Lesson: Simplifying Fractions

Say we want to reduce the fraction: $\frac{55}{77}$

To do so, we need to understand what numbers must be multiplied to make this fraction. Write the steps below as outlined in the videos:

Example 1

Reduce the following fractions.

a. $\frac{15}{35}$

b. $\frac{12}{18}$

c. $\frac{18}{42}$

d. $\frac{42}{54}$

e. $\frac{105}{135}$

f. $\frac{113}{125}$

How do you convert the mixed number $3\frac{4}{5}$ to an improper fraction?

Example 2

Convert the following to improper fractions

a. $5\frac{1}{2}$

b. $7\frac{2}{3}$

How do you convert the improper fraction $\frac{66}{5}$ to a mixed number?

Example 3

Convert the following to improper fractions

a. $\frac{15}{2}$

b. $\frac{25}{3}$

Lesson: Multiplying Fractions

The Rule for Multiplying Fractions

Let a, b, c, and d be real numbers where b $\neq 0$ and d $\neq 0$. Then:

What is actually happening with this rule? Write down a simple non-math explanation of how to multiply fractions:

Is a common denominator required for this? Yes No

Example 1

Multiply the following. There will be no reducing required, just make sure you understand how to multiply these together:

a. $\frac{2}{5} \times \frac{3}{7}$

d. $\frac{1}{2} \times \frac{3}{5} \times \frac{1}{7}$

b. $\frac{7}{9} \times \frac{2}{5}$

e. $5 \times \frac{2}{3}$

c. $\frac{12}{7} \times \frac{10}{7}$

f. $\frac{2}{3} \times \frac{5}{7} \times 2$

Reducing Fractions Before Multiplying

Occasionally you will multiply fractions that will need to be reduced. It can be easier to reduce them before multiplying them out. We now practice this.

Say you have this problem: $\frac{55}{54} \times \frac{45}{77}$

In this case, multiplying first would be messy so it is more advantageous to reduce before we multiply. Write down the steps below:

Example 2

Try this in the next problem. If you aren't sure what factor goes into each number, then break the numbers into their prime factorizations.

$$\frac{21}{28} \times \frac{16}{35}$$

Example 3

Cancel out any common factors before multiplying.

a. $\frac{35}{24} \times \frac{18}{49}$

c. $12 \times \frac{5}{6} \times \frac{1}{10}$

b. $\frac{60}{64} \times \frac{56}{72}$

d. $\frac{30}{75} \times 3\frac{2}{5}$

Lesson: Dividing Fractions

The Rule for Dividing Fractions

Let a, b, c, and d be real numbers where b$\neq 0$, c$\neq 0$, d$\neq 0$. Then:

What is actually happening with this rule? Write down a simple non-math explanation of how to multiply fractions:

Is a common denominator required? Yes No

Example 1

Complete the following exercises:

a. $\frac{2}{3} \div \frac{5}{7}$

b. $\frac{2}{5} \div 3$

c. $7 \div \frac{2}{3}$

Similar to multiplicationn, we can make cancelations *after we multiply by the reciprocal of the divisor*.

Example 2

Divide.

a. $\frac{15}{16} \div \frac{9}{14}$

c. $\frac{6}{7} \div 4\frac{1}{2}$

b. $20 \div \frac{4}{5}$

d. $\frac{24}{65} \div \frac{28}{70}$

Lesson: Intro to Adding and Subtracting Fractions

Adding and Subtracting Fractions When Fractions Have a Common Denominator

When fractions have the same denominator, we do not change the denominator but add the numerators as shown ($c \neq 0$):

Is a common denominator required? Yes No

Note this is the first instance where the denominator actually matters in the discussion.

Example 1

Add or subtract.

a. $\frac{3}{5} + \frac{1}{5}$

b. $\frac{3}{10} + \frac{7}{10}$

c. $\frac{5}{3} + \frac{4}{3}$

Adding and Subtracting A Whole Number with a Fraction

Suppose you want to complete the following example: $5 + \frac{1}{2}$

What denominator does the fraction have?

In this case, this is the common denominator we will want.

How to Turn a Whole Number into a Fraction

One property of all whole numbers is that we can rewrite them as fractions, so we know that:

$$5 = \frac{5}{1}$$

Once we convert 5 into this fraction form, we can then convert it to an equivalent fraction with the common denominator of 2. In this case, we say "whatever we do to the bottom, we also do the top." Show how to do this below:

Example 2

a. $5 - \frac{2}{7}$

c. $2 - \frac{11}{4}$

b. $3 + \frac{2}{3}$

d. $8 - \frac{1}{8}$

Lesson: Adding and Subtracting Fractions when You Need to Find a Common Denominator

This a is a much longer process. The ultimate goal is to make two fractions have the same denominator so we can combine them like we did in the first case.

To add two fractions that do not have the same denominator, we must complete the following three steps:

1) Find the LCD between the two fractions.
2) Convert each fraction as a fraction with the LCD.
3) Add or subtract the fractions and reduce is necessary.

Example 1

Find the LCD between: $\frac{1}{6}, \frac{1}{10}$

What is the largest number that divides both 6 and 10?

Now show how you can factor 6 and 10 by using this number that divides each:

Show how to find the LCD:

Example 2

Find the LCD between: $\frac{1}{12}, \frac{1}{18}$

What is the largest number that divides both 12 and 18?

Now show how you can factor 12 and 18 by using this number that divides each:

Show how to find the LCD:

Example 3

Find the LCD of the following fractional pairs. Put a box around your final answer to clearly show what the LCD is.

a. $\frac{3}{28}; \frac{5}{42}$

b. $\frac{5}{24}; \frac{1}{42}$

A Complete Example Showing How to Add Two Fractions with Unlike Denominators

Add the fractions: $\frac{1}{12} + \frac{4}{15}$

First, find the LCD between the denominators first:

What do you have to multiply 12 by to get the LCD?

What do you have to multiply 15 by to get the LCD?

Show how to finish the problem below:

Example 4

Use the guided steps to add $\frac{3}{10} + \frac{2}{15}$

What is the LCD between the fractions?

What must you multiply 10 by to get to the LCD?

What must you multiply 15 by to get to the LCD?

Complete the problem and put a box around the final answer:

Example 5

Add $\frac{1}{24} - \frac{3}{18}$ using the steps in the previous example to guide you.

Examples : Adding and Subtracting Fractions when You Need to Find a Common Denominator

Add or subtract the following:

a. $\frac{5}{16} - \frac{5}{6}$

b. $\frac{7}{12} + \frac{5}{18}$

c. $\frac{9}{10} - 3\frac{1}{2}$

d. $\frac{4}{15} + \frac{6}{25}$

Lesson: Exponents and the Order of Operations

Exponents

Label the parts of this exponent:

$$2^3$$

What does 2^3 literally mean?

Example 1

Evaluate the following exponents

a. 3^3

b. 5^3

c. 2^4

We need to understand exponents so that we can fully understand the order of operations. The order of operations is one of the handiest concepts in arithmetic that is often confused.

Order of Operations

Write down what the order of operations is:

Classic Example of the Order of Operations

Evaluate: $10 \div 5 \times 6$

Example 2

Evaluate the following. This page and the next page are both blank so that you have plenty of space to work out your calculations by hand.

a. $3 + 2^3 - 6 \div 3 \times 7$

b. $2 + 3^2 + 4 \times 2 - 5 \times 2$

c. $3(1 + 2^2 - 3)^2$

Examples: The Order of Operations

a. $10 - 3(1 + 2 \times 3) - 2^2 \times 4$

b. $3(1 + 2^2 - 3)^2 + 5(4 - 2 \times 3)$

c. $\frac{1}{2}(5 + 3) - 2^2(5 - 6)$

Lesson: Advanced Notes on Exponents

Multiplicative Property of Negative One

Explain what is the difference between -5^2 and $(-5)^2$ from the standpoint of the order of operations:

Example 1

Evaluate the following.

a. $(-3)^2$

b. $(-3)^3$

c. -3^2

d. -3^3

e. -7^2

f. $(-2)^5$

Example 2

What is the sign on the following problems? Do not worry about the final answer, you only need to circle if the final answer is positive or negative.

a. $(-4)^7$ Positive Negative

b. $(-58)^{12}$ Positive Negative

c. -72^{57} Positive Negative

d. $(-13)^{72}$ Positive Negative

e. -64^{14} Positive Negative

f. $(-23)^{91}$ Positive Negative

There is one final rule we need to discuss.

Zero Exponents

For any real number a,

Example 3

Evaluate the following:

a. 5^0

b. x^0

c. 35^0

d. -6^0

e. $(-7)^0$

f. $3^0 + 7^0$

g. $-5^0 + 2^0$

h. $\left(-3 + \sqrt{52}\right)^0$

Lesson: Algebraic Expressions

First we will define the terminology, which is already written out for you.

- **Constant:** any real number in an expression that is not attached to a variable
- **Variable:** a letter than can stand for any set of real numbers, and we can substitute values in for it if desired
- **Coefficient:** a number that is being multiplied by a variable
- **Term:** this is a single number, variable, or a number and a variable multiplied together, often separated by a plus + or minus – sign.
- **Factor:** a number or variable that is being multiplied with another number or variable
- **Algebraic Expression:** a mathematical string of constants and terms that can be added, subtracted, multiplied or divided together

This terminology will be used often, so it's important to understand what it means.

Example 1

Classify each part of the expressions

Expression	Constants	Variables	Coefficients	Terms	Factors
a. 5x					
b. 2x + y					
c. 8					
d. x					
e. $2x^2 - 4x + 7$					

Algebraic expressions are often used in everyday situations; we just don't realize it.

Example 2

You want to go to the movies, and on Tuesdays a theatre runs a deal so that all tickets cost $8. How much would it cost for each of the following situations?

a. A ticket for only you?

b. A ticket for you and 2 of your friends?

c. Tickets for 5 people, a a giant tub of popcorn that costs $20.

d. Tickets for z people?

In the last part of this example, we wrote an algebraic expression where z represented some quantity. Notice that if you input any of the previous number of tickets from examples a, b, and c into z, you would get the same answers.

To evaluate expression, you can simply plug in the desired values for a variable.

Example 3

Evaluate each expression for $x = -2$, $y = 4$, and $z = \frac{1}{3}$ as applicable:

a. $3x - 7y$

b. $6z - x^2 + 2y$

c. $x + z^2$

Lesson: The Commutative, Associative, and Distributive Properties

Your math homework and English homework are due on the same day. Does it matter which one you do first?

This situation illustrates the idea of the Commutative Law**,** which we can now state below:

Commutative Law

Let a and b represent any two real numbers. Now we can define the following properties:

- The Commutative Law for Addition:

- The Commutative Law for Multiplication:

Example 1

Rewrite the following using the commutative law.

a. $3 + y$

b. $3c$

c. $\frac{2}{5}k$

d. $2 - z$

You are buying chips, a drink, and a notebook. Does it matter what order the cashier rings up this items in?

These situations illustrate the idea of the **Associative Law**, which we can now state below:

Let a, b, and c represent any three real numbers. Now we can define the following properties:

- The Associative Law for Addition:

- The Associative Law for Multiplication:

Example 2

Rewrite the following at least 2 different ways using the associative law.

a. $4xz$

b. $8 + 2 + 7 + 3$

These rules are helpful when we are computing longer expressions.

How to Simplify Calculations Using the Commutative and Associative Laws

a. Suppose we want to calculate: $2 + 3 + 8 + 7$
   Using the commutative law, we more cleverly group these numbers together like this:
   $$2 + 8 + 3 + 7$$
   Usually we recognize 10s faster than some other quantities, so from here we can rearrange and group like this to find the final answer:

b. How can you rearrange $\left(\frac{1}{5} * \frac{1}{3}\right) \times (10 * 6)$ to make the calculations simpler to work with?

The Distributive Law

Let a, b, and c represent any three real numbers. Then the following property holds:

The distributive property effectively gives us permission to either add or multiply certain expressions in the order that simplifies the calculation.

Example 3
Use the distributive property as needed to simplify the following calculations.

a. $10\left(\frac{2}{5}-\frac{1}{2}\right)$

b. $\frac{2}{3}(6+9)$

c. $20\left(\frac{1}{4}+\frac{2}{5}\right)$

d. $\frac{2}{5}(14-4)$

The distributive law works in the opposite direction as well. If we start with $ab+ac$, according to the distributive law what would this equal?

Another name for this process is called **factoring.**

Example 4
Use the distributive law to factor the following:

a. $km+kn$

b. $5x+10y$

There are a few other properties we need to mention:

The Additive Identity

For any number a, $0 + a = a + 0 = a$

The Multiplicative Identity

For any number a, $1 \times a = a \times 1 = a$

The Additive Inverse (aka the opposite of a number)

For any number a, the addition of its opposite will result in zero, ie: $a + (-a) = 0$.

The Multiplicative Inverse (aka the reciprocal)

For any number a, it's inverse can be written in the form $\frac{1}{a}$ and has the property: $a \times \frac{1}{a} = 1$

It is interesting to note that there exist number systems (not discussed in this class) where these properties do **not** exist!

Lesson: Like Terms and Combining Like Terms
In algebra, we often want to simplify expressions if possible to make our lives easier. This is where the discussion of like terms comes into play.

Like Terms
Like terms are terms that share the exact same variables with the exact same exponents. Only like terms may be added.

Do like terms work in real life? Ie, can you come up with an example of like terms that could still be added in the real world?

Building off of your example, what wouldn't be like terms and could NOT be added in that scenario?

Example 1
In the following list below, classify which terms are like terms.

Discuss the two examples with the class to ensure they understand how to identify like terms. Students may color code, underline, or draw shapes around anything in the lists to indicate they are like terms.

a. $2x, 7y, 6x^2, 8y, x, -2x^2$

b. $3x^2y^2, 7x^2y, 2xy^2, 8x^2y, x^2y^2, -3x^2, 5xy^2$

The idea of like terms only comes up when we are talking about addition or subtraction. We do not worry about this when we want to multiply two things together. You may only add or subtract terms that are like terms.

Example 2

Combine like terms. The next page is also left blank if you need more space.

a. $3x\ +\ 2y\ - 5x\ + 9y$

b. $4x^2 + 2x + 6y^2 + 2x^2 + 8x$

c. $\frac{1}{2}x + \frac{2}{3}y - \frac{3}{2}x - 2\frac{1}{3}y$

d. $3x^2y^3 + 7x^2y^2 - 2x^2y^3 + x^2y^2$

e. $\frac{1}{4}x + \frac{2}{5}y - \frac{1}{2} + \frac{1}{5}y$

Lesson: Translating English into Math

As we move through this course, one major goal is to learn how to recognize and interpret certain situations as math problems. There are particular words in English that denote certain operations in mathematics, and we will classify those here.

<u>**Addition**</u>	<u>**Subtraction**</u>	<u>**Multiplication**</u>	<u>**Division**</u>

<u>Example 1</u>

Translate the following sentences into an algebraic expression. There is nothing to solve for you, you are merely setting up expressions.

a. 3 more than Lucy's height

b. Two less than a number

c. Three more than twice his salary.

d. Five less than three times a number.

e. Two-thirds of the cost.

f. Three more than twice his and her salary combined.

Lesson: Solving Linear Equations, A Review of the Basics

To begin, let's review some of what we've learned about working with equations and using graphs to solve them.

Linear Equation in One Variable
An equation that can be written in the form ax+b=0 where a and b are real numbers

Solution
A value that when substituted into the variable of an equation makes the equation true

Example 1
Determine if $x = -1$ is a solution to the following equations.
a. $5 - 4x = 9$

b. $2x + 3 = 1 - 3x$

We will now work with some basic ideas of solving linear equations. There are two key concepts we must understand to do this.

The Addition Principle
Let a, b, and c be real numbers. If a=b then:

Example 1
Use the addition principle to solve the following:
Choose at least one from each category. Label the examples you choose as a, b, c, etc to keep the class organized.
a. $x - 4 = 2$

b. $x + 3 = 3$

c. $x - \frac{1}{3} = \frac{1}{2}$

The Multiplication Principle
Let a, b, c be real numbers. If a=b then:

Example 2
Use the multiplication principle to solve the following:
a. $3x = 15$

b. $5x = 0$

c. $-\frac{2}{5}\mathrm{x} = 10$

Example 3

Solve the following examples:

a. $5 - 4x = 33$

b. $3x + 2 = 2 + 4x - 8$

c. $3(4 - 2x) = 3x + 6$

Lesson: Solving Linear Equations, No Solution and Infinite Solutions

There are 2 special outcomes we have to watch out for:

An Example Where There Is No Solution

$3(2x - 1) = 2(2x + 1) - 2(3 - x)$

An Example Where There Are Infinite Solutions

$4x + 5 = 3(2x + 1) - 2(x - 1)$

A Summary of the 3 Possible Solution Types for Linear Equations

Examples: Solving Linear Equations, Basic Examples with No Fractions or Decimals

a. $3x - 5 = 7$

b. $6 - 2x = 6$

c. $4x + 7 = 2x - 9$

d. $3 - 4x - 7 + 2x = 4 - 6\text{x-}8 + 4x$

e. $4x + 7 + 6x = 7(1 - 2x)$

f. $3(5 - 2x) - 6(3 - 2x) = 5(1 + 2x)$

g. $4(2x - 1) - 2(6 - 3x) = 2(3x - 4)$

h. $3(2x - 5) - 4x + 1 = 4(x - 3) - 2(1 - 3x)$

Lesson: Solving Linear Equations with Fractions

To begin, we will look at 3 examples to demonstrate how to clear fractions without parentheses. Take notes as shown in the video.

a. $\frac{1}{2}x + \frac{1}{5} = \frac{6}{5}$

b. $\frac{1}{2}x + \frac{1}{5} - \frac{1}{10}x = 1 + \frac{3}{10}x - \frac{2}{5}$

c. $\frac{1}{2}x + 1 + \frac{1}{4}x = \frac{3}{4} + \frac{2}{3} + \frac{1}{3}x$

Now we will look at how to clear fractions in equations with parentheses.

a. $\frac{3}{4}(3+2x)-\frac{1}{2}(5-3x)=\frac{5}{4}$

b. $\frac{1}{2}(x+2)+\frac{1}{3}(2x-5)=\frac{1}{3}(4+2x)$

Examples: Solving Linear Equations with Fractions (No Parentheses)

a. $\frac{1}{5}x - \frac{1}{3} = \frac{2}{3}$

b. $\frac{1}{3}x - 1 + \frac{1}{5}x + \frac{2}{5} = \frac{2}{5}x + \frac{1}{5}$

c. $\frac{1}{4}x - \frac{1}{10} + \frac{1}{5}x - \frac{4}{5} = \frac{3}{20}x + \frac{1}{10} + \frac{3}{10}$

d. $\frac{2}{3}x + \frac{1}{3} - \frac{1}{2}x = \frac{5}{6}x - \frac{1}{6} - \frac{2}{3}x + \frac{1}{2}$

Examples: Solving Linear Equations with Fractions (With Parentheses)

a. $\frac{1}{2}(3x-2)-\frac{2}{5}(2x-3)=\frac{1}{5}(x+1)$

b. $\frac{3}{4}(4x+5)+\frac{1}{2}(2-3x)=\frac{3}{4}(2x+1)$

c. $\frac{1}{2}(2x+3)-\frac{1}{3}(2x-1)=\frac{1}{4}(x+7)$

d. $\frac{3}{4}(2x-1)+\frac{5}{8}(2-4x)=\frac{1}{4}(2-4x)$

Lesson: Solving Linear Equations with Decimals

To begin, we will look at 3 examples to demonstrate how to clear decimals without parentheses. Take notes as shown in the video.

a. $0.03x + 0.04 = 0.19$

b. $0.05x + 0.1 - 0.03x = 0.08$

Now we will look at 3 examples to demonstrate how to clear decimals with parentheses. Take notes as shown in the video.

a. $0.4(x + 2) - 0.3(2x + 1) = 0.09$

b. $0.1x + 0.03(4 - 2x) = 0.5(x + 3)$

Examples: Solving Linear Equations with Decimals (no parentheses)

a. $0.1x - 0.06 - 0.2x - 0.04 = 0.3$

b. $0.3x - 0.6 + x - 0.2 = 0.6x - 0.8 + 0.7x$

c. $1 + 0.2x - 0.7 + 2x = 0.5 - 0.4x + 1.8 + 0.6x$

Examples: Solving Linear Equations with Decimals (with parentheses)

a. $0.4(3x+1)-1.5(2x+1)=0.6(3-3x)$

b. $0.2(3x+1)-0.05(2x+8)=0.1(2x+4)$

c. $0.09(2-4x)-0.1(2x-1)=0.2(2-x)-0.06(2+6x)$

Lesson: The Basics of Applications and Word Problems

As a reminder, here is a review of some of the key words that imply certain operations in math:

Words That Imply Addition	**Words That Imply Subtraction**	**Word That Imply Multiplication**	**Words That Imply Division**
Add	Subtract	Multiplied by	Divided by
Increased by	Difference	Times	Quotient of
Altogether	Less than	Of	Out of
Sum	Fewer	Product of	Ratio
Longer than	Decrease	Twice	
Plus	Minus		
Above	Take away		
More than	Shorter than		

Example Juan is memorizing a list of names for a history class quiz. So far, he has memorized 8 of the names, which is one-fourth of the list. How many names are on this list?

