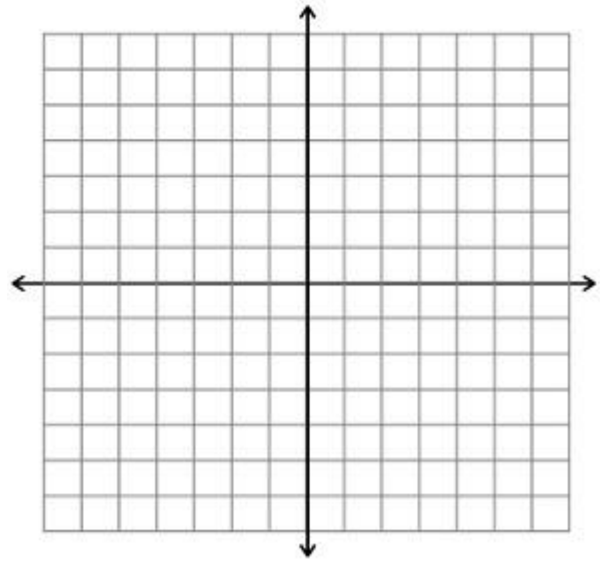
An exponential function is a function of the form ______________________________ where $a > 0, a \neq 1$.

<u>Examples</u> Graph the following functions, then determine their domain and range.

a. $f(x) = 2^x$



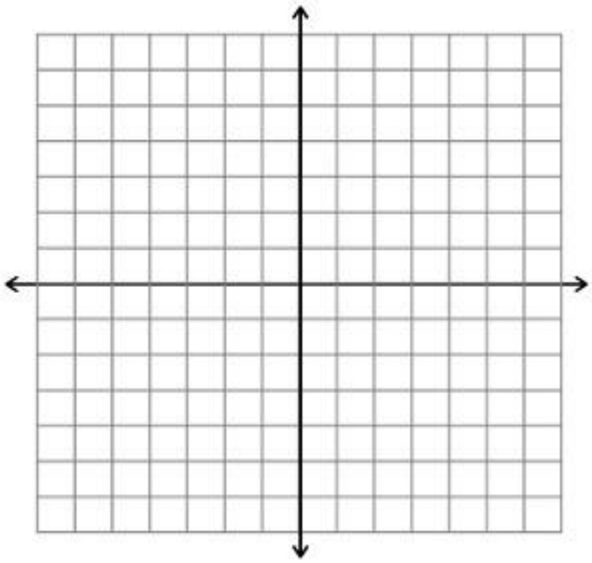
b. $g(x) = \left(\frac{1}{2}\right)^x$



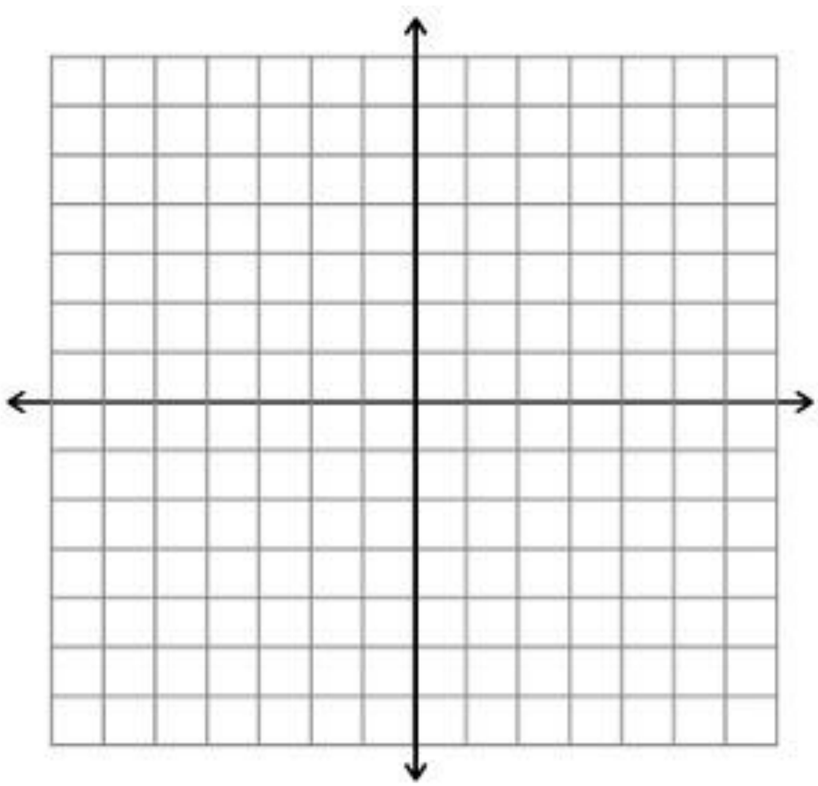
Using the graphs above, summarize some of the characteristics of these graphs:

The number e is an irrational number that is frequently used in math. What is an approximation of e?