Example Corey wants to rent a truck for the day. It costs \$30 for the base fee, plus \$0.23 for each mile driven. When he returned the truck, he owed \$41.19. How many miles did he drive?

Example A café serves both tea and coffee. On Sunday, the café sold 95 teas and coffees combined. The café sold 9 more coffees than teas. How many teas were sold on Sunday?

Example Latoya bought a new sweater and a pair of jeans for $64. The jeans cost $8 less than the sweater. How much was the sweater? How much were the jeans?

Example There are red, orange, and green balls in a ball pit. There are 12 more orange balls than red balls, and there are 9 more green balls than red balls. How many are there of each color?

Lesson: Consecutive Integer Word Problems

How do you represent consecutive integers with variables?

Example The sum of 2 consecutive integers is 73. Find the integers.

Example The sum of 3 consecutive integers is -33. Find the integers.

How do you represent even consecutive integers with variables?

How do you represent odd consecutive integers with variables?

What is noteworthy about how you represent even or odd consecutive integers?

Example The sum of 2 consecutive even integers is 110. Find the integers.

Example The sum of 3 consecutive odd integers is -87. Find the integers.

Example Find 3 consecutive odd integers such that their sum is seven more than four times the largest integer.

Lesson: A Review of Percentages

What is a percent? What does it represent?

<u>Examples</u> Convert the following percentages to decimals:

a. 34%

b. 122%

c. 4.57%

<u>Examples</u> Convert the following decimals to percentages:

a. 0.052

b. 0.995

c. 0.00078

<u>Examples</u> Convert the following fractions to percentages:

a. $\frac{3}{4}$

b. $\frac{3}{5}$

Examples

a. Find 34% of 87

b. What is 17.5% of 32?

c. 12 is what percent of 36?

d. What percent of 18 is 3?

Lesson: Applications with Percents – General Applications and $ Applications

Example In the incoming freshman class of English majors, 52% of them were female. If there were 2,300 females majoring in English that year, what was the total number of English majors?

Example When Hector bought his house, it was worth $232,000. After a few years, the house's value increased by 22%. What is the increase in value of the house and what is the actual current value?

Example Devin wanted to buy a shirt that normally costs $18. When he go to the register to buy it, he found out there was a 55% discount. What is the price of the shirt now?

Example Rachel buys dresses for her clothing store at $8 wholesale. When she gets to the store, she marks them up 23%. How much does she sell each dress for?

Example Mia is buying a new laptop for $800. There is an 6.25% sales tax in her county. What is the final price of the laptop?

Example Rose found a shirt she wanted for $24. When the cashier rang up the shirt, she owed $25.98. What percentage of tax did she pay?

Lesson: Applications with Percents – Mixing with Percentages

<u>Example</u> Lala deposited $500 into a simple interest account that paid 7% interest per year, and $1500 into another account that paid 9% interest per year. After one year, how much interest did she make? What is the total percent of interest earned on the 2 deposits?

<u>Example</u> Steff works at a coffee shop and she is mixing coffee roasts. The Breakfast Blend contains 20% of the beans from Columbia. She mixes 3 pounds of the Breakfast Blend with 7 pounds of another roast that is 100% pure Colombian coffee. How many pounds of this mixture contain Colombian beans? What percentage of this new blend contains Colombian coffee?

<u>Example</u> Larry has mixture A with 55% acid and mixture B with 12% acid. He has 200 ml of mixture A. How much of mixture B did he use if his new solution is 48% acid?

Example A company owns 2 different factories to make furniture. Factory A produced 2 times as many dining chairs as Factory B. Five percent of the chairs from Factory A and 3% from Factory B were defective. How many chairs did Factory B produce if 770 of the chairs were defective?

Lesson: Solving a Linear Equation with Multiple Variables

Examples

a. Solve for x: $x + y = 7$

b. Solve for y: $4y = z$

c. Solve for a: $3a - 6b = 2c$

d. Solve for b: $4a + 8b = 16c$

e. Solve for k: $\frac{1}{3}k = m$

f. Solve for a: $3(a - 2b) = 6c$

g. Solve for b: $\frac{2}{3}(3a + 2b) = c$

h. Solve for a: $-\frac{2}{5}(2a - 4b) = 4$

Lesson: Ratios and Proportions

What is a ratio?

Example The unit price of an item is the ratio of the price of an item to the amount of the item. Find the unit price if 5oz of peanuts costs $1.

Example Express the ratio of 12 boys to 14 girls.

What is a proportion?

Examples Solve the following proportions:

a. $\frac{x}{5} = \frac{6}{10}$

b. $\frac{x+2}{12} = \frac{x-4}{8}$

Examples: Proportions

a. $\frac{3}{x}=\frac{9}{21}$

b. $\frac{3x+10}{15}=\frac{2}{5}$

c. $\frac{2x-1}{6}=\frac{4x}{10}$

Lesson: Proportion Word Problems

It 3 cookies cost $1.36, how much will 7 cookies cost?

At a motorcycle race, the ratio of parents to children was 5 to 3. If 500 children attended the motorcycle race, how many parents were at the race?

One euro is equivalent to $1.18 US dollars. How many euro would you get for $25?

Lesson: Distance, Rate, and Time Problems

Two cars leave Glen Ellyn, IL, with one driving north and the other driving south. The northbound car is traveling 8mph faster than the southbound car, and after 2 hours they are 420 miles apart. Find the speed of each car.

Marshall and Lilly live together. Marshall leaves the house at 2pm, and then Lilly leaves the house at 2:30pm. If Marshall is driving 30 mph and Lilly is driving 35 mph in the same direction, how long will it be until their cars meet?

A Lamborghini and a Ferrari pass one another at noon on a highway in California. The cars are going opposite directions. If the Lamborghini is traveling 75 mph, and the Ferrari is traveling 80 mph, how long will it take for the cars to be 310 miles away from one another?

Lesson: Money Word Problems

Sarah has five more nickels than quarters. When combined, the coins are worth $4.75. How many nickels and quarters does she have?

Maria has $75 worth of $1 and $5 bills. In total, she has 27 bills. How many $5 bills does she have?

Tickets at the movie theater course $15 for an adult and $10 for a child. In the last hour, sales were $550. How many adult tickets versus child tickets were sold?

Lesson: Number Lines, Interval Notation, and Set Notation

Show how to set up the number line, interval notation, and set notation for the following:

a. $x > 5$

b. $x \leq 7$

c. $x \geq 0$

d. $-3 < x < 5$

e. $-7 \leq x < -1$

f. $3 < x \leq 8$

Lesson: How to Solve Linear Inequalities in One Variable

Linear inequalities in one variable have the form $ax + b < 0$. Alternatively, they could use the symbol $\leq, >, \geq$.

Solving linear inequalities has some similarities and some differences when compared to solving for linear equations.

The Addition and Subtraction Properties

Let a, b, c be real numbers. If $a < b$, then:

This also holds for $\leq . >, \geq$

<u>**Examples**</u> Solve the following. Put your final answer both on a number line and in interval notation.

a. $x + 4 > 9$

b. $x - 8 \leq 12$

The Multiplication and Division Properties

Let a, b, c be real numbers. If $a < b$, then:

- if c is positive ($c > 0$):

- if c is negative ($c < 0$)

This also holds for $\leq . >, \geq$

Why is this rule different? Use the following example.

To begin, is $-2 < 4$ a true statement? True False

Now multiply both sides of the inequality by 3 and write down the new result below:

This new statement is True False

Now multiply both sides of the inequality by -3 and write down the new result below:

This new statement is True False

Now divide both sides of the inequality by -2 and write down the new result below:

This new statement is True False

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $3x \leq 15$

b. $-2x > 8$

c. $5x < -10$

d. $-4x \geq -12$

e. $-9x \leq 27$

From here, you can start to combine the rules to solve more types of inequalities.

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x - 4 < 6$

b. $3x + 7 > 5x - 9$

c. $\frac{1}{2}x + \frac{1}{5} > \frac{3}{4}x + \frac{7}{10}$

Examples: Solving Linear Inequalities – Basic Examples

Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x + 7 \geq 13$

b. $4x - 6 < 6x - 10$

c. $3(5 - 2x) > 5(3 + 2x)$

d. $2(4 - 2x) - 3(4 - 4x) \leq 4(3x + 3)$

Examples: Solving Linear Inequalities – Examples with Fractions Decimals

Solve the following. Put your final answer both on a number line and in interval notation.

a. $\frac{1}{3}x - \frac{1}{5} \le \frac{1}{3} + \frac{2}{5}x$

b. $0.03x + 0.2 > 0.02$

c. $\frac{3}{5}(2x-1) - \frac{3}{4}(2-3x) < \frac{1}{10}(4-3x)$

d. $0.2(2x-4) + 0.03(6-10x) \ge 0.06(x+9)$

Lesson: Three-Part Inequalities

Show how to solve the following three-part inequalities. Put your final answer both on a number line and in interval notation.

a. $4 < x + 2 \leq 10$

b. $-3 \leq 2x - 1 < 11$

c. $-5 < 3x + 4 < 19$

d. $4 \leq -1 - 5x < 14$

Lesson: The Basics of Compound Inequalities

The word “and” has special implications in mathematic. One place it is often used is with sets, and we represent “and” with the intersection symbol ∩.

The word “or” also has special implications in mathematic. It is also used is with sets, and we represent “or” with the intersection symbol ∪.

<u>Examples</u> Given $A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 6, 8, 10\}, C = \{1, 3, 5, 7\}$, find the following:

a. $A \cap B$

b. $A \cap C$

c. $A \cup C$

d. $B \cup C$

We use similar ideas with compound inequalities.

Show how to represent a solution to $x \geq -2$ and $x < 5$

<u>Example</u> Represent the following with a number line and interval notation.

a. $x > 0$ and $x \leq 7$

b. $x > 8$ and $x \leq -2$

Show how to represent a solution to $x \geq 4$ or $x < -3$

Example Represent the following with a number line and interval notation.

a. $x \leq 0$ or $x > 5$

b. $x > 0$ or $x \leq 5$

We can create more complicated problems where we first have to isolate x, but then we can represent in the same manner as shown above.

Example Represent the following with a number line and interval notation.

$3x - 1 > 5$ and $5x + 4 \leq 24$

Examples: Compound Inequalities

Put your final answer both on a number line and in interval notation.

a. $x - 4 > 8$ or $x + 2 < -3$

b. $2x + 1 \geq 7$ and $5x - 10 < 20$

c. $4x + 2 > 18$ or $3x - 1 < 17$

d. $2 - 3x > 8$ and $5x - 2 > 12$

e. $5x + 9 < 16$ or $4 - 3x > 14$

f. $3 - 2x > 9$ and $5 - x < -12$

Lesson: Intro to Linear Equations in Two Variables, Graphs, and Plotting Points

A **linear equation in two variables** is an equation of the form ax+by=c, where a, b, and c are real numbers, and both a and b are not zero at the same time.

Solutions for equations with two variables come as an ordered pair. Unless otherwise stated, you generally view the ordered pair in the alphabetical order of the equation.

Examples Decide if the following ordered pairs are solutions to the equations.

a. $\left(2, -\frac{1}{2}\right)$ for $3x - 2y = 4$

b. $(-5, -1)$ for $3a + 2b = 10$

c. $(3, 2)$ for $y = 5 - x$

Examples Complete the ordered pairs for the following equations

a. $(2, b)$ for $4a - 6b = 2$

b. $(x, -3)$ for $y = 2x + 1$

Another way to represent ordered pairs is with a table.

Examples Complete the following tables for the linear equations:

a. $y = 2x + 4$

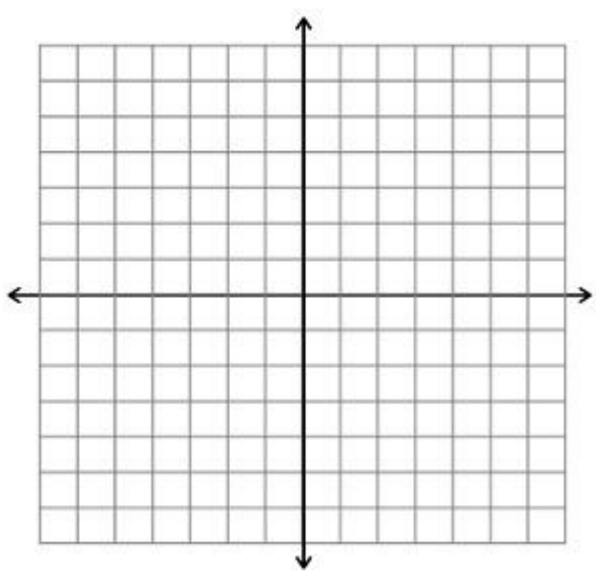
x	y
-3	
	10
0	

b. $3a - 2b = 12$

x	y
	0
0	
3	

Order pairs are often represented in coordinate systems. Below, draw the Cartesian (aka rectangular) coordinate system and label its parts.

Examples Plot the following points on the graph:



a. $(-2, 3)$

b. $(4, -1)$

c. $(1, 4)$

d. $(0, -4)$

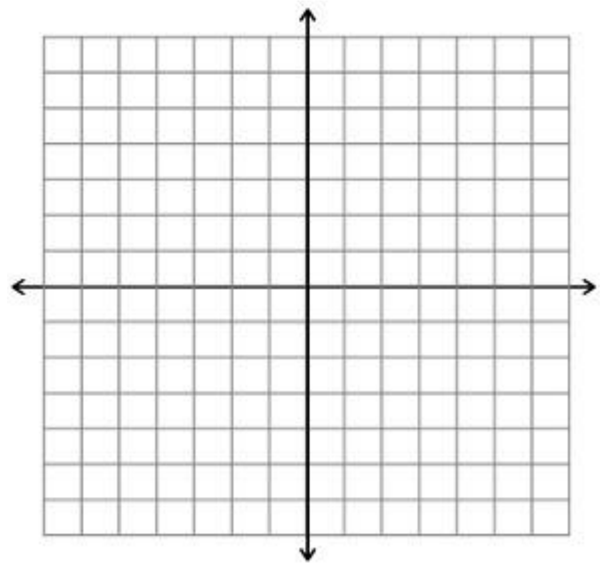
Lesson: Graphing Equations by Using a Table

One way to graph lines is by finding three points, and the plotting those points.

Examples Complete the following tables of ordered pairs, and the plot the points to form a line.

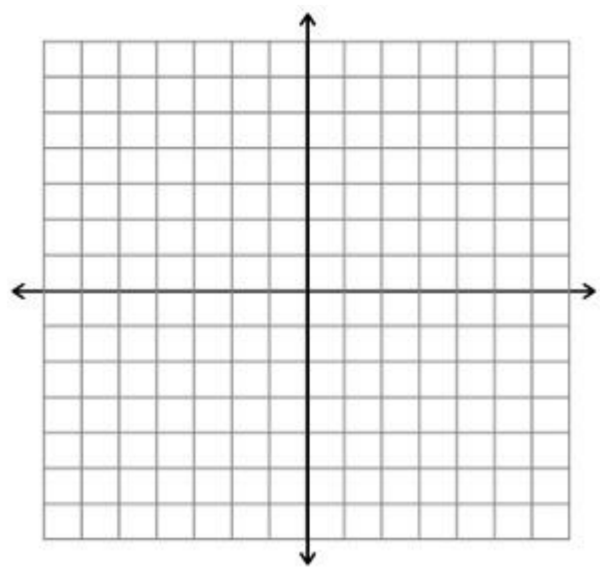
a. $y = 2x - 1$

x	y
2	
-2	
	0



b. $-2x + 6y = 12$

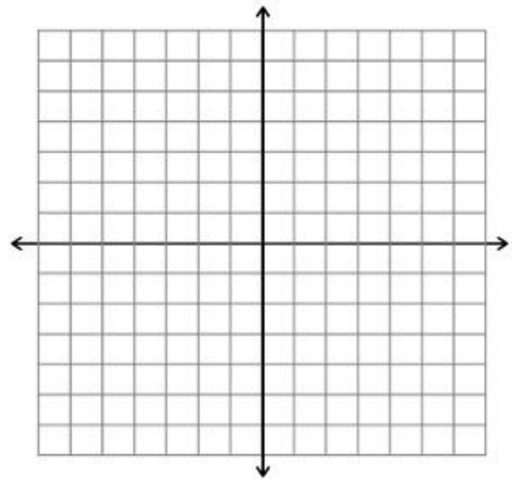
x	y
-3	
	0
3	



Examples Find three points for the following equations, then plot those points to make a line.

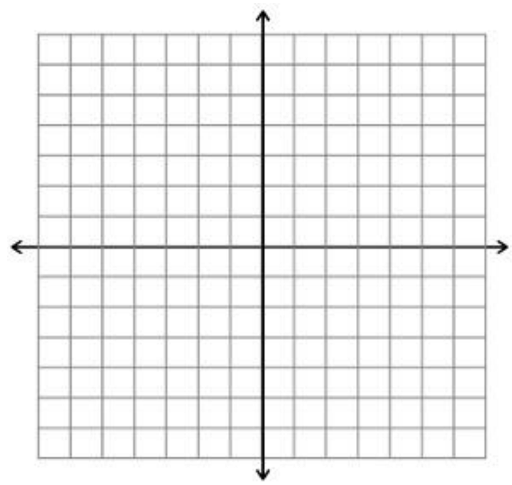
a. $y = -3x + 2$

x	y



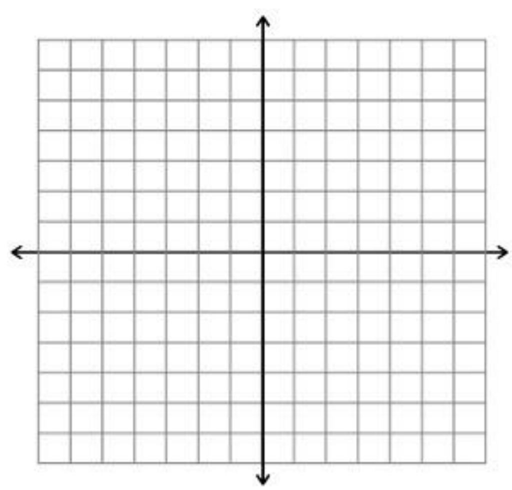
b. $y = -\frac{1}{2}x + 3$

x	y



c. $3x - 5y = 15$

x	y



Lesson: Graphing Equations by Using Intercepts

What are x-intercepts? What are y-intercepts? Draw an illustration.

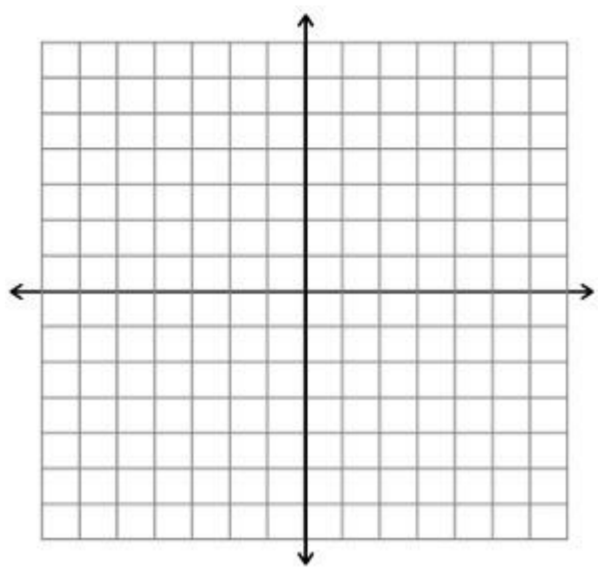
How do you find x-intercepts?

How do you find y-intercepts?

<u>Examples</u> Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$3x - 2y = 6$

x-intercept



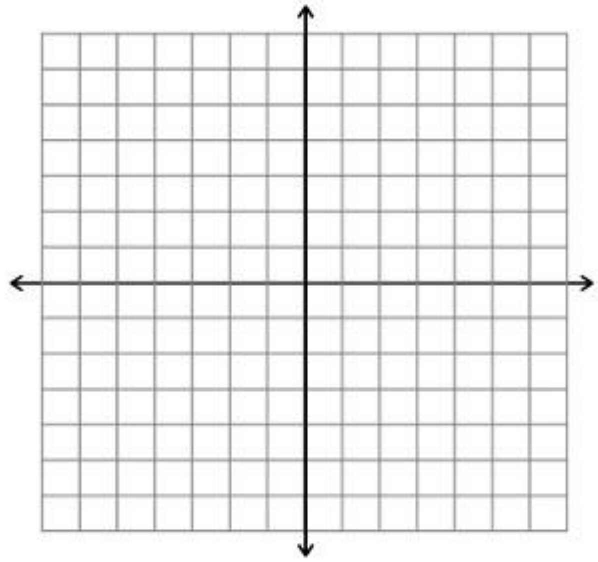
y-intercept

Third point:

Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$x - 4y = 4$

x-intercept



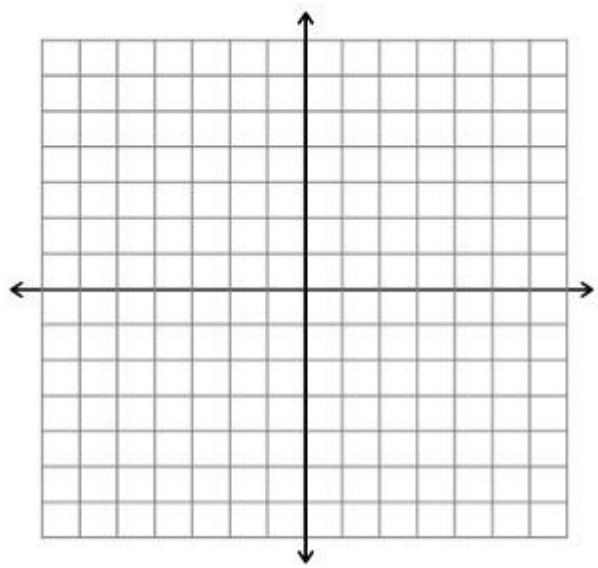
y-intercept

Third point:

Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$y = \frac{1}{2}x - 2$

x-intercept

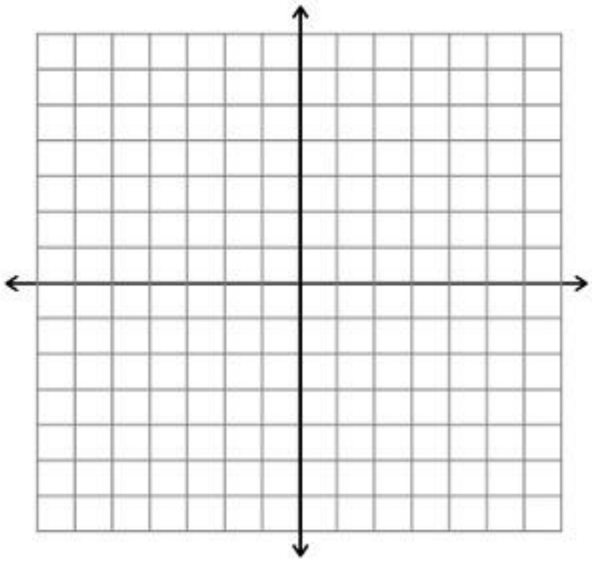


y-intercept

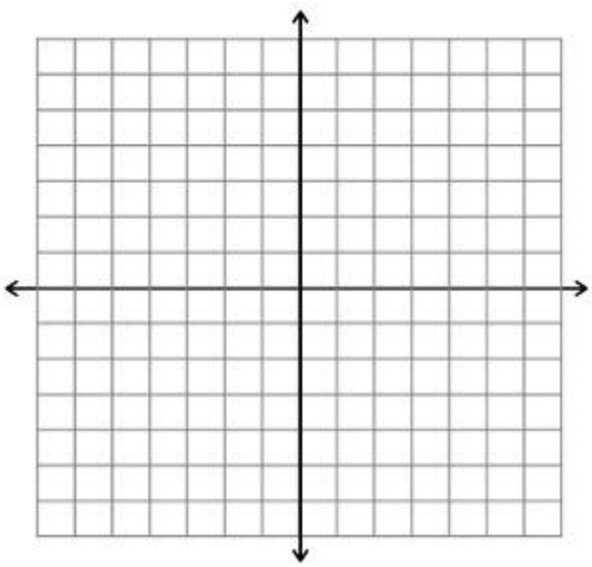
Third point:

Lesson: How to Graph Vertical and Horizontal Lines

Show how to graph $y = 3$



Show how to graph $x = -2$

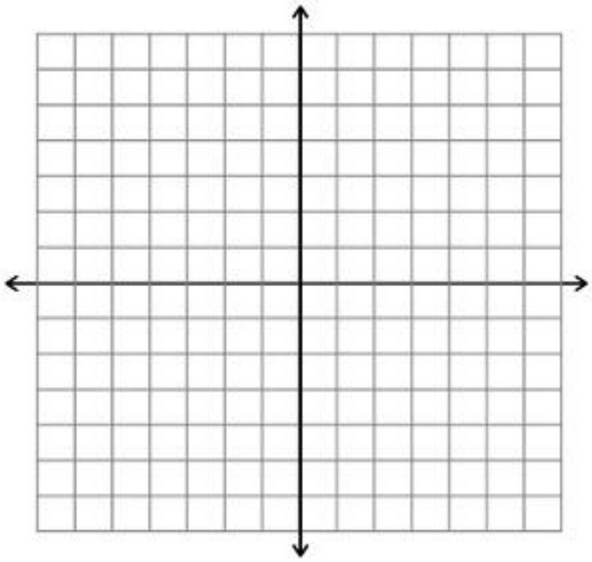


How do you know if you are working with a vertical line? How do you graph it?

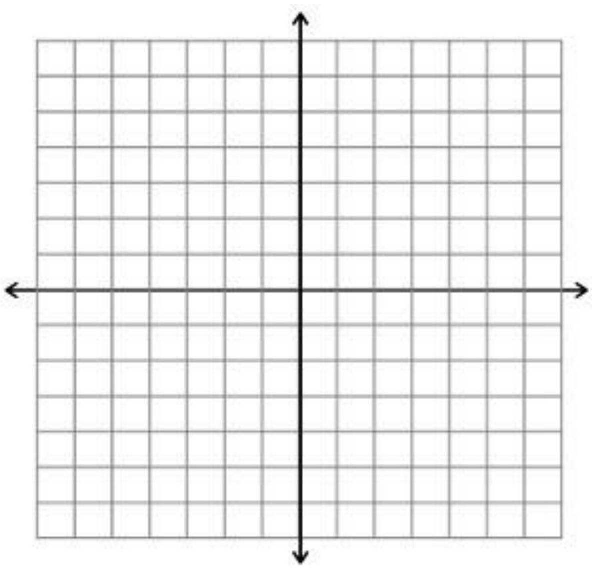
How do you know if you are working with a horizontal line? How do you graph it?

Examples Graph the following:

a. $x = 4$



b. $y = -1$



Lesson: The Slope of a Line

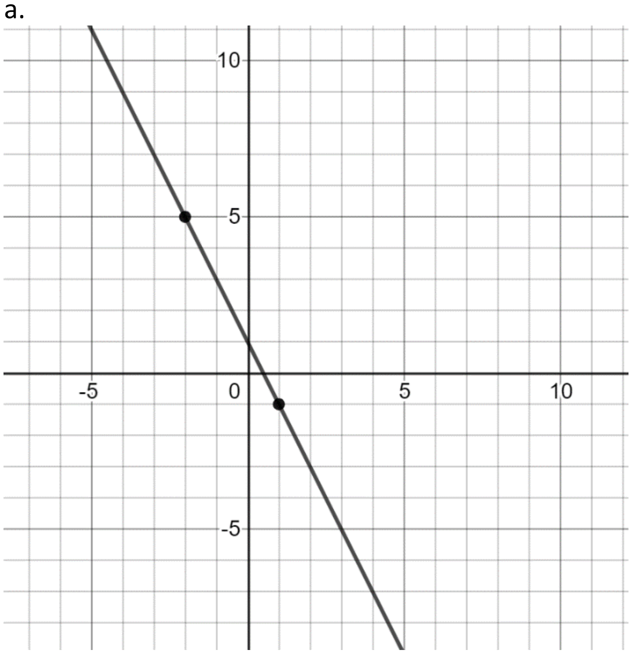
What does slope represent?

Draw the various possible slopes of lines:

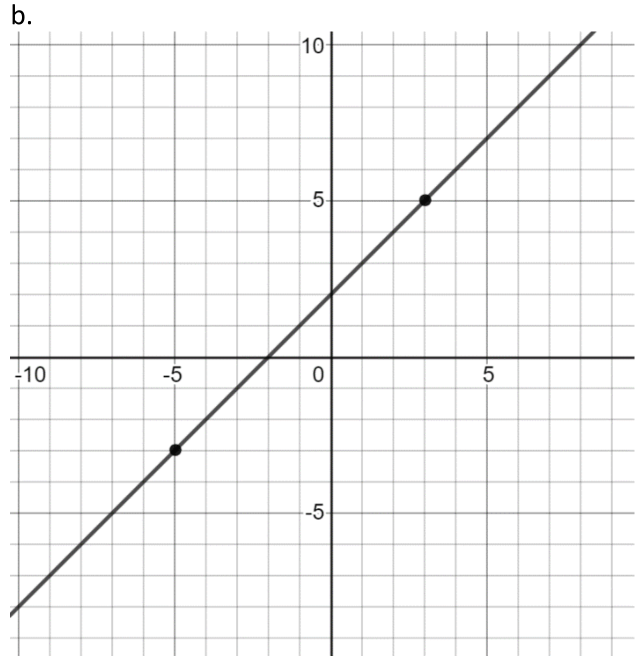
How do find the slope of a line?

Examples Find the slope of the following lines:

a.



b.



What is the slope formula?

Examples Use the slope formula to find the slope between the pairs of points:

a. $(3, 5)$ and $(4, 8)$

b. $(6, -2)$ and $(-2, 4)$

c. $(7, 2)$ and $(7, 4)$

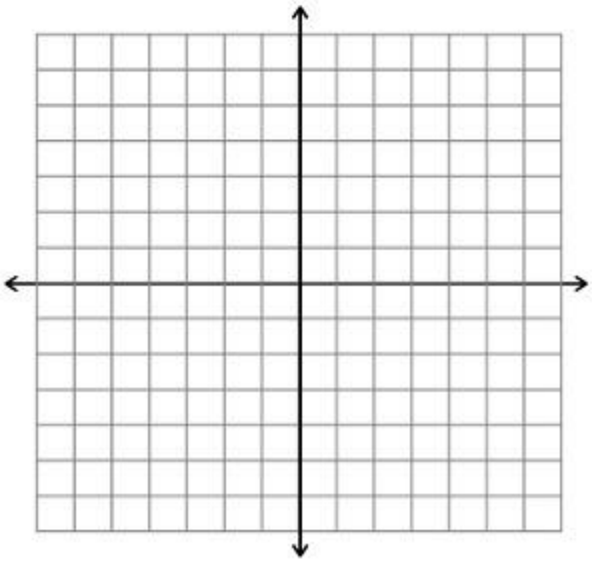
d. $(2, 4)$ and $(-1, 4)$

Lesson: Using Slope to Graph Lines; Intro to Slope-Intercept Form

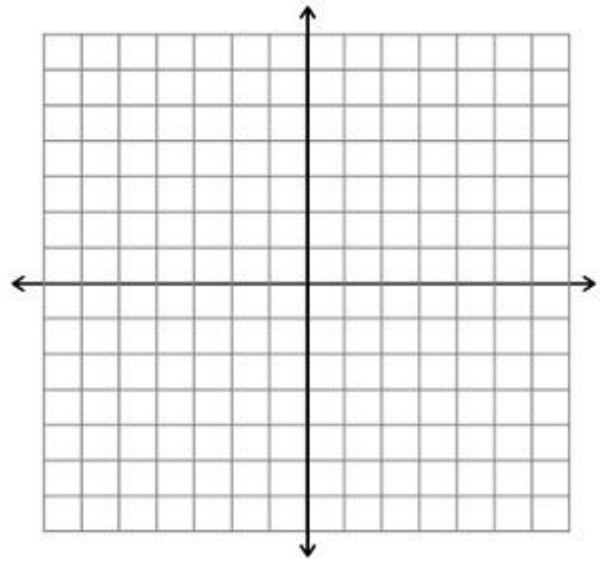
When the slope of a line and a point are known, a line can be easily graphed.

Examples Given the slope and the point, graph the following lines:

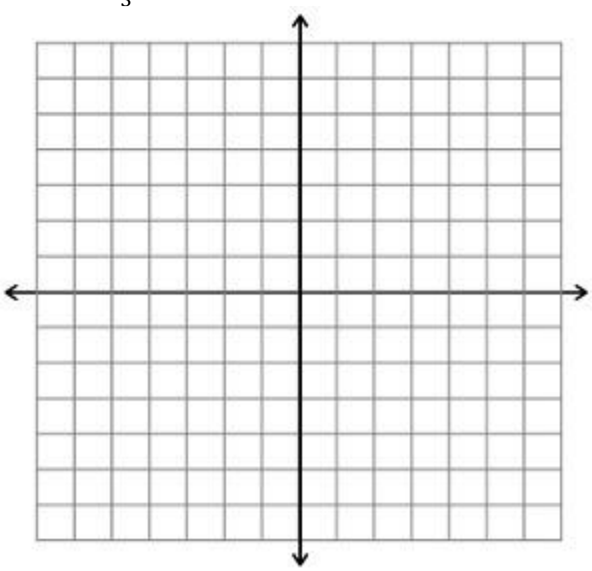
a. $m = 2$ and $(-1, 2)$



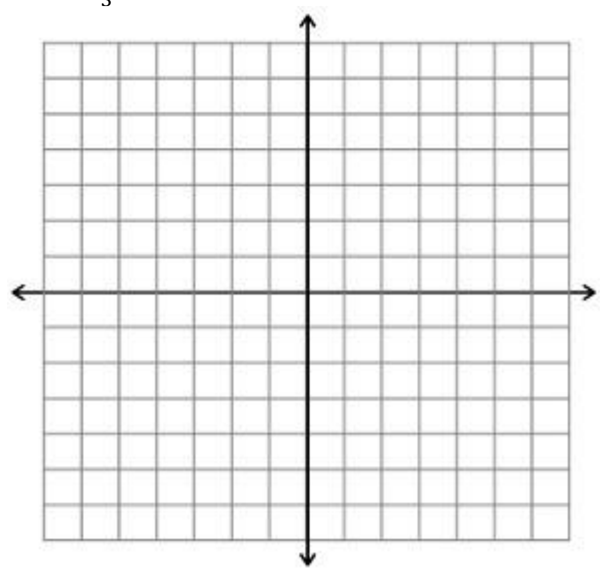
c. $m = 1$ and $(2, -1)$



b. $m = -\frac{1}{3}$ and $(-3, 4)$



d. $m = \frac{2}{3}$ and $(0, -3)$



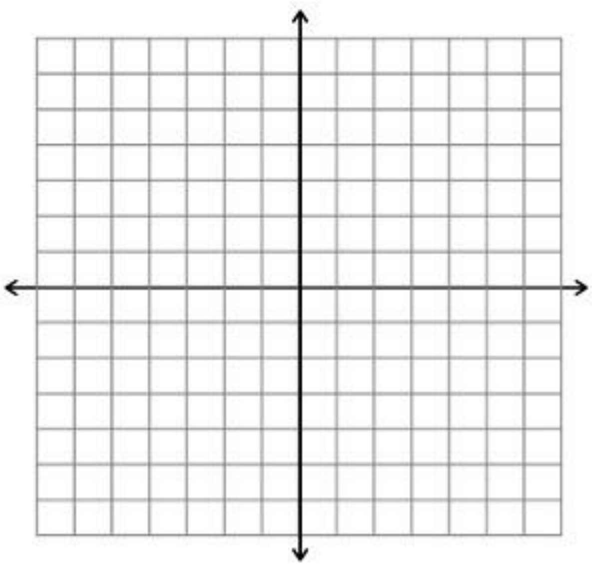
What is the slope-intercept form of a line?