Example Graph $f(x) = 2^x, g(x) = e^x, h(x) = 3^x$ all on the same graph below for comparison:



Translations work similarly to other graphs we have discussed. Using the graph of $f(x) = 2^x$ as a “base graph”, draw the following graphs together on the same graph: $f(x) = 2^x, g(x) = 2^x + 3, h(x) = 2^{x-1}$



Lesson: Solving Exponential Equations

In this lesson, we will solve equations that look like this:

$$3^x = 27 \qquad \text{or} \qquad 4^{x+3} = 8^{x-2}$$

To do this, we will write directions around the following example: $2^{3x} = 8^{x+1}$

<u>Examples</u> Solve:

a. $49^{x-3} = 7^{2x}$

b. $9^{4x} = 27^{x+4}$

Examples Solve:

a. $5^{x-2} = \frac{1}{25}$

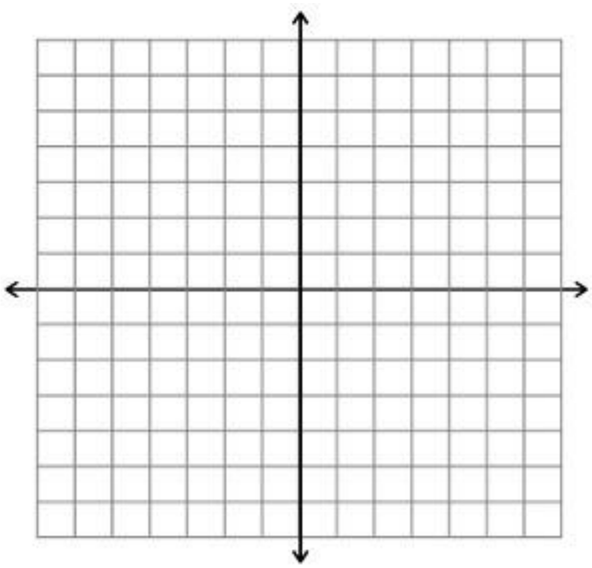
b. $\left(\frac{3}{5}\right)^{2x} = \left(\frac{27}{125}\right)^{x+2}$

c. $27^{3x+1} = 9^{4x+2}$

Lesson: Intro to Logarithmic Functions

First, we want to review how to function inverses of functions. Find the inverse of: $f(x) = 2x + 8$

Graph the exponential function $f(x) = 3^x$ below:



Recall the exponential functions are one-to-one, which means that they have inverses. Attempt to find the inverse of $f(x) = 3^x$ below:

As we can see, there is an issue with finding the inverse – we have no way to solve for y! As such, we need new mathematics to help solve this.

Write down the definition of a logarithm:

Examples Write the following in exponential form:

a. $\log_3 81 = 4$

b. $\log_2 \frac{1}{8} = -3$

c. $\log_5 125 = 3$

d. $\log_9 9 = 1$

Examples Write the following in logarithmic form:

a. $2^4 = 16$

b. $3^0 = 1$

c. $6^{-2} = \frac{1}{36}$

d. $8^{\frac{1}{3}} = 2$

Now we can finally determine what is the inverse of $f(x) = 3^x$:

There are a few common logarithms:

- $\log x$ means:

- $\ln x$ means:

There are also 2 key properties of logarithms:

1. $\log_a a = 1$: explain why this makes sense

2. $\log_a 1 = 0$: explain why this makes sense:

Lesson: Basics of Solving Logarithmic Equations and Evaluating Logarithms

We want to solve equations that look like:

$$\log_3 x = 4 \qquad \text{or} \qquad \log_x 36 = 2$$

Before you begin, you should already be able to do the following:

a. Put 3^5 in logarithmic form:

b. Put $\log_5 5 = 1$ in exponential form:

Now we can explain how to solve logarithmic equations using the example $\log_3(2x + 1) = 3$

<u>Example</u> Solve: $\log_x 16 = 4$

Example Solve:

a. $\log_9 x = 2$

b. $\log_6(2x + 4) = 2$

Now we can also discuss how to evaluate logarithms. Explain how to evaluate: $\log_3 27$:

Examples Evaluate:

a. $\log_5 \frac{1}{25}$

b. $\log 100$

Examples Evaluate:

a. $\ln e$

b. $\log_3 27$

c. $\log_{25} 5$

d. $\log \frac{1}{10}$

Lesson: How to Graph a Logarithm

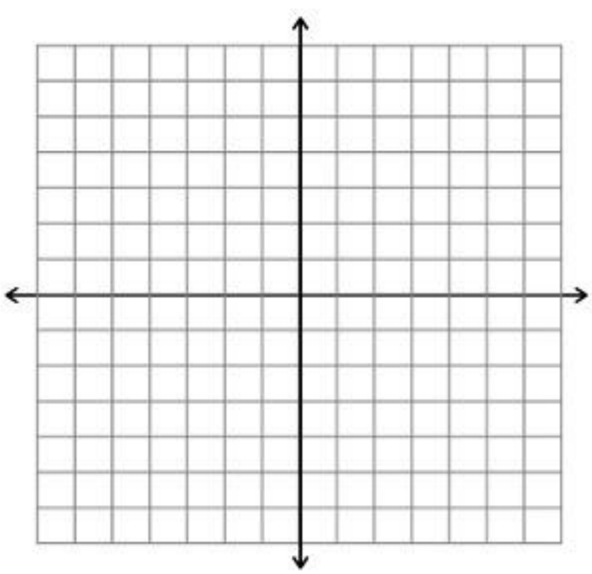
Before we begin, you should be able to do the following:

a. Convert to logarithmic form: $25 = 5^2$

b. Convert to exponential form: $\log_2 8 = 3$

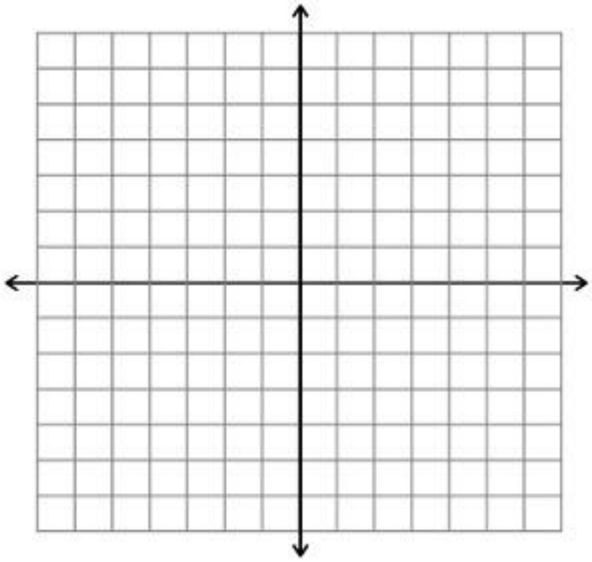
c. Find the inverse of $y = 2^x$

Now we can explain how to graph $y = \log_3 x$



For comparison, also graph $f(x) = 3^x$ on the same graph above.

Example Graph $y = \log_{\frac{1}{2}} x$ below:



Not included (no notes file):

004 (Examples of the Greatest Common Factor)

005 (Examples of Factoring Basic Trinomials via Trial and Error)

006 (Examples of Grouping)

007 (Examples of the Sum and Difference of Cubes)

008 (Examples of Factoring Harder Trinomials)

013 (Examples of Simplifying Rational Expressions)

014 (Examples of How to Use the Multiplicative Property of -1 to Simplify Rational Expressions)

015 (Examples of Finding Domains of Rational Expressions)

018 (Examples of Multiplying and Dividing Rational Expressions)

022 (Examples of Finding Common Denominators for Rational Expressions)

037 (Examples of Solving Absolute Value Equations)

042 (Examples on Absolute Value Inequalities)

878 (Examples: Putting Everything Together and Another Explanation of Special Cases, Slower-Paced)