Examples Identify the slope m and the y-intercept $(0, b)$ in each of the following equations:

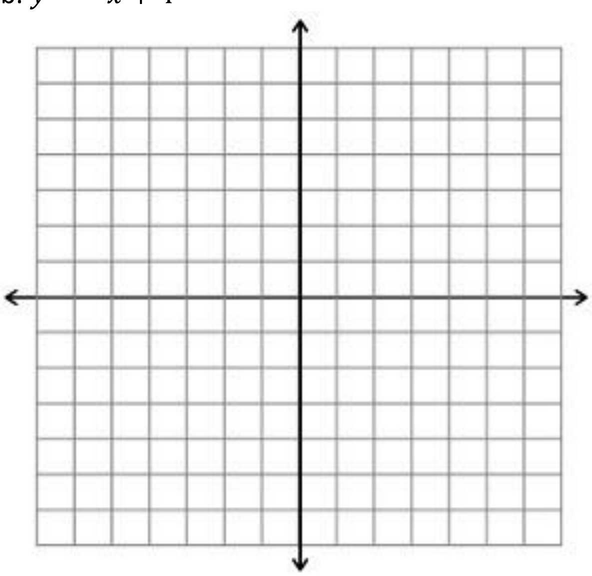
Equation	Slope, m	What stands for b	State the y-intercept (as a point)
a. $y = -2x + 3$			
b. $y = \frac{1}{4}x - 7$			
c. $y = 3x + 9$			
d. $y = x$			

Examples Graph the following lines:

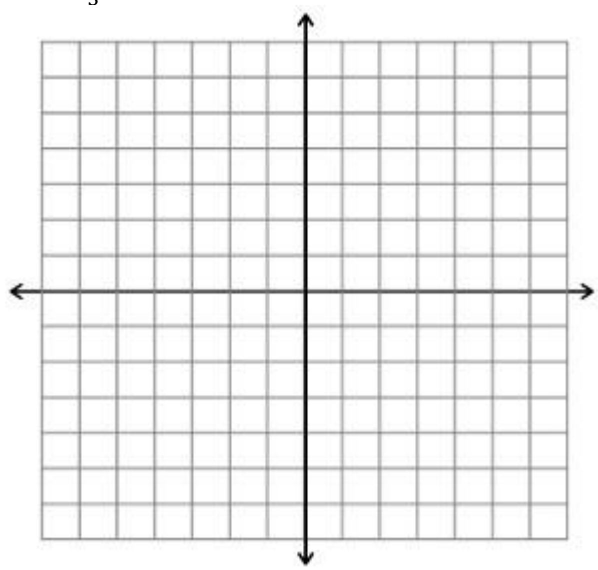
a. $y = 2x + 3$



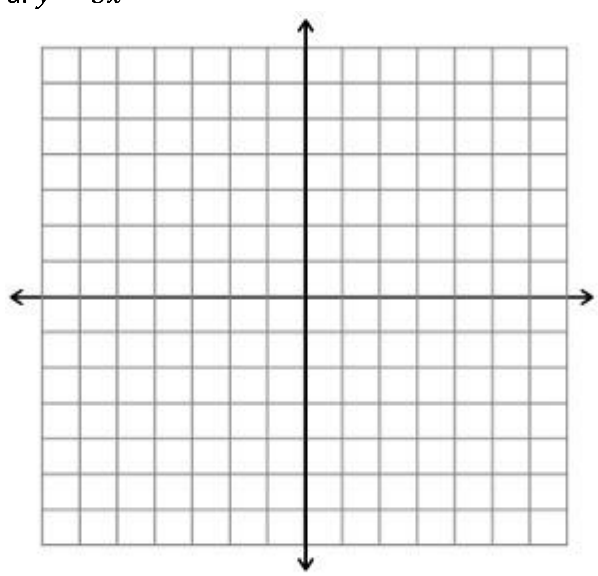
b. $y = -x + 4$



c. $y = \frac{1}{3}x - 4$



d. $y = 3x$



Lesson: Rewriting Linear Equations into Slope-Intercept Form of a Line

What is the slope-intercept form of a line?

Explain how to put the equation $2x - 4y = 8$ into slope-intercept form.

<u>Examples</u> Put the following equations in slope-intercept form:

a. $6y - 3x = 12$

b. $-5x + 2y = -4$

Lesson: Determining Whether Two Lines are Parallel or Perpendicular

What makes 2 lines parallel?

<u>Examples</u> Are the equations $y = \frac{2}{3}x + 5$ and $2x - 3y = 6$ parallel?

What makes 2 lines perpendicular?

<u>Examples</u> Determine the negative reciprocal of each of the following:

a. $\frac{2}{3}$

b. $-\frac{1}{5}$

c. -7

d. 1

Example Are the equations $y = \frac{3}{5}x + 2$ and $-3x + 5y = -15$ perpendicular?

Examples Are the following pairs of lines parallel, perpendicular, or neither?

a. $3x - 2y = 6$ and $4y = -6x - 8$

b. $y - 3x = 0$ and $12x - 4y = 24$

c. $y = 3$ and $x = -2$

Examples Lines A and B contain the given points. Determine whether the lines are parallel, perpendicular, or neither.

a. Line A: $(2, 11), (5, 10)$ Line B: $(1, 3), (-6, -18)$

b. Line A: $(-5, 5), (-7, -2)$ Line B: $(0, 11), (-4, 9)$

Lesson: Putting Equations in Standard Form

What is the standard form of a line?

<u>Examples</u> Write the following equations in standard form:

a. $y = 2x - 4$

b. $y = -5x + 3$

c. $y = -\frac{1}{2}x + 1$

d. $y = \frac{2}{3}x - 2$

Lesson: Writing Equations of Lines with The Point-Slope Form

What is the point-slope form of a line?

<u>Examples</u> Given the point and the slope, write the point-slope form of a line.

a. $m = -2, (1, 3)$

b. $m = -\frac{2}{3}, (5, -2)$

What is the slope-intercept form of a line?

What is the standard form of a line?

Examples Given the point and the slope, write the equation of the line in the desired format.

a. $m = -1, (2, -5)$ in slope-intercept form

b. $m = 3, (5, -4)$ in standard form

c. $m = -\frac{1}{2}, (6, 3)$

d. $m = \frac{2}{3}, (-5, -1)$

Lesson: Writing the Equation of a Line from Two Points

Write the equation of the line that contains the following 2 points in the desired format.

a. $(4, 2)$ and $(5, 5)$ in slope-intercept form

b. $(-2, 4)$ and $(-4, -2)$ in standard form

c.$(5, -3)$ and $(3, 5)$ in slope-intercept form

d. $(-1, -3)$ and $(1, 5)$ in standard form

Lesson: Writing the Equation of a Line Parallel/Perpendicular to Another Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

Examples Write the equation of a line with the given conditions.

a. Parallel to $y = 3x + 1$ that goes through $(-2, -5)$ in slope-intercept form

b. Perpendicular to $y = -\frac{2}{3}x - 5$ that goes through $(6, -2)$ in standard form

c. Parallel to $2x - 3y = 6$ that goes through $(1, -2)$ in slope-intercept form

Examples Write the equation of a line with the given conditions.

a. Perpendicular to $2x - 4y = 8$ that goes through $(-4, -2)$ in standard form

b. Parallel to $3x - y = 12$ that goes through $(-1, -5)$ in slope-intercept form

c. Perpendicular to $-2x + 3y = 12$ that goes through $(2, -5)$ in standard form

Lesson: Linear Inequalities in Two Variables

What is a linear inequality in 2 variables?

Show how to graph $y \leq 2x + 1$

Examples Graph the following:

a. $3x + 2y > -6$

b. $5x - 2y \leq 10$

c. $-3x + 4y < -12$

Lesson: Compound Linear Inequalities in Two Variables

What is a compound linear inequality?

What does "and" mean in compound inequalities?

What does "or" mean in compound inequalities?

Show how to graph $y > 2x + 1$ and $y \leq -x - 3$

Show how to graph $y \leq -2x + 1$ or $y > x + 2$

Examples Graph the following:

a. $3x - 2y > 6$ and $2x + y \leq 3$

b. $5x - 2y \leq 10$ or $2x + 6y > 12$

Examples Graph the following:

a. $y \geq 4$ and $y < -2x + 1$

b. $x < 3$ or $4x - 2y \leq 8$

c. $x > -1$ and $y \geq 2x + 2$

Lesson: An Intro to Functions

Define a relation:

Define a domain:

Define a range:

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, define the domain and range of the relation.

Define a function:

What is the visual explanation that explains what a function is or is not?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, is this a function?

Example Are the following functions?

a. $\{(4,-1),(3,2),(7,-1)\}$

b. $\{(2,4),(3,5),(2,-1)\}$

What is the notation for a function? How do we describe/use this notation?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, what is $f(4)$?

Example Suppose $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find the value of the following:

a. $f(2)$

b. $g(1)$

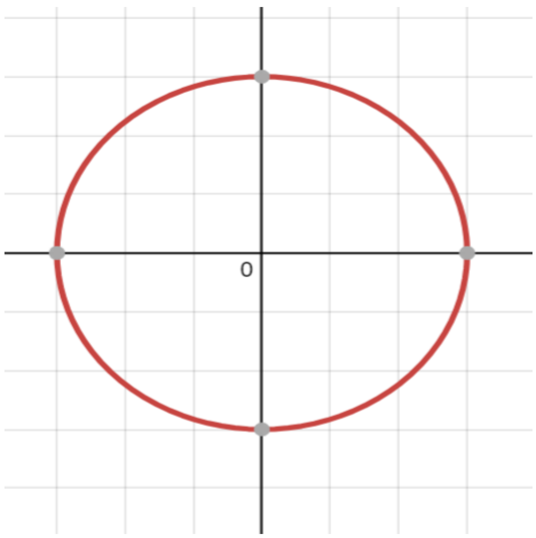
c. $f(a)$

d. $g(h + k)$

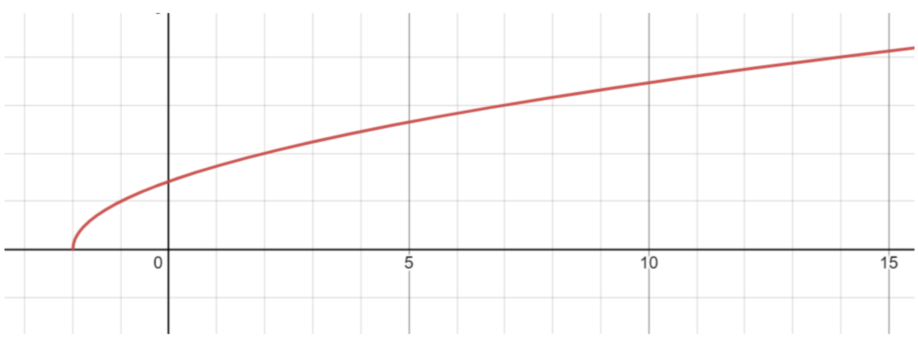
What is the vertical line test?

<u>Example</u> Do the following graphs represent a function?

a.



b.



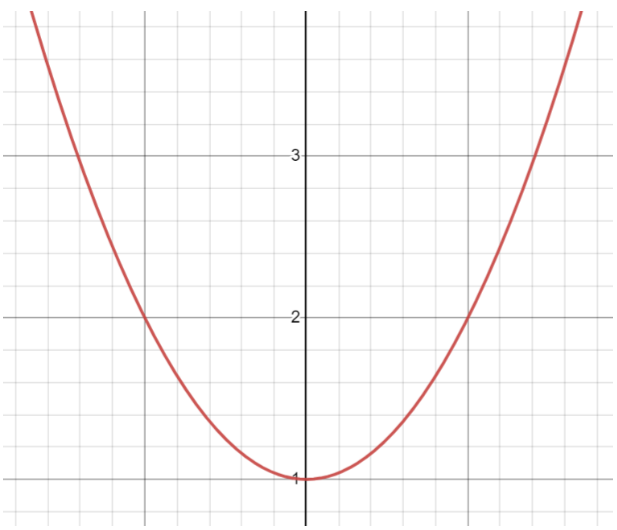
Lesson: Determining the Domain and Range of Functions In General

How do you determine the domain and range of a set of points like $\{(2,4),(5,8),(4,6)\}$?

What is the domain and range of that set of points?

In general, it is easiest to determine the domain and range when we have the graph in front of us.

For the following graph, write out how you should determine the domain and range.



Write a sentence on how you should find the domain:

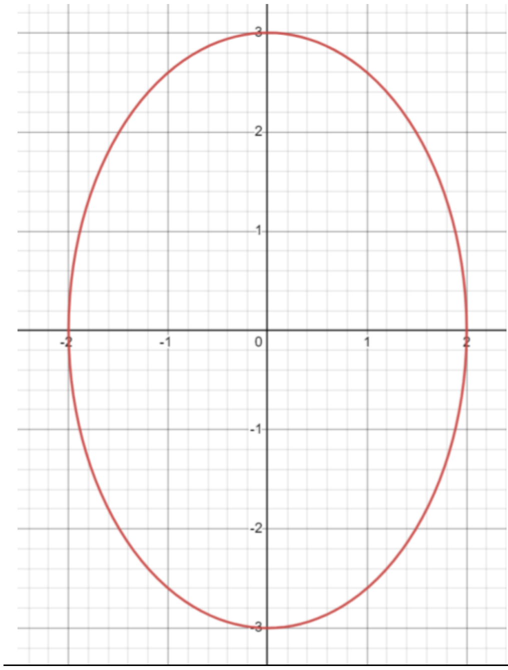
State the domain:

Write a sentence on how you should find the range:

State the range:

Examples Determine if the following graphs represent functions, and then state the domain and range.

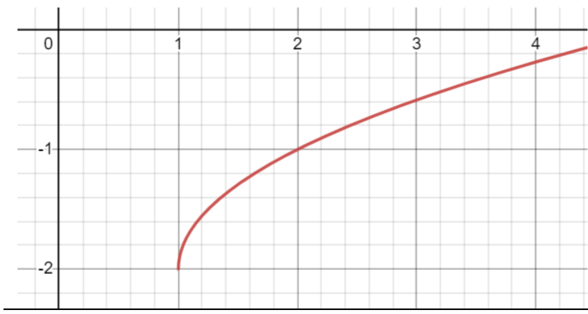
a.



Domain:

Range:

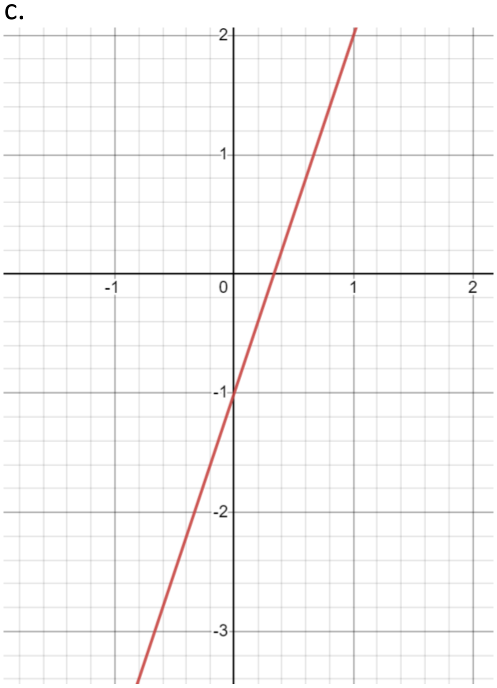
b.



Domain:

Range:

c.



Domain:

Range

Lesson: The Domain of Some Specific Types of Functions

What is a polynomial? Write some examples.

What is the domain of any polynomial function?

What is a rational function?

What is the domain of any rational function?

Examples Determine the domain of the following.

a. $f(x) = 3x - 1$

b. $g(x) = \frac{4}{x+2}$

c. $h(x) = 3x^3 - 2x^2 + 4x - 1$

d. $f(x) = \frac{2x+1}{x^2-9}$

What is a square root function?

What is the domain of a square root function?

<u>Examples</u> Find the domain of the following:

a. $f(x) = \sqrt{x-2}$

b. $g(x) = \sqrt{4x-1}$

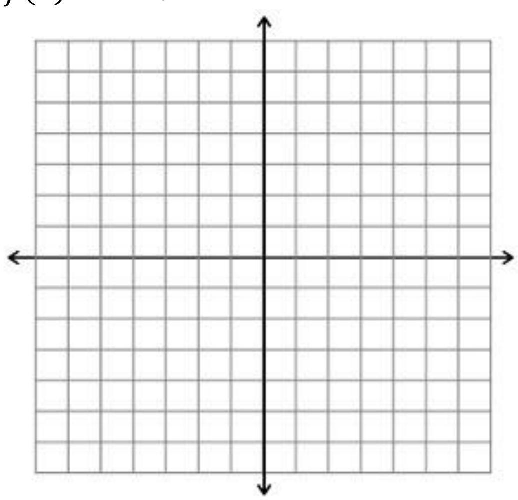
c. $h(x) = \sqrt{4-x}$

Lesson: How to Graph a Function

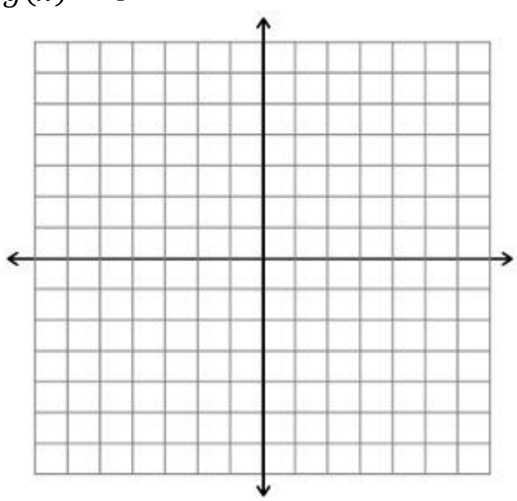
How do you graph a function?

Examples Graph the following functions.

a. $f(x) = 3x + 1$



b. $g(x) = 4$



Lesson: Solving Systems of Equations by Graphing

We will now turn our attention to systems of equations.

System of Equations

A system of equations refers to 2 or more equations that share 2 or more variables.

Solution of a System of Equations

A solution of a system of equations is an ordered pair that is true in all equations.

Determine if the following pairs are solutions to the systems.

a. $(1,3)$
$x - 2y = -5$
$2x - 3y = -4$

b. $(-4, 3)$
$x + 3y = 5$
$2x + 3y = 1$

There are several ways to solve systems. The first method we will focus on is how to solve by graphing.

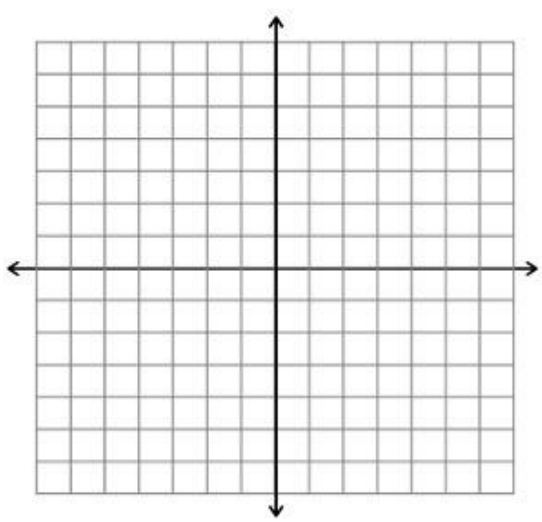
Solving by Graphing

In this method, we graph both lines and determine the point of intersection if possible. We will examine all the possibilities.

Example

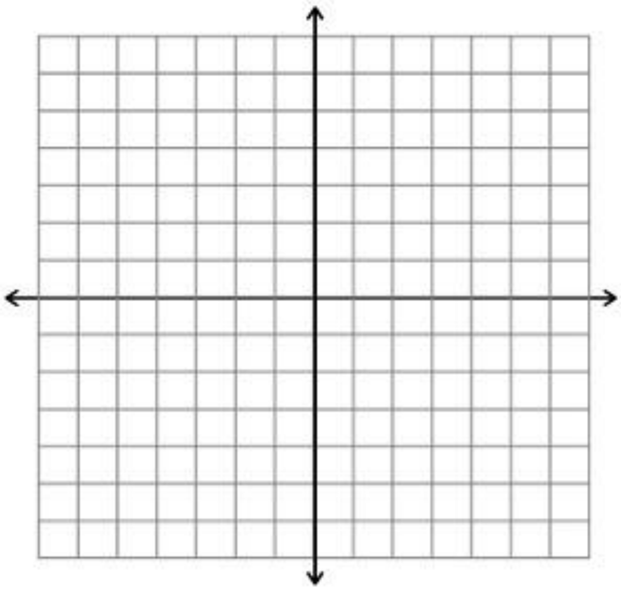
Graph each line and determine the point of intersection to find the solution.

$y = 2x + 3$
$y = -\frac{1}{2}x - 2$

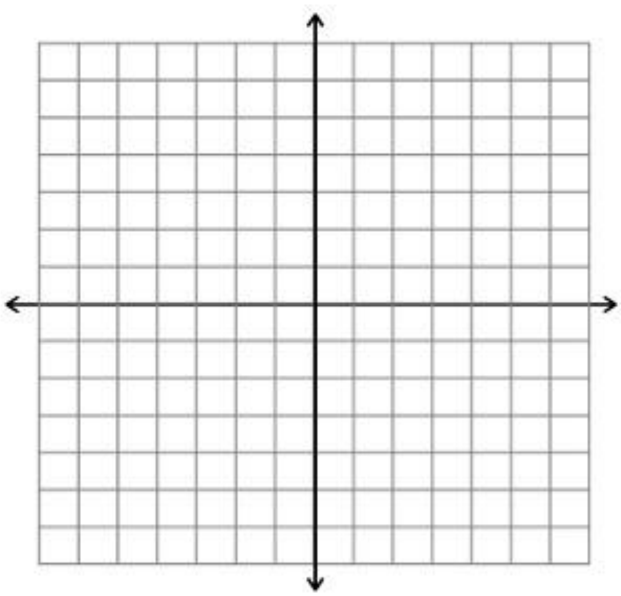


Example

a. $y = -3x + 1$
$y = 2x - 4$



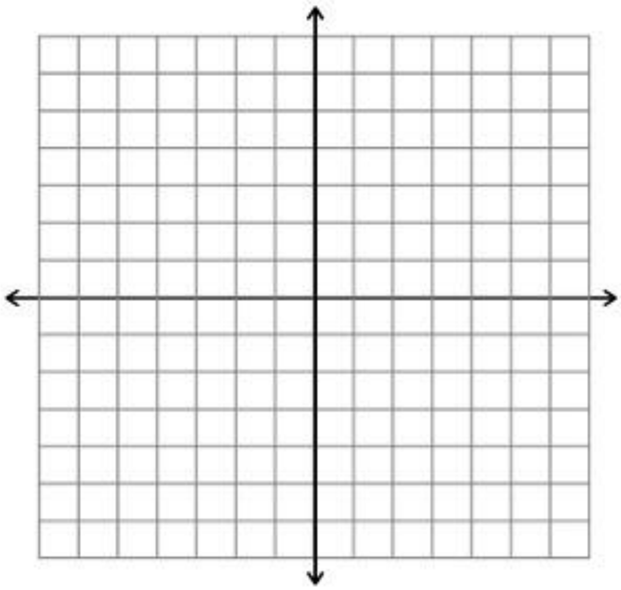
b. $y - \frac{1}{2}x = -4$
$y + 2x = 1$



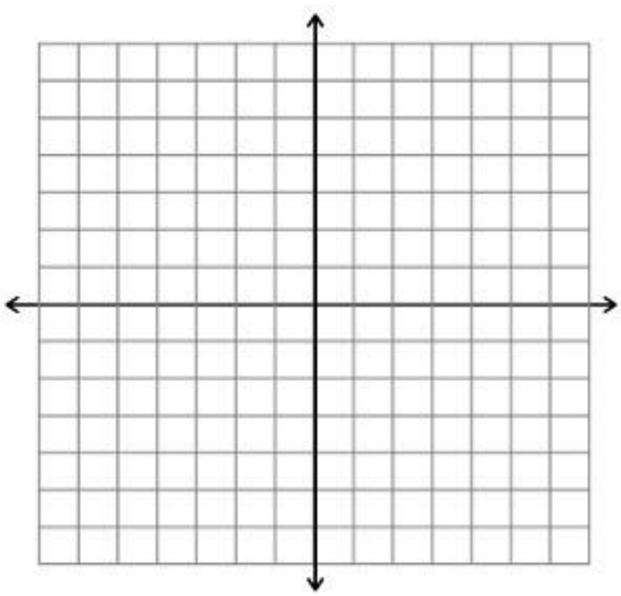
There are two special situations that can occur with graphing.

Example

a. $y = -3x + 2$
   $9x + 3y = 6$



b. $y = -\frac{1}{2}x + 4$
   $2x + 4y = 12$



Lesson: Solving Systems of Equations by Substitution

In this lesson, we will focus on the second method for solving systems: substitution.

Show how to use substitution with this example:

$2x + y = 6$

$3x + 5y = 9$

Example Solve the system using substitution

$x - 2y = 10$

$5x - 7y = 20$

Below are 2 more complicated problems using substitution. Outline the steps shown in the video:

$-2x + 4y = 4$

$-2x + 6y = 6$

$2x - 5y = -4$

$8x - 9y = 6$

Now let’s see what happens when we get a problem with no solution or infinite solutions.

a. $9y - 18x = 5$

$2x - y = 3$

b. $x + 2y = -3$

$4x + 12 = -8y$

Lesson: Solving Systems of Equations by Elimination

In this lesson, we will focus on the third method for solving systems: elimination.

Show how to use elimination with this example:
$3x - y = 8$

$2x + y = -3$

Example Use elimination to solve the system.

$4x + 2y = -6$
$3x - 2y = 6$

Now let's examine a more complicated case of elimination. Write down how to approach these problems:

a. $3x - 6y = 12$

$5x - 2y = -4$

b. $5x - 10y = 5$

$2x - 3y = 6$

Now let's investigate what happens in the situation where there is no solution or infinite solutions.

a. $5x - 2y = 1$
$-10x + 4y = 3$

b. $3x + y = 5$
$-9x - 3y = -15$

Lesson: Solving Systems of Equations that Require Clearing of Fractions

<u>Examples</u>

a. $\frac{1}{6}x - \frac{1}{3}y = \frac{1}{2}$
$\frac{2}{5}x - \frac{3}{5}y = -\frac{6}{5}$

b. $-\frac{1}{2}x - \frac{3}{4}y = 1$
$\frac{1}{3}x + \frac{3}{2}y = -\frac{2}{3}$

c. $\frac{1}{2}x + \frac{3}{5}y = 6$

$\frac{1}{4}x - \frac{1}{2}y = -9$

Lesson: Solving Systems of Equations that Require Clearing of Decimals

<u>Examples</u>

a. $0.02x + 0.1y = 0.05$
$4x + 20y = 10$

b. $0.1x - 0.05y = 0.2$
$0.3x - 0.25y = 0.2$

c. $-0.4x + 1.3y = 1.$
$1.6x - 0.9y = 6.9$

Lesson: Applications of Systems

The top of a rectangular desk is twice as long as it is wide. The perimeter of the desk is 180 inches. What are the dimensions of the desk?

Sarah bought 3 shirts and a scarf for $53. Marie bought 2 shirts and 3 scarves for $54. Find the cost of each item.

How many milliliters of a 5% acid solution and how many milliliters of a 17% acid solution must be mixed to obtain a 60 mL solution that is 15% acid?

Lesson: Systems of Equations with 3 Equations

Show how to solve the system:

$x + 2y + 3z = -11$

$3x - y + z = 0$

$-2x + 3y - z = 4$

Sometimes you will have missing variables. Show how to solve the following system that is missing some variables:

$x - 2y = -3$

$x + 6z = 3$

$2y - 4z = 6$

Now let's look at an example where there is no solution versus infinite solutions:

$8x + 20y - 4z = 16$

$-2x - 5y + z = -4$

$10x + 25y - 5z = 20$

$x + 4y - 3z = 2$

$2x - 3y + 2z = -5$

$3x + 12y - 9z = 2$

Examples: More Examples of Systems of 3 Equations

Example Solve the system:

$x + 3y - z = -2$

$-x + 3y - 2z = 5$

$2x + 4y + 2z = 4$

Example Solve the following system:

$x - 2z = 2$

$-4y + 2z = -2$

$-x + 3y = -3$

Example Solve the following system:

$x - 4y + 2z = -7$

$3x - 8y + z = 7$

$6x - 12y + 3z = 12$

Lesson: Basic Exponent Rules – Product, Power, and Quotient Rules

When working with rules of exponents, it is very important to be to identify the base of each exponent. To begin, will will practice this.

Example 1

State the base for each exponent in the problems below.

Expression	Exponent	Base for this Exponent
5^3		
-5^4		
$-5x^6$		
$3y^7$		

Product Rule

First we want to remember what the actual definition of an exponent means.
We want to find: $2^3 * 2^2$. We will calculate this a few ways to understand the rules.

First we will work towards find the exact answer. Evaluate each exponent below as you normally would with the **order of operations**, and then multiply the numbers together:

Now we will re-examine this result solely from the viewpoint of exponents. Write out each exponent as a multiplication problem, and notice the results you get from this exercise.

What do you notice about the exponents?

Product Rule (formal mathematical definition)

Let x a real number, and let a and b be integers. Then:

Example 2

Use the product rule to evaluate the following.

a. $x^2 * x^4$

c. $x^3 + x^7$

b. $3x^3 * -1x^4 * 2x$

d. $\frac{2}{3}x^4y^2z^5 * 6x^3y^2 * \frac{1}{2}y^3z^4$

Power Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$(2^3)^2$ =

Now rewrite this entire result just utilizing what we know about the definition of exponents:

What is the relationship between the exponents in this case?

The Power Rule (formal mathematical definition)

Let x a real number, and let a and b be integers. Then:

Example 3

Use the power rule to evaluate the following:

a. $(x^3)^5$

c. $(3x^4y^5)^3$

b. $(x^2y^7z)^4$

d. $\left(\frac{2}{3}x^5y^2\right)^3$

Is there a difference between $(x+y)^n$ and $(xy)^n$? Explain.

The Quotient Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$\frac{2^5}{2^3}$

First we will evaluate and find the final answer by using the order of operations below?

Notice the final answer of this question. How can we rewrite this number in it's exponent form?

What relationship do you see between the exponents in the original question and the final answer?

The Quotient Rule (formal mathematical definition)

Let x a real number, and let a and b be integers. Then:

Example 4

Use the quotient rule to simplify the following:

a. $\frac{x^7}{x^3}$

c. $\frac{15x^4y^3z^{10}}{2x^2y^2z^5}$

b. $\frac{x^5y^8z^{10}}{xyz}$

d. $\frac{12x^7y^3}{8x^3z^4}$

Examples: Exponent Rules, Basic Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not contain negative exponents.

a. $x^2y^7 * 5x^3y^5z$

b. $(4x^7y^3)^2$

c. $\frac{15x^7y^3}{10x^2y}$

d. $\frac{2}{5}x^2y^5 * \frac{1}{3}x^7yz^3$

e. $\left(\frac{5}{3}x^3y^2z^3\right)^3$

f. $(-3x^5)^2$

g. $(-3x^5)^3$

h. $(2x^4y^2z^6)^2 * 3x^2z^3$

i. $\frac{(x^3y^7)^2}{x^4y}$

j. $\frac{(x^4y^2)^3}{(x^2y)^2}$

Examples: Exponent Rules, With Numbers Only

Simplify the following using only the product, power, and quotient rules. Your final answer should be a number.

a. $3^2 * 3$

b. $(-4)^2 * 2$

c. $\frac{2^7}{2^4}$

d. $\left(\frac{2}{3}\right)^3 * 3^2$

e. $\frac{6^2}{2^3}$

f. $\left(\frac{1}{2}\right)^4 * 2^2$

Examples: Exponent Rules, Trickier and Harder Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not include negative exponents.

a. $(3x^2y^4)^2 * (x^4y^5)^3$

b. $\frac{(2x^5y^2)^3}{(x^2yz^5)^2}$

c. $\left(\frac{2}{3}x^5y^7z^3\right)^2 * 9x^3y^3$

d. $\left(\frac{3}{5}x^5y^2z^7\right)^2 * \frac{xy^3}{z^5}$

e. $\frac{(3x^5y^2z^4)^3}{(6x^2yz^3)^2}$

f. $\frac{(4x^3y^2z^5)^2}{(2xz^2)^4}$

Lesson: Negative Exponents

Simplify: $\frac{25}{125}$

If we wrote these numbers as exponents, this calculation would look like this:

$$\frac{5^2}{5^3} = \frac{1}{5}$$

Consider what would happen if we used the quotient rule on this problem:

$$\frac{5^2}{5^3} = 5^{2-3} - 5^{-1} = \frac{1}{5}$$

This problem shows us that exponents can also be negative.

<u>Three Rules for Negative Exponents</u>

Let x and y be a real numbers, and let n be an integer. Then:

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{1}{x^{-n}} = x^n$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$$

In general your final answer cannot have a negative exponent, so it is essential that you understand what to do with it.

Example 1

Rewrite the following expressions without negative exponents.

a. 4^{-2}

b. $\frac{1}{3^{-3}}$

c. $\left(\frac{2}{5}\right)^{-2}$

d. $\left(\frac{1}{3}\right)^{-2}$

One of the biggest challenges with negative exponents is identifying how to apply them to the proper base. As a quick review, remember how we identified the bases in the last section:

Example 2

Identify the base of the following exponents:

Expression	First Base	First Exponent	Second Base	Second Exponent
3^5				
$-2x^5$				
$5x^3y^7$				

This is extremely important to remember when working with negative exponents. In general, our final answer can't have negative exponents so you need to be used to reformatting your answers properly if negative exponents come up.

Example 3

Simplify the following expressions, and write your final answer without negative exponents.

a. $\frac{a^3b^{-2}}{c^{-4}d^6}$

b. $\frac{3a^{-3}b^4c^{-5}}{d^2e^{-2}f^5}$

c. $\frac{a^5b^3c^4}{4d^{-2}e^{-4}f^{-2}}$

d. $\frac{a^{-3}b^{-2}c^{-6}}{d^4e^7f}$

With this in mind, we should now be able to work on combining all of the rules of exponents we've learned.

Example 4

Simplify the following. Your final answer cannot contain negative exponents.

a. $(3x^{-2}y^{4})^{3}$

b. $\left(\frac{2x^{-4}y^{6}}{x^{5}y^{-3}}\right)^{4}$

c. $(-2x^{3}y^{-2}z)^{-3}$

d. $\left(\frac{2x^{3}y^{-4}}{3x^{-2}y^{-5}}\right)^{-3}$

Examples: Negative Exponents, Basic Examples

Your final answer should not contain negative exponents.

a. $\left(\frac{4}{5}\right)^{-2}$

b. $3x^{-6}y^4z^{-2}$

c. $\frac{4x^{-4}y^7}{z^{-5}}$

d. $\frac{6x^3y^{-5}}{x^7y^2}$

e. $\frac{14x^{-3}y^5z^3}{21x^{-2}y^{-3}z^9}$

f. $(3x^{-4}y^{-2}z^4)^2$

g. $\left(\frac{5x^2y^{-4}}{z^2}\right)^5$

h. $(x^{-3}y^2z^{-4})^{-3}$

Examples: Negative Exponents, Tricky Examples

Your final answer should not contain negative exponents.

a. $(3x^{-4}y^5z^{-2})^{-3}$

b. $(-5x^2y^{-3}z^4)^{-2}$

c. $\left(\frac{4x^{-5}y^2}{x^{-3}y^{-5}}\right)^{-2}$

d. $\left(\frac{3x^3y^{-3}z^4}{2x^{-2}y^4z^{-2}}\right)^{-3}$

e. $\frac{(2x^{-4}y^5)^4}{x^{-3}y^7}$

f. $\frac{(3x^{-3}y^2z^{-3})^{-3}}{(2x^4y^{-5}z^3)^{-4}}$

Examples: Negative Exponents versus Negative Numbers

<u>Examples</u> Simplify:

a. 6^{-2}

b. -6^2

c. $(-6)^2$

d. $(-6)^{-2}$

<u>Examples</u> Simplify:

a. $5x^{-2}$

b. $(5x)^{-2}$

c. $-5x^2$

d. $(-5x)^{-2}$

Examples Simplify

a. -7^2

b. $(-7)^{-2}$

c. 7^{-2}

d. -7^{-2}

Examples Simplify

a. $3x^{-2}$

b. $(-3x)^{-2}$

c. $(-3x)^{-3}$

Lesson: Scientific Notation

Scientific Notation
A number in scientific notation is of the form: $A \times 10^n$ where $1 \le |A| < 10$ and n is an integer

Before we can write numbers in this form, it is best if we can identify examples that are or are not in this format.

Example 1
Identify if the following examples are in scientific notation. If not, circle and explain why.

a. 5.34×10^7 Yes, it's in scientific notation No, it's not in scientific notation

b. $-2.7832 \times 10^{\frac{1}{2}}$ Yes, it's in scientific notation No, it's not in scientific notation

c. -4.5382×10^{-8} Yes, it's in scientific notation No, it's not in scientific notation

d. 0.8×10^{23} Yes, it's in scientific notation No, it's not in scientific notation

We will now examine how to rewrite the following example in scientific notation.
Example 1
Rewrite the following in scientific notation.

Example 2
Rewrite the following examples in their expanded form.

Example 3
Use the rules of exponents to multiply the following results together.

a. $(2 \times 10^{6}) \times (3 \times 10^{8})$

b. $(2 \times 10^{17}) \times (2.1 \times 10^{23})$

c. $(8 \times 10^{7}) \times (2 \times 10^{-3})$

Example 4
Use the rules of exponents to divide the following results.

a. $\frac{8 \times 10^{12}}{4 \times 10^{3}}$

b. $\frac{6 \times 10^{7}}{2 \times 10^{19}}$

c. $\frac{3 \times 10^{-12}}{6 \times 10^{-5}}$

Example 5
Add the following:

a. $(5 \times 10^{78}) + (2 \times 10^{78})$

b. $(8 \times 10^{6}) + (2 \times 10^{6})$

c. $(3 \times 10^{4}) + (2 \times 10^{3})$

Lesson: A Review of Exponent Rules for Algebra Students

Power Rule: $x^m * x^n =$

Product Rule: $(x^m)^n =$

Quotient Rule: $\frac{x^m}{x^n} =$

Negative Exponents

$x^{-n} =$

$\frac{1}{x^{-n}} =$

$\left(\frac{x}{y}\right)^{-n} =$

Simplify the following using exponent rules. Your final answer should not contain negative exponents.

a. $3x^7y^{-4} * 5x^{-2}y^9$

b. $\left(\frac{2x^4y^5}{3z^5}\right)^3$

c. $\frac{10x^4y^7}{2x^2y^3z^{-5}}$

d. $\frac{4x^3y^{-2}}{6x^{-4}y^{-5}}$

e. $\frac{8x^{-3}y^4z^{-3}}{10x^5y^{-2}z^{-7}}$

f. $\left(\frac{3x^{-3}y^{-2}}{6x^2y^4}\right)^{-2}$

g. $\left(\frac{2x^{-7}z^3}{3x^{-2}y^{-3}z^5}\right)^{-3}$

h. $\left(\frac{5x^{-2}y^{-3}z^3}{x^3y^4z^{-7}}\right)^{-2}$

Lesson: An Intro to Polynomials

We can extend of knowledge of exponents to another algebraic topic – polynomials

Before we define what a polynomial is, it is usually easier to see a few examples of what is or is not a polynomial.

The following ARE polynomials:

- $x^2 - 3x + 8$
- $x^7 - 5x^4 + 2x^3 + x$
- 9
- $7x + \pi$
- $x^3 + \frac{2}{3}$

The following ARE NOT polynomials:

- $\sqrt{x} - 4$
- $4x^{-4} - 7x^{-3}$
- $\frac{4}{x} - \frac{2}{x+3}$

Now we can more formally define a polynomial.

Polynomial
An expression with real numbers, variables, and whole number exponents.

Descending Order
Ordering a polynomial from the largest exponent to the smallest exponent, and any freestanding number without a variable is last

Example 1
Put the polynomials in descending order.

a. $x^3 + 4x^5 - 2x^7 + 3x^2 + 5x^8$

b. $x^2 - x^3 + x^4 - x^5 - x^6 + 3$

Term
Any part in a polynomial that is separated by a "+" or "-" sign is considered to be its own term

Variables
The letters in a polynomial that represent numbers

Coefficients
The numbers attached to the variables

Constant Term
The freestanding number within a polynomial that is not attached to any variables

Example 1
Examine the polynomials, then determine the variables, coefficients, and constant terms

Polynomial	Terms	Variable(s)	Coefficient(s)	Constant
$2x + 3$				
$3x^2 - x + 5$				
5				
$2x^2y + 3y - x$				

Degree of a Term
The degree of a term is the sum of the exponents of that term.

Example 2
Identify the degree of each term

Term	Degree	Term	Degree
x^4		x^4y^3	
$3y^6$		x^7y^9	
x		$x^4y^2z^5$	
6		xy^4	

Degree of a Polynomial

The degree of a polynomial is the same as the degree of the term of highest degree within that polynomial

Leading Coefficient
The leading coefficient is the coefficient attached to the term of highest degree in a polynomial

Trinomial
A polynomial with 3 terms

Binomial
A polynomial with 2 terms

Monomial
A polynomial with 1 term

Example 3
Identify the degree of each polynomial and the type of polynomial it is.

Polynomial	Degree of Each Term, Respectively	Deg of Polynom	Leading Coefficient	Type of Polynomial
$x^5 + 4x^3 - 2x$				
$-5x^2 + 18x$				
$6x + 2x^4 - 3x^2$				
$x^5 + 2x^7 - 4$				
8				

Evaluating Polynomials

Evaluating polynomials is not unlike evaluating other expressions we've seen in this course.

Example 4

Evaluate the polynomials for $x = -2$.

a. $x^2 + 3x + 1$

b. $x^3 - x^2 - 3x + 4$

c. $\frac{1}{4}x^3 - \frac{1}{2}x^2 + 5x$

Lesson: How to Add and Subtract Polynomials

To add or subtract polynomials, combine like terms.

<u>Examples</u>

Perform the following operations.

a. $(2x + 4) + (5x^2 - 3x + 1)$

b. $(5x^2 - 3x + 1) - (x^3 - 2x^2 - x + 1)$

c. $(3x^3 + 2x^2 - 5x) + (4x^3 + 5x^2 + 2x)$

d. $(2x^2 + 5x - 4) - (x^3 - 2x^2 + 3x - 5)$

Lesson: How to Multiply Polynomials

To begin, let's first focus on the process of distributing a monomial with a polynomial.

<u>Examples</u> Multiply the following:

a. $3x(4x^2 + 5x - 7)$

b. $x^2y^4(3x^2 + 2xy - 8y^4)$

c. $\frac{1}{2}x^3y^4(4xy^3 - 2x^3y + 8xy)$

Now write down the process for how to multiply $(x + 2)(x^2 + 3x)$

<u>Examples</u> Multiply the following:

a. $(x+3)(x+2)$

b. $(3x+1)(x-6)$

c. $(x^2+4)(x-2y)$

d. $(x+4)(x-4)$

e. $(x+3)(x^2+4x+5)$

f. $(2x^2+3x+1)(3x^2-4x+2)$

How do you multiply $(x+5)^2$?

Examples Multiply the following:

a. $(x+2)^2$

b. $(2x^2+1)^2$

c. $(3x^2+x+5)$

How do you multiply 3 polynomials together, such as $(x+2)(x+1)(x-3)$?

Examples Multiply the following together:

a. $(x-3)(x+4)(x-1)$

b. $(2x+1)(x-2)(x-1)$

Lesson: The Basics of Dividing Polynomials

In this section, we will discuss how to divide polynomials that are in these forms:

$\frac{x^2+3x}{x}$ or $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$

To divide in these cases, you can just use rules of exponents. Sometimes you will want to separate the problem into separate fractions.

<u>Examples</u>

a. $\frac{x^2+3x}{x}$

b. $\frac{5x^4-10x^3+15x^2}{5x}$

c. $(27x^4y^2 + 18x^3y) \div 9x^2y$

d. $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$

Lesson: Long Division for Polynomials

In this section, we will discuss how to divide polynomials that are in these forms:

$(x^2 + 5x + 6) \div (x + 3)$ or $(2x^2 + 5x + 3) \div (2x + 3)$

In these types of problems, we need to use long division. In another video, we talked about how to divide polynomial problems like these:

$\frac{4x^3+2x^2}{2x}$

But we did not need long division. What is different about these types of problems that require long division?

Show how to use long division to divide $(x^2 + 5x + 6) \div (x + 3)$

<u>Examples</u> Divide the following.

a. $(x^2 + 7x + 10) \div (x + 5)$

b. $(2x^2 + 5x + 3) \div (2x + 3)$

c. $\frac{x^2 - 11x + 30}{x - 5}$

Now we will look at what to do when there is a remainder.

Divide the following: $(x^2 + 3x + 1) \div (x + 2)$

Now show how to divide when there are "missing" terms, such as with $(x^4 - 81) \div (x^2 - 9)$

Examples Divide.

a. $(5x^2 - 10x^3 + 8 - 9x) \div (x + 2)$

b. $(x^3 + 1) \div (x + 1)$

c. $(5x^3 - 4x^2 + 1) \div (x + 2)$

Lesson: The Greatest Common Factor

The Greatest Common Factor (GCF)

The Greatest Common Factor is the biggest common factor between a set of terms.

Examples

Find the greatest common factor in the following polynomials, then factor the polynomial.

a. Polynomial: $3x + 3y$

GCF:

Factor out the GCF:

b. Polynomial: $3xy + 6xz$

GCF:

Factor out the GCF:

c. Polynomial: $10x^2 - 25xy + 15xz$

GCF:

Factor out the GCF:

d. Polynomial: $3x^7y + 8x^3z^5 - 5y^4z^3$

GCF:

Factor out the GCF:

Examples

Factor the following polynomials.

a. $12xy^2 + 10x^2y^4 - 14xy^5$

b. $5xy - 10yz + 12xz$

c. $3x + 6x^2 - 12x^3$

d. $5x^4y^2z^5 + 10x^2y^4z^6 - 25x^5yz^3$

Lesson: Factoring by Grouping

Grouping can best be understood with the following example.

Factor $ax + ay + bx + by$ by grouping:

Examples

Finish the following examples where the grouping had been started.

a. $3(x - y) + 2z(x - y)$

b. $2a(3b + 2c) - 5(2c + 3b)$

c. $4x(y - z) - 3(z + y)$

d. $4z(4x - 7) - (4x - 7)$

Examples

Factor the following.

a. $5x + 10y + xz + 2yz$

b. $10x^2 + 15x^3y - 2 - 3xy$

c. $3xy + 4y - 27x - 36$

d. $3xy + 6x + 21y + 42$

e. $3x^2 - 48y + 8x - 18xy$

Lesson: The Basics of Factoring Trinomials of the Form $x^2 + bx + c$

Factoring trinomials will require us to multiply polynomials, so first we will review this skill.

Examples

Multiply.

a. $(x+3)(x-5)$

b. $(2x-y)(5x+2y)$

Definition of a Trinomial

A trinomial is a polynomial with three terms.

How to Factor a Trinomial Using Trial and Error

Show how to factor $x^2 + 3x + 2$

Show how to factor $x^2 + 7x + 12$

Examples

Factor the following.

a. $x^2 + 6x + 5$

b. $x^2 + 12x + 11$

c. $x^2 - 8x + 15$

d. $x^2 - 13x + 30$

e. $x^2 + 10x - 24$

f. $x^2 - 5x - 14$

g. $2x^2 + 16x + 30$

h. $5x^4 + 55x^3 - 45x^2$

Lesson: The Basics of Factoring Trinomials of the Form $-x^2 + bx + c$

How can you factor $-x^2 - 5x - 6$?

What about $-x^2 - 15x - 36$?

<u>Examples</u>

Factor the following.

a. $-x^2 + 13x - 30$

b. $-x^2 + 16x - 55$

c. $-x^2 - 2x + 3$

Lesson: The Basics of Factoring Trinomials of the Form $\boldsymbol{x^2 + bxy + cy^2}$

How do you factor $x^2 + 5x + 6$?

How do you factor $x^2 + 5xy + 6y^2$?

<u>Examples</u> Factor the following.

a. $x^2 + xy - 12y^2$

b. $x^2 - 2xy + y^2$

c. $3x^2 + 21xy + 18y^2$

Lesson: The Basics of Factoring Trinomials of the Form $ax^2 + bx + c$

In this lesson we will focus on trinomials that do not have a leading coefficient.

To begin, first let's get warmed up by factoring the following using trial and error:

a. $x^2 + 7x + 12$

b. $x^2 - 3xy + 10y^2$

How do you factor $2x^2 + 5x + 3$ using trial and error?

How do you factor $3x^2 + 16x + 5$ using trial and error?

Another way to factor $3x^2 + 16x + 5$ is by grouping. Show how to use this technique below:

Examples Factor the following:

a. $7x^2 - 11x + 4$

b. $3x^2 + 10x + 8$

c. $5x^2 - 11x - 36$

d. $3x^2 - 17xy - 6y^2$

e. $-10x^2 + 19x - 6$

Lesson: Factoring Special Trinomials

How do you factor $x^2 + 6x + 9$?

What's another way to write this factorization?

What do you call this type of trinomials?

Examples Factor the following:

a. $x^2 + 12x + 36$

b. $9x^2 - 12x + 16$

c. $x^2 - x + \frac{1}{4}$

d. $36x^2 - 60xy + 25y^2$

How do you factor $x^2 - 4$?

What is this type of factoring called?

<u>Examples</u> Factor the following:

a. $x^2 - 25$

b. $x^2 - 16$

c. $x^2 - 1$

d. $25x^2 - 36$

e. $16x^2 - 81y^2$

f. $16 - x^2$

g. $25y^2 - 64x^2$

h. $\frac{49}{64} - x^2$

Lesson: How to Factor The Sum and Difference of Cubes

Write down the formula for the sum of cubes

$a^3 + b^3 =$

Write down the formula for the difference of cubes

$a^3 - b^3 =$

Show how to factor $x^3 + 8$

Show how to factor $8x^3 - 27$

<u>Examples</u> Factor:

a. $x^3 - 1$

b. $x^3 - \frac{1}{8}$

c. $x^3 + 64$

Review: An Overview of All the Types of Factoring

We will review all of the different types of factoring by presenting an example of each type.

Greatest Common Factor

1. $4x^2 + 8xy$

2. $6x^3y^2 - 8x^2y^5 + 10x^5y$

Grouping

3. $3x + 6y - xz^2 - 2yz^2$

4. $12x^2 - 2y + 3x - 8xy$

Basic Trinomials of the Form $\boldsymbol{x^2 + bx + c}$

5. $x^2 + 7x + 10$

6. $x^2 - 2x - 15$

Trickier or Special Forms of Trinomials of the Form $x^2 + bx + c$

7. $-x^2 - 3x + 28$

8. $6x^2 + 54x + 120$

9. $x^2 - 6xy + 8y^2$

10. $x^2 - 8x + 16$

11. $x^2 - 25$

12. $x^2 - \frac{9}{4}$

Trinomials of the Form $ax^2 + bx + c$

13. $2x^2 + 11x + 5$

14. $7x^2 - 15x + 8$

15. $-3x^2 - 15x^2 + 198x$

16. $16x^2 - 24xy + 9y^2$

More Practice with the Difference of Squares

17. $x^2 - 81$

18. $25x^2 - 4$

19. $\frac{1}{4}x^2 - 9y^2$

Sum and Difference of Cubes

20. $x^3 + 8$

21. $x^3 - 27$

22. $27x^3 - 1$

23. $8x^3 + 64y^3$

Lesson: How to Solve Quadratic Equations with Factoring

A **quadratic equation** is an equation of the from $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

There are several ways that you can solve quadratic equations, but in this lesson we will focus on how to use factoring as a way to solve them.

Solving a quadratic equation by factoring requires a rule called the **Zero Product Rule.**

Write down the Zero Product Rule:

<u>**Examples**</u> Solve the following equations using the zero product rule:

a. $3x = 0$

b. $3(x + 1) = 0$

c. $(x + 3)(x + 1) = 0$

d. $x(2x + 1)(3x + 5) = 0$

Show how to use factoring to solve $x^2 + 5x + 6 = 0$.

Examples Solve the following by using factoring:

a. $x^2 + 12x - 45 = 0$

b. $x^2 - 12x + 11 = 0$

c. $2x^2 + 5x + 3 = 0$

d. $55x = -20x^2 - 30$

e. $x^2 = 25$

f. $x^2 = 9x$

Lesson: Tips and Tricks to Help with Harder Quadratics Solved by Factoring

A **quadratic equation** is an equation of the from $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

In the previous lesson, we talked about the basics of using factoring to solve quadratics.

In this lesson, we will look at some trickier examples. First, let's warm up by solving the following:

a. $4x = 14x^2 - 48$

b. $3x^3 = 12x^2$

Sometimes we will have to do further work to simplify the problem before we factor.

Examples Use factoring to solve.

a. $(x + 3)(x + 4) = 6$

b. $5x(x + 2) - 4x = 4(x^2 - 2)$

c. $(x + 4)^2 - 2x = 11$

What can be done to solve $\frac{1}{3}(x+1)^2 - \frac{1}{4}x = \frac{1}{6}(x-2)^2 - \frac{1}{3}$?

<u>Examples</u> Use factoring to solve.

a. $\frac{1}{4}(2x-3)^2 + \frac{1}{4}x = \frac{1}{4}(x-5)^2 - \frac{3}{2}$

b. $\frac{1}{4}x(2-x) - \frac{3}{4} = \frac{1}{5}x(x+1) - \frac{7}{10}$

Lesson: Applications of Quadratic Equations

The area of a rectangle is 15ft squared, and the length of the rectangle is 7ft less than twice the width. Find the dimensions of the rectangle.

An 12ft ladder leans against the side of a house. The bottom of the ladder is 6ft from the side of the house. How high is the top of the ladder from the ground? If necessary, round your answer to the nearest tenth.

A kite flying in the air has a 14ft line attached to it. Its line is pulled taut and casts a 9ft shadow. Find the height of the kite. If necessary, round your answer to the nearest tenth.

The longer leg of a right triangle is 2in longer than the shorter leg. The hypotenuse is 4in longer than the shorter leg. Find the side lengths of the triangle.

Rational Expressions
Video 1: The Basics

What are rational expressions?

What do rational expressions look like? Write down some examples:

Simplifying Rational Expressions
Take down some notes on this:

<u>Examples</u> <u>together:</u>

a. $\frac{25x^7y^9}{15x^3y^2z^7}$

<u>Examples</u> <u>on your own:</u>

b. $\frac{21x^7y^9}{14y^8z^3}$

c. $\frac{16x^4y^7z^3}{15x^3y^4z^8}$

Examples together:

a. $\frac{x^2+5x+6}{x^2-9}$

Examples on your own:

b. $\frac{x^2+7x+10}{x^2+10x+25}$

c. $\frac{x^2-x-6}{x^2-9x+14}$

d. $\frac{x^2+10x+16}{x^3+2x^2+x+2}$

Play the video and check your work.

Examples on your own:

e. $\frac{x^3-8}{x^2-9x+14}$

f. $\frac{10x^3+15x^7}{10x^3+40x^2+40x}$

Rational Expressions
Video 2: Other Things to Know

When it comes to fractions, where are you allowed to have a zero? In the numerator (top) or denominator (bottom)?

Therefore, what do we have to worry about with rational expressions?

Examples on your own:

Evaluate the following rational expressions for $x = 2$.

a. $\frac{x^2+6x+8}{x^2+8x+15}$

b. $\frac{x^2+9x+20}{x^2+3x-10}$

<u>Example</u> <u>on your own: (evaluate for x=2)</u>

c. $\frac{x^2-4}{x^2+9x+14}$

To summarize…

When a rational expression has a zero in the numerator, what does this mean?

When a rational expression has a zero in the denominator, what does this mean?

Define the term **domain:**

What are we concerned with for the domain of rational expressions?

A review of interval notation:

Examples:

Write the following in interval notation:

a.

b.

c.

<u>**Example** together:</u>

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

a. $\frac{x+3}{x-5}$

b. $\frac{2x+1}{x^2+7x+10}$

<u>**Examples** on your own:</u>

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

c. $\frac{x-3}{x-1}$

d. $\frac{x^2-81}{x^2+5x+6}$

Rational Expressions
Video 3: The Negative One Trick with Simplifying

In its current form, can you simplify: $\frac{x-3}{3-x}$? Why or why not?

Show how you can manipulate this expression to simplify it.

There is no formal name for this trick. Some might refer to it as a *multiplicative property of -1,* but I refer to as “that -1 trick.” I know, it’s not exactly formal sounding!

Let’s do one more example of this. Simplify $\frac{5-x}{3x-15}$

The first thing to get used to it how to rewrite expressions using this trick.

Examples

Rewrite the order of each expression by factoring out -1 first.

a. $4-x$

b. $3x-1$

c. $x-6$

Now let's use this to simplify rational expressions.

Examples Simplify the following.

a. $\frac{6x-12}{2-x}$

<u>Examples</u> Simplify the following.

b. $\frac{4x-20}{30x-6x^2}$

c. $\frac{x^2-4}{2-x}$

d. $\frac{x-6}{36-x^2}$

e. $\frac{5-x}{x^2-7x+10}$

<u>Examples</u> Simplify the following.

f. $\frac{x^2-10x+25}{25-x^2}$

g. $\frac{27-x^3}{x^2-5x+6}$

h. $\frac{x^3-3x^2+5x-15}{6-3x}$

Examples Simplify the following.

i. $\frac{54-9x}{x^2-36}$

j. $\frac{8-x^3}{x^2-9x+14}$

k. $\frac{y^3-x^3}{x^2-y^2}$

Example/Question: Troubleshooting the -1 Trick

Explain the actual DETAIL behind WHY $\frac{4}{5-x} = -\frac{4}{x-5}$

<u>Examples</u> Write an equivalent fraction for the following:

a. $\frac{5}{x-8}$

b. $\frac{x-3}{x+2}$

Lesson: Multiplying Rational Expressions

How do you multiply the fractions $\frac{a}{b} \times \frac{c}{d}$? Include a one-sentence explanation.

<u>Examples</u> Multiply the following.

a. $\frac{4}{5} \times \frac{7}{9}$

b. $\frac{2x^3}{7y^2} \times \frac{5x^5}{9y^7}$

Show how to make cancellations before multiplying $\frac{12}{25} \times \frac{15}{16}$

We can apply this same logic to rational expressions.

Examples Multiply the following and simplify when possible.

a. $\frac{21x^7}{15y^3} \times \frac{5y^8}{42x^4}$

b. $\frac{2x^2+5x+3}{x^2-25} \times \frac{x^3+125}{4x^2+6x}$

Lesson: Dividing Rational Expressions

How do you divide $\frac{a}{b} \div \frac{c}{d}$? Write a one sentence explanation of how to do this too.

<u>Example</u> Divide the following: $\frac{5x}{9} \div \frac{2}{7x^4}$

When can you make cancellations in the process?

Examples Divide.

a. $\frac{12x^7}{45} \div \frac{42x^3}{15}$

b. $\frac{x^2+4x+4}{x+5} \div (x+2)^5$

c. $\frac{x^3+1}{6x^3-6x^2+6x} \div \frac{x^2-1}{8}$

Explain why $\frac{\frac{5x+5y}{24}}{\frac{15x^2+15xy}{12}}$ is both a fraction and a division problem.

<u>Example</u> Divide: $\frac{\frac{16x^2-25}{x^5}}{\frac{36x-45}{9x^7}}$

Lesson: Finding the Least Common Denominator for Addition and Subtraction – A Quick Review for Fractions (with only numbers)

We are going to work on adding and subtracting rational expressions with different denominators. Before we do that, we are going to remind ourselves how to find common denominators for fractions that only have numbers.

The simplest case: what is the least common denominator between the fractions $\frac{1}{2}$ and $\frac{1}{3}$?

Why is that the common denominator?

Now let's look at something slightly more complicated. What is the least common denominator between $\frac{1}{6}$ and $\frac{1}{10}$?

First, take a guess. It's totally fine if this guess is wrong, but just make a guess.

Why is this your guess? What did you do?

There is a specific procedure that you want to use when finding least common denominators in this class. Write down instructions that were given in the video, but write them our in a way that makes sense to you.

Repeat this procedure to find the least common denominator $\frac{1}{15}$ and $\frac{1}{10}$.

Repeat this procedure to find the least common denominator $\frac{1}{12}$ and $\frac{1}{8}$.

Now we will focus on how to write fractions with this desired denominator.

The simplest case: using the least common denominator you found for $\frac{1}{2}$ and $\frac{1}{3}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Using the least common denominator you found for $\frac{1}{6}$ and $\frac{1}{10}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Using the least common denominator you found for $\frac{1}{12}$ and $\frac{1}{8}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Lesson: Finding the Least Common Denominator for Rational Expressions

The simplest case: what is the least common denominator between the fractions $\frac{1}{2x}$ and $\frac{1}{3y^2}$?

<u>Examples</u> Find the least common denominator between the following fractions.

a. $\frac{1}{2x^2}, \frac{1}{3xy^2}$

b. $\frac{1}{2x^2y^5}, \frac{1}{3x^7y^2}$

c. $\frac{1}{2x^3y^5z^2}, \frac{1}{3x^5y^7z^8}$

Examples Find the least common denominator for the following.

a. $\frac{1}{14x^3y^2}, \frac{1}{21x^2y^7}$

b. $\frac{1}{24x^3y}, \frac{1}{12x^2y^5}$

What is the least common denominator between $\frac{1}{x+2}$ and $\frac{1}{x}$?

What is the least common denominator between $\frac{1}{x^2-4}$ and $\frac{1}{x^2+5x+6}$?

Examples Find the least common denominator between the following fractions.

a. $\frac{3}{5x-10}, \frac{1}{7x-14}$

b. $\frac{1}{x^2-12x+35}, \frac{1}{x^2+x-20}$

Examples Find the least common denominator between the following fractions.

a. $\frac{1}{x-3}, \frac{1}{3-x}$

b. $\frac{1}{x^2-4}, \frac{1}{2-x}$

c. $\frac{1}{x^2+9x+20}, \frac{1}{x^2-25}, \frac{1}{x^2-x-20}$

Lesson: Writing Equivalent Rational Expressions with the Same Denominator

Examples Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{2}{15x^2y^5}, \frac{4}{9x^3y^2}$

b. $\frac{5}{x+3}, \frac{4}{x}$

Examples Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{x}{x^2+9x+18}, \frac{2}{x^2+5x+6}$

b. $\frac{x}{5x^2-30x}, \frac{x-2}{2x^2-17x+30}, \frac{2}{2x^2-5x}$

Lesson: Adding and Subtracting Rational Expressions

Show how to add or subtract the following:

a. $\frac{5}{x+2} - \frac{3}{x+2}$

b. $\frac{5}{x-4} + \frac{7}{x}$

c. $\frac{5}{12xy^2} - \frac{7}{8x^3y}$

d. $\frac{x}{x+3} - \frac{2}{x^2-9}$

e. $\frac{x}{x^2-25} - \frac{5}{x^2-3x-40}$

Example/Question: When Are You Allowed to Cancel with Rational Expressions?

What is the wrong way to cancel $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and explain why you can't cancel like that? How can you tell it's wrong?

What is the difference between $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and $\frac{3(x+1)(x-3)}{2(x-3)(x+1)}$?

How do you simplify $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$?

Lesson: Complex Fractions

First let's review how to simplify a complex fraction we've already discussed. Show how to simplify:

$$\frac{\frac{2x}{x+5}}{\frac{2}{x^2-25}}$$

Explain what makes the problem below different from the one we just reviewed above.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

Outline the steps required to simplify. Write out the instructions as they were explained in the video.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

Example Simplify the following

$$\frac{\frac{7}{9}-\frac{1}{3}}{1+\frac{5}{9}}$$

<u>Examples</u> Simplify the following.

a. $\dfrac{\frac{5}{x}+\frac{2}{y}}{\frac{1}{y}-\frac{3}{x}}$

b. $\dfrac{\frac{6}{x+3}-\frac{4}{x-1}}{\frac{2}{x-1}+\frac{5}{x+3}}$

Example Simplify the following.

$$\frac{1-\frac{4}{t+5}}{\frac{4}{t^2-25}+\frac{t}{t-5}}$$

Complex Fractions
Video 2: More Complicated Examples

Complex fractions look like this:

$$\frac{\frac{4x}{15y^2}}{\frac{6x^4}{25y^8}} \quad \text{or} \quad \frac{\frac{1}{3}-\frac{1}{6}}{\frac{3}{2}+\frac{5}{6}} \quad \text{or} \quad \frac{\frac{4}{x^2}-\frac{4}{y^2}}{\frac{1}{xy}+\frac{5}{y}}$$

First we will discuss the basics.

Examples

a. $\frac{\frac{1}{x^2}-\frac{1}{y}}{\frac{3}{x}-\frac{4}{y^2}}$

b. $\frac{\frac{3}{x}-x}{\frac{2}{x}+x}$

Examples

a. $\dfrac{\frac{3}{x-1}-\frac{2}{x+3}}{\frac{5}{x+3}-\frac{2}{x-1}}$

b. $\dfrac{\frac{5}{x-2}-\frac{3}{x+5}}{\frac{1}{x+5}+\frac{2}{x-2}}$

Examples

a. $\dfrac{1-\frac{1}{x+5}}{\frac{x}{x-5}+\frac{4}{x^2-25}}$

b. $\dfrac{1+\frac{12}{x^2-16}}{\frac{x}{x+4}-\frac{6}{x-4}}$

c. $\dfrac{\frac{1}{x+2}+\frac{4}{x^2-4}}{\frac{3}{x-2}+1}$

Lesson: Solving Rational Equations

How do you know that you need to solve a problem versus simplify it?

Explain how to solve the rational equation. Include some instructions in your notes.

$$\frac{2}{x-1}+\frac{1}{x+4}=\frac{4}{x+4}$$

What do we have to watch out for when solving rational equations?

<u>Examples</u> For the following, which solutions would NOT work for these rational equations?

a. $\frac{4}{x+1} - \frac{2}{x+3} = \frac{5}{x+7}$

b. $\frac{4}{x^2-9} + \frac{3}{x-3} = \frac{2}{x+3}$

<u>Examples</u> Solve.

a. $1 - \frac{8}{x} = \frac{10}{x}$

b. $\frac{5}{x-3} - \frac{2}{x^2-9} = \frac{5x+13}{x^2-9}$

c. $\frac{x^2}{2} = \frac{x^2-6x}{3}$

d. $\frac{4}{x+1} = \frac{10}{x^2-1} - \frac{5}{x-1}$

Solving Equations with Rational Expression

We will learn how to solve equations that look like this:

$$z = \frac{x+3y}{4w} \quad \text{or} \quad \frac{1}{a} - \frac{2}{b} = \frac{3}{c}$$

Examples

a. Solve for y: $z = \frac{3xy}{5w}$

b. Solve for x: $w = \frac{7+2y}{xz}$

c. Solve for z: $y = \frac{4xw}{z-w}$

Examples

a. Solve for x: $\frac{3}{x}+\frac{1}{y}=\frac{2}{z}$

b. Solve for b: $\frac{1}{a}-\frac{1}{b}=\frac{3}{c}$

c. Solve for z: $\frac{5}{x}-\frac{3}{y}=\frac{7}{z}$

Lesson: Three Ways to Use the LCD with Rational Expressions

We have spent quite a bit of learning how to find the LCD between rational expressions and then using it in very different ways. We are going to compare the 3 techniques for how we used the LCD in this video.

First, simplify completely:

$$\frac{9}{x+5}-\frac{4}{x+1}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

Simplify completely:

$$\frac{\frac{9}{x+5}-\frac{4}{x+1}}{\frac{10}{x^2+6x+5}-\frac{3}{x+1}}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

Solve:

$$\frac{9}{x+5} - \frac{4}{x+1} = \frac{10}{x^2+6x+5}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

Example/Question: When Can You Cross Multiply

Explain what a ratio is:

When can you cross-multiply?

Solve: $\frac{8}{x+2} - \frac{12}{x-4} = \frac{2}{x+2}$. Show that you CANNOT cross-multiply in this situation.

Now show why cross-multiplying works with $-\frac{x}{5} = \frac{3}{x+8}$

Rational Expression Applications
Video 1: Proportion Type Problems

What is a proportion?

A bakery can make 300 cookies in 1.5 hours. How long would it take to make 525 cookies?

The ratio of patrons at a café who purchase coffee versus those who purchase tea is 5 to 3. On Monday, the café sold 45 teas. How many coffees were sold?

A painter makes a particular shade of orange by using 3 parts red and 4 parts yellow. The painter has four more gallons of yellow paint than he has of red paint. How many gallons of red paint should he use?

Rational Expression Applications
Video 2: Distance, Rate, and Time Problems

In these types of problems, we need to use the equation:

$$d = rt$$

Where d=distance, r=rate/speed of a vehicle, and t=time

We can also see that we can rearrange this formula in a few ways:

A boat travels 30 miles downstream in the same time that it takes to travel 20 miles upstream. The current of the river is 5mph. How fast would the boat travel in still water?

A boat travels 16 miles downstream in the same time that it travels 12 miles upstream. In still water, the boat can travel 6mph. What is the speed of the current?

A plane travels 550 miles with the wind. In the same amount of time, it travels 375 miles against the wind. If the wind is blowing at a speed of 25 mph, what is the speed of the plane when there is no wind?

Rational Expression Applications
Video 3: Work Problems

What is the setup for work problems?

It takes Maria 3 hours to paint a room, while it takes her younger sister Lupe 5 hours to paint the same room. How long would it take if the girls painted the room together?

It takes Max 30 minutes to shovel the driveway, while it takes his brother Darrell 45 minutes to do the same job. How long would it take for the guys to do it together?

The Basics of Roots
Video 1: Square Roots

What are the square roots of 9? Write down what it means to be a square root.

<u>Examples</u> Find the square roots of the following numbers.

a.

b.

What does the symbol $\sqrt{}$ refer to?

How can we refer to the negative square root?

Draw a square root and label its parts:

Examples Evaluate the following:

a. $\sqrt{25}$

b. $\sqrt{81}$

c. $\sqrt{36}$

d. $\sqrt{\frac{1}{36}}$

e. $\sqrt{\frac{16}{49}}$

f. $\sqrt{\frac{9}{25}}$

g. $\sqrt{-36}$

h. $-\sqrt{64}$

Finally, how would you estimate a root like $\sqrt{5}$?

Example Estimate $\sqrt{14}$

The Basics of Roots
Video 2: Higher Roots

How would you evaluate the following? Explain the rationale for how to evaluate these:

$\sqrt[3]{8}$

$\sqrt[4]{81}$

$\sqrt[5]{32}$

The most general symbol for a root is $\sqrt[n]{a}$. Explain what this symbol means and once again, label it.

Now we can create a list of the most likely roots that we will encounter. You will very likely want to create another list of this on a page you can easily access.

Squares	**Cubes**	**Other**

Examples Evaluate the following. Be careful when you evaluate.

a. $\sqrt[3]{125}$

b. $\sqrt[3]{-125}$

c. $\sqrt[4]{16}$

d. $\sqrt[4]{-16}$

d. $-\sqrt[3]{8}$

e. $-\sqrt[3]{-8}$

f. $\sqrt[5]{-32}$

g. $\sqrt[6]{-64}$

Now let's generalize. For all roots of the form $\sqrt[n]{a}$, what can we say if....

1. The index (n) is an **odd** number?

2. The index (n) is an **even** number and a is a **negative** number?

3. The index (n) is an **even** number and a is a **positive** number?

The Basics of Roots
Video 3: The "Ultimate" Test and A Few Other Notes

To begin, we will first see how well we can put everything we've learned so far about roots together.

Examples Evaluate the following, if possible.

a. $\sqrt[3]{-8}$

b. $\sqrt{-16}$

c. $-\sqrt[4]{16}$

d. $\sqrt[3]{-125}$

e. $-\sqrt{100}$

f. $\sqrt[4]{-81}$

g. $-\sqrt{25}$

h. $-\sqrt[3]{-8}$

i. $-\sqrt[4]{-16}$

j. $-\sqrt[5]{-32}$

And now that you have completed this … it's time for my "ultimate" test … drum roll please:

The "Ultimate" Test for Roots

Evaluate. Note: at this moment in time, we do not know anything about imaginary numbers.

a. $\sqrt{1}$

b. $\sqrt[3]{1}$

c. $\sqrt[5]{1}$

d. $\sqrt[4]{-1}$

e. $\sqrt[3]{-1}$

f. $-\sqrt{1}$

g. $-\sqrt[4]{-1}$

h. $-\sqrt[3]{1}$

i. $-\sqrt{-1}$

j. $-\sqrt[7]{-1}$

k. $-\sqrt[3]{-1}$

l. $\sqrt[8]{-1}$

m. $-\sqrt[10]{-1}$

n. $\sqrt{-1}$

Are you having fun yet???

We have one last thing to discuss: the general rule for evaluating $\sqrt[n]{a^n}$

If n is even ….

If n is odd ….

<u>Examples</u> Evaluate the following:

a. $\sqrt[3]{y^3}$

b. $\sqrt[4]{(x-4)^4}$

c. $\sqrt[7]{(2y-4)^7}$

d. $\sqrt[6]{(3x+2)^6}$

Simplifying Radicals
The Basics

We will look at how to simplify radicals like this:

$$\sqrt{50} = 5\sqrt{2} \quad \text{or} \quad \sqrt[3]{250} = 5\sqrt[3]{3}$$

You might want to have the list of squares / cubes / quartics ready for this exercise. If you don't, pause the video and write them all down on a separate sheet of paper so that you can have it handy for this entire lesson.

First we look at **the multiplication property of radicals:**

<u>Examples</u> Multiply the following:

a. $\sqrt[3]{2} * \sqrt[3]{3}$

b. $\sqrt{7} * \sqrt{5}$

c. $\sqrt[3]{3} * \sqrt[3]{9}$

d. $\sqrt[3]{6} * \sqrt[3]{9}$

How are examples c and d related?

Being able to multiply radicals together is powerful, because it also allows us to break apart radicals and simplify them. Using the example $\sqrt{12}$, write down how to simplify:

<u>Examples</u> Simplify the following:

a. $\sqrt{50}$

b. $\sqrt{20}$

c. $\sqrt[3]{54}$

d. $\sqrt[4]{32}$

e. $\sqrt[3]{24}$

At this point, we should probably define when a radical is actually simplified.

A simplified radical will ……

<u>Examples</u> Explain why the following radicals are not simplified:

a. $\sqrt{24}$

b. $\frac{5}{\sqrt{7}}$

c. $\sqrt[4]{\frac{3}{5}}$

d. $\sqrt[3]{8x^8y^2}$

Examples Simplify the following:

a. $\sqrt{24}$

b. $\sqrt[3]{32}$

c. $\sqrt[4]{64}$

d. $\sqrt[5]{64}$

e. $\sqrt{250}$

f. $\sqrt[3]{250}$

Lesson: Simplifying Radicals with Variables

We are going to rely on a skill we've already learned to help with simplifying radicals.

<u>Examples</u> Show you can simplify the following utilizing rational exponents.

a. $\sqrt{x^6}$

b. $\sqrt[3]{x^{15}}$

c. $\sqrt{x^6 y^8}$

d. $\sqrt[4]{x^{12} y^{20}}$

How can we simplify $\sqrt{x^5}$? Watch the 2 explanations in the video and summarize the explanation that makes more sense to you.

Example Simplify the following.

a. $\sqrt[3]{x^{13}}$

b. $\sqrt[4]{x^{22}}$

c. $\sqrt{12x^6y^7}$

d. $\sqrt[3]{250x^8y^{16}}$

e. $\sqrt[4]{\frac{32x^7}{y^8}}$

Lesson: Multiplying Radicals

Write a one-sentence explanation of how to multiply $\sqrt[3]{3} * \sqrt[3]{4}$

<u>Examples</u> Multiply the following. Simplify if possible.

a. $\sqrt[3]{6} * \sqrt[3]{4}$

b. $\sqrt[4]{4x^3} * \sqrt[4]{12x^2}$

c. $\sqrt{5x} * \sqrt{10xy}$

Simplifying Radicals
Multiplying and Dividing Radicals with Different Indices

We will look at how to perform operations like:

$$\sqrt{x} * \sqrt[3]{x} \qquad \text{or} \qquad \frac{\sqrt[4]{x}}{\sqrt[3]{x}}$$

Before we begin, you should be comfortable with doing problems like this:

$8^{\frac{1}{3}}$

$25^{\frac{3}{2}}$

Write down the approach for performing this operation: $\sqrt{x} * \sqrt[3]{x}$

Examples Perform the following operations:

a. $\sqrt[3]{x} * \sqrt[4]{x}$

b. $\frac{\sqrt[5]{x^2}}{\sqrt[3]{x}}$

Examples Perform the following operations:

a. $\sqrt[4]{x^3} * \sqrt[5]{x^3}$

b. $\frac{\sqrt{x}}{\sqrt[3]{x^2}}$

c. $\sqrt[3]{x^2} * \sqrt{x}$

Adding and Subtracting Radicals

In this video we will look at how to perform operations like:

$$\sqrt{3} + \sqrt[3]{5} - 5\sqrt{3} + 7\sqrt[3]{3} \qquad \text{or} \qquad \sqrt{24x^4y^3} - 3x^2y\sqrt{6y}$$

Before you begin, you should already be able to simplify radicals like this:

a. $\sqrt{24x^6y^9}$

b. $\sqrt[4]{16x^7y^{11}}$

What are like terms?

Add the like terms in this problem: $3x + 7y - 9x + 7y^2 - 2x - 2y$

What are like radicals?

Add the like radicals in this problem: $5\sqrt{3} - 2\sqrt[3]{3} + 8\sqrt[3]{3} - 2\sqrt{5} + 8\sqrt{3}$

Examples Perform the following operations:

a. $4\sqrt{7} - 5\sqrt[3]{3} + 2\sqrt{3} - 3\sqrt[3]{7} - 9\sqrt{3} + \sqrt[3]{3}$

b. $2x\sqrt{7} - 3\sqrt{7} + 8x\sqrt{7} - 2\sqrt{7}$

c. $3x\sqrt[3]{5} + 2x\sqrt[3]{4} - 3\sqrt[3]{4} + 5x\sqrt[3]{4} - 7x\sqrt[3]{5} + 5\sqrt[3]{4}$

It is possible that you might have to simplify a radical before you can add it. Note how this works with the example:

$$\sqrt{24} + 2\sqrt{54}$$

Examples Perform the following operations:

a. $5\sqrt{8} - \sqrt{18}$

b. $4\sqrt[3]{54} - 7\sqrt[3]{16}$

Examples Perform the following operations:

a. $\sqrt{8x^2y^4} - 9xy^2\sqrt{2}$

b. $4x\sqrt[3]{16x^6y^7} + 9x^3y\sqrt[3]{250y^4}$

c. $5x^2y^3\sqrt[4]{32x^7z^{10}} - 2x^3z^2\sqrt[4]{2x^3y^{12}z^2}$

d. $3x\sqrt[3]{24x^5y^7} + 3xy\sqrt[3]{3x^2y} + y^2\sqrt[3]{375x^8y}$

Lesson: Multiplying Radicals Expressions

First, let's review how to multiply radicals in general. Show how to multiply:

a. $\sqrt[3]{4x^2y} * \sqrt[3]{6x^2y}$

b. $\sqrt{6xy} * \sqrt{10y}$

Now we will apply that logic to larger rational expressions.

Write a one-sentence explanation of how to multiply: $\sqrt{2}\left(\sqrt{3} + \sqrt{10}\right)$, then multiply.

<u>**Example**</u> Multiply $2\sqrt[3]{3}\left(\sqrt[3]{9} + 5\sqrt[3]{6}\right)$

Example Multiply $3x\sqrt{5}\left(\sqrt{10x} - 5x\sqrt{2}\right)$

Write out an explanation of how to multiply: $\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{6} + \sqrt{3}\right)$

Example Multiply: $\left(3\sqrt{5} - \sqrt{2}\right)\left(\sqrt{10} + 4\sqrt{2}\right)$

Example Multiply: $\left(3\sqrt[3]{4}-\sqrt[3]{6}\right)\left(5\sqrt[3]{2}+\sqrt[3]{6}\right)$

How do you multiply $\left(\sqrt{2}+\sqrt{3}\right)^2$

Examples Multiply: $\left(\sqrt[3]{9}+\sqrt[3]{6}\right)^2$

Lesson: Rationalizing a Denominator with One Square Root

What does it mean to rationalize the denominator?

How would you rationalize $\frac{2}{\sqrt{3}}$? Explain *why* we can do this.

Can every fraction with a radical in the denominator be rationalized like this? Circle: Yes No

What types of problems can be rationalized this way?

Examples Rationalize the following.

a. $\frac{20}{\sqrt{8}}$

b. $\sqrt{\frac{40}{60}}$

c. $\sqrt{\frac{81x^7}{2y}}$

d. $\frac{\sqrt{22}}{\sqrt{x^9}}$

Dividing Radicals/Rationalizing the Denominator
Part 2: Higher Roots

In this video we will look at how to simplify expressions like:

$$\frac{5}{\sqrt[3]{3}} \quad \text{or} \quad \frac{\sqrt[4]{2}}{\sqrt[4]{9}}$$

Before you begin, you should already be able to simplify problems like this:

a. $\frac{5}{\sqrt{3}}$

b. $\frac{\sqrt{6}}{\sqrt{18}}$

First, let's observe why higher roots are a different situation. Write down what happens if we use the same methods as above to rationalize the following: $\frac{5}{\sqrt[3]{3}}$

We have to pivot our thinking to "count powers". Note how this works with this examples: $\sqrt[3]{3} * \_\_\_ = 3$

Examples Determine what to fill in the blank to create a true statement:

a. $\sqrt[3]{25} * ? = 5$

b. $\sqrt[4]{2} * ? = 2$

c. $\sqrt[4]{27} * ? = 3$

d. $\sqrt[5]{4} * ? = 2$

e. $\sqrt[3]{7} * ? = 7$

f. $\sqrt[4]{25} * ? = 5$

g. $\sqrt[5]{3} * ? = 3$

h. $\sqrt[3]{36} * ? = 6$

Now you should have a sense of what number you want this to equal. This is a very strange exercise, but using the pattern above, determine what to multiply by to get to the desired root:

a. $\sqrt[3]{4} * ? =$

b. $\sqrt[4]{5} * ? =$

c. $\sqrt[5]{9} * ? =$

d. $\sqrt[4]{36} * ? =$

Now we should have a decent sense of how to rationalize and what the answer should equal. Explain how to rationalize the following:

a. $\frac{5}{\sqrt[3]{2}}$

b. $\frac{7}{\sqrt[4]{5}}$

Examples Rationalize the following denominators:

a. $\frac{6}{\sqrt[3]{5}}$

b. $\frac{2}{\sqrt[4]{9}}$

c. $\frac{2}{\sqrt[5]{125}}$

d. $\frac{7}{\sqrt[4]{8}}$

Examples Rationalize the following denominators:

a. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

b. $\frac{2\sqrt[3]{3}}{\sqrt[3]{4}}$

c. $\frac{5\sqrt[4]{2}}{\sqrt[4]{27}}$

d. $\frac{6\sqrt[3]{5}}{\sqrt[3]{3}}$

e. $\frac{3}{\sqrt[5]{x^3}}$

f. $\frac{\sqrt[5]{25}}{\sqrt[5]{16}}$

Lesson: Rationalizing a Denominator with Two Square Roots

How do you find the conjugate of $4+\sqrt{2}$? Write one sentence to explain what you need to do.

<u>Examples</u> What is the conjugate of the following?

a. $3-\sqrt{5}$

b. $-4-\sqrt{2}$

Show how to rationalize: $\frac{5}{\sqrt{2}-\sqrt{3}}$

<u>Examples</u> Rationalize the following.

a. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}}$

b. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{5}}$

c. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Solving Equations with Radicals
Part 1: One Square Root in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{3x+1} = 2 \qquad \text{or} \qquad \sqrt{4x-2} = 5 - x$$

Use the example $\sqrt{3x+1} = 4$ to show the general procedure for solving these types of equations:

Why must we check the solutions?

Example Solve $\sqrt{3x+1} = 8$

Examples Solve

a. $\sqrt{5x-4}+2=8$

b. $\sqrt{1+3x}+5=2$

Explain what is different about solving $\sqrt{5x+4}=x+2$

Examples Solve the following.

a. $\sqrt{3x-6} = x - 2$

b. $\sqrt{2x+4} = x - 2$

c. $\sqrt{5x+1} + 5 = 3x$

Solving Equations with Radicals
Part 2: Two Square Roots in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{4x-2} = 3\sqrt{2x+6} \quad \text{or} \quad \sqrt{3x-1}+2 = \sqrt{4x-2}$$

Let's begin by explaining how to solve: $\sqrt{5x+2} = \sqrt{3x+6}$

Examples Solve:

a. $\sqrt{3-2x} = \sqrt{2x+7}$

b. $\sqrt{5+3x} = \sqrt{3+2x}$

c. $4\sqrt{3-2x} = 2\sqrt{1+3x}$

Example Solve: $6\sqrt{x} = 4\sqrt{2x+1}$

Now write out the detailed explanation of how to solve $\sqrt{4x+1} - 2 = \sqrt{2x+1}$

Examples Solve:

a. $\sqrt{3x+6} = 2 + \sqrt{2x-4}$

b. $\sqrt{3x+1} = 2 + \sqrt{2x-2}$

Examples Solve:

a. $\sqrt{2x+4}+\sqrt{1-2x}=3$

b. $\sqrt{2x+4}=2+\sqrt{2x+3}$

Solving Equations with Radicals
Part 3: Two Cube Roots of Higher in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt[3]{3x+1} = 2 \qquad \text{or} \qquad \sqrt[4]{2x-1} = 5$$

Explain how to solve: $\sqrt[3]{3x+1} = 2$

Explain how to solve: $\sqrt[4]{3x+4} = 2$

When is it more likely a solution will not work?

Examples Solve the following:

a. $\sqrt[3]{4x+7}=3$

b. $\sqrt[3]{3-5x}=2$

c. $\sqrt[4]{3-2x}=3$

d. $\sqrt[4]{4x-1}=-7$

Solving Quadratic Equations with Factoring

In this video, we will look at how to solve equations by factoring, which looks like this:

$$x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0$$

Quadratic equations are equations of the form:

Where a, b, and c are real numbers and $a \neq 0$.

How would you solve the quadratic equation $x^2 + 7x + 6 = 0$ by factoring?

<u>Examples</u> Solve the following by factoring:

a. $x^2 - x - 12 = 0$

b. $x^2 + 9 = -6x$

c. $x^2 = 25$

Examples Solve the following by factoring:

a. $x^2 + 7x = 0$

b. $\frac{1}{10}x^2 + \frac{1}{2}x + \frac{3}{5} = 0$

c. $2x^3 + 28x^2 + 90x = 0$

Solving Quadratic Equations using the Square Root Property

In this video, we will look at how to solve equations by using the square root property, which looks like this:

$$(x+2)^2 = 9 \quad \rightarrow \quad x+2 = \pm 3$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

What are the solutions to $x^2 = 25$?

In general, how do you solve equations of the form $Q^2 = k$, where Q is an expression and k is a constant? In other words, what is the **square root property**?

Examples Solve the following equations using the square root property:

a. $(x+3)^2 = 25$

b. $(x+3)^2 = 12$

Examples Solve the following equations using the square root property:

a. $(5x-2)^2+5=29$

b. $(6x+7)^2+4=-41$

Solving Quadratic Equations by Completing the Square

In this video, we will look at how to solve equations by factoring, which looks like this:

$$x^2 + 6x = 7 \qquad b = 6,\ \ \frac{1}{2}b = 3,\ \ \left(\frac{1}{2}b\right)^2 = 9$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

Before we begin, you should be familiar with the square root property. Solve $(2x + 1)^2 = 49$ using the square root property.

First let's talk about how to complete the square. Show how to complete the square on $x^2 + 6x$ below:

Examples Complete the square on the following:

a. $x^2 + 8x$

b. $x^2 - 10x$

c. $x^2 + 3x$

d. $x^2 - \frac{1}{2}x$

e. $x^2 + \frac{2}{3}x$

Show how to solve $2x^2 + 12x - 16 = 0$ by completing the square:

Examples Solve the following by completing the square:
a. $x^2 + 4x - 6 = 0$

b. $x^2 + 5 = 2x$

Examples Solve the following by completing the square:

a. $3x^2 + 15x = -6$

b. $\frac{1}{2}x^2 + 3x - 2 = 0$

Examples: The Distance Formula

Find the distance between the following sets of points.

a. (6, 2) and (2, 11)

b. (3, -1) and (-4, -2)

Solving Quadratic Equations by the Quadratic Formula

Complete the square on: $ax^2 + bx + c = 0$

State the quadratic formula:

Examples Use the quadratic formula to solve the following:

a. $x^2 + 3x + 2 = 0$

b. $2x^2 + 3x = 5$

c. $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{5}{6} = 0$

Lesson: The Discriminant

Restate the Quadratic Formula, and then label the discriminant.

Value of the Discriminant	Interpretation

Examples Find the values of the discriminant for the following equations. Then solve the equations (using the most efficient method) to determine if this makes sense.

a. $x^2 - 4x + 4 = 0$

b. $x^2 + 6x + 5 = 0$

c. $3x^2 + 2x + 5 = 0$

Choosing the Best Method to Solve a Quadratic

There are 4 ways we can solve quadratics:

1.	3.
2.	4.

Using each of these methods only once, determine the best way to solve these quadratics:

a. $x^2 + 6x = 2$
b. $x^2 + 3 = 2x$
c. $(2x + 1)^2 = 49$
d. $3x^2 + 2 = 3x$

Examples Using each of these methods only once, determine the best way to solve these quadratics:
a. $(3x-2)^2=-24$
b. $2x^2+5x-1=0$
c. $x^2-12=x$
d. $x^2-2x+5=0$

Solving Formulas with Squares and Square Roots

In this lesson, we will look at how to solve for formulas that look like this:

$$y = 3x^2 \qquad \text{or} \qquad M = \sqrt{\frac{AB}{C}} \qquad \text{or} \qquad kx^2 + cx - n = 0$$

Solving Formulas with Squares

Solve the following for the indicated letters:

a. for x: $y = \frac{4x^2}{z}$

b. for k: $b = \frac{3}{k^2}$

Solving Formulas with Square Roots

Solve the following for the indicated letters:

a. for b: $a = \sqrt{\frac{b}{c}}$

b. for z: $x = \sqrt{\frac{3y}{z}}$

Solving Formulas in other forms

Solve the following for the indicated letters:

a. for x: $kx^2 + cx - d = 0$

b. for b: $ab^2 - 5b + m = 0$

c. for k: $\frac{1}{2}k^2 - 5nk + m = 0$

Solving Applications of Quadratic Equations

An object is thrown upwards off a cliff that is 48 feet above the ground. The height of the object t seconds after the object is released is given by:

$$h = -16t^2 + 32t + 48$$

How long does it take the object to hit the ground?

How long does it take the object to reach a height of 20 feet?

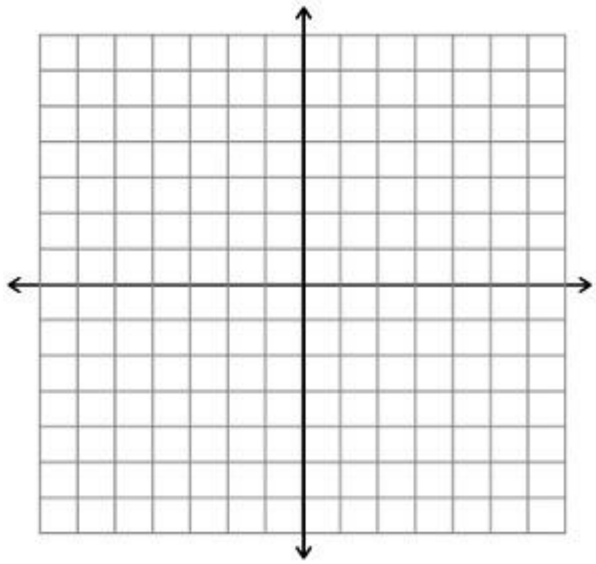
A 12 ft by 16 ft pond is going to be surround by a border of grass of uniform width all the way around. If the area of the enclosure must be 480 feet squared, what is the width of the grass border?

Lesson: Understanding the Graphs of Quadratic Functions

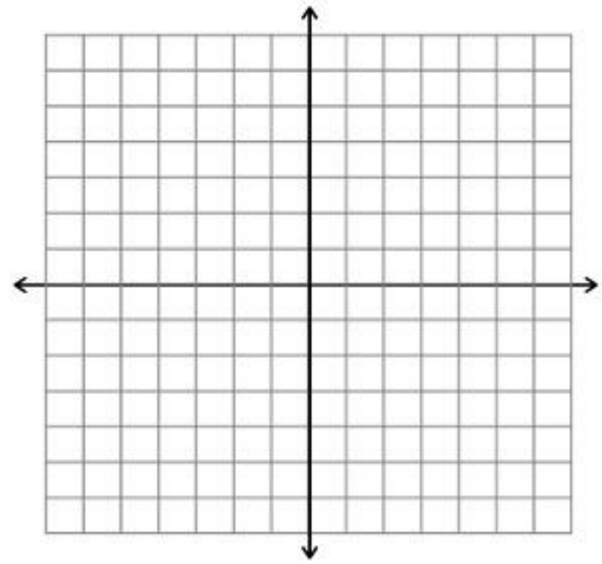
Quadratic functions are functions of the form ________________________________, where $a \neq 0$. But in previous lessons we worked on graphing quadratic functions, ie parabolas in another way. Let's review that first.

<u>Examples</u> Graph the following:

a. $f(x) = (x+2)^2 - 3$



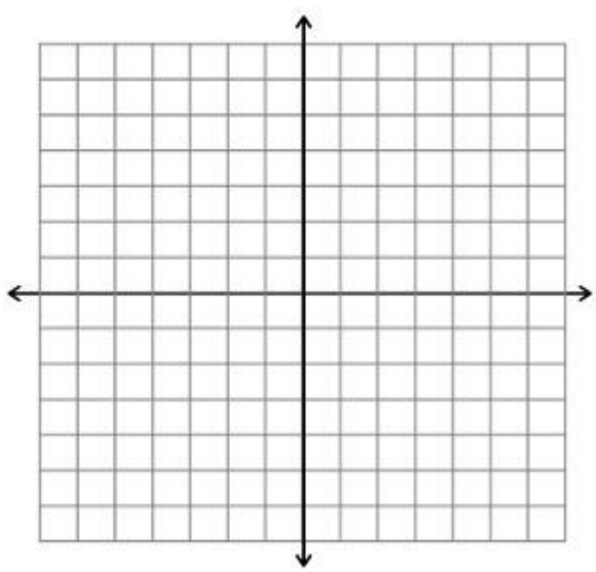
b. $g(x) = -(x-3)^2 + 1$



To begin, we will add on one more way to manipulate the graph of a parabola. Explain how to graph the follow:

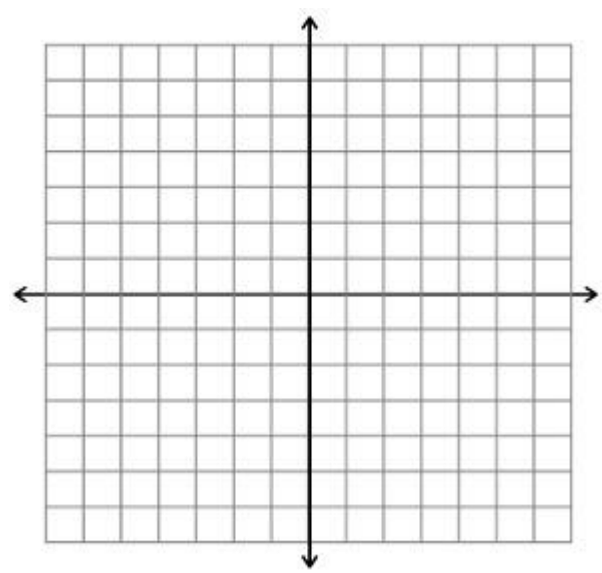
a. $f(x) = 3x^2$

How do you interpret the 3?



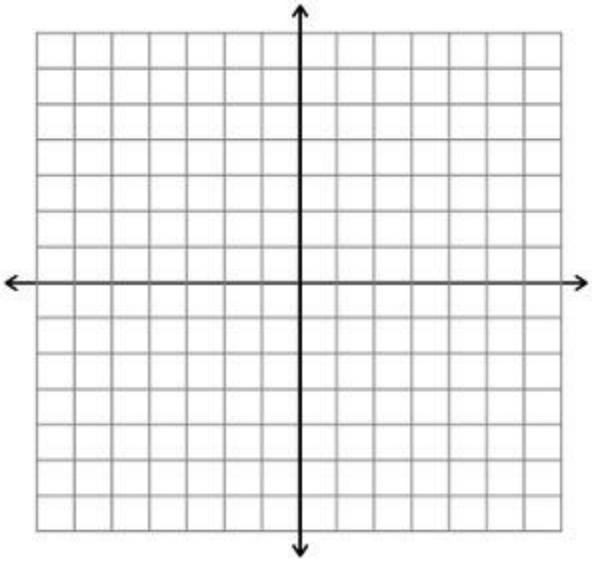
b. $g(x) = -\frac{1}{2}(x-2)^2 + 1$

How do you interpret the $-\frac{1}{2}$?

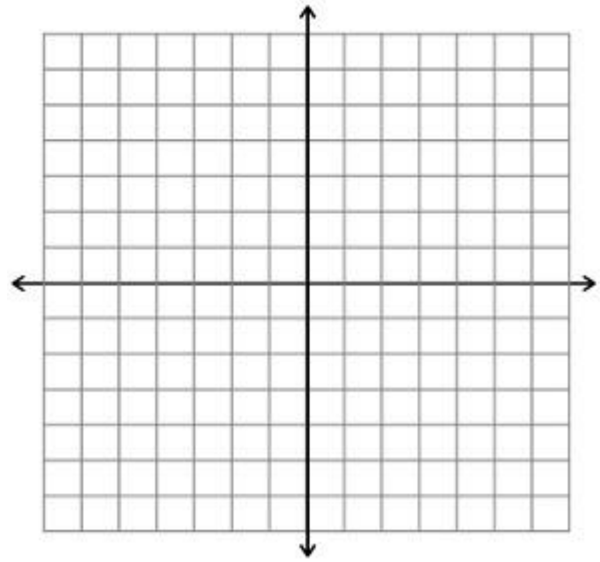


Examples Graph the following:

a. $f(x) = -2(x+1)^2 + 3$



b. $g(x) = \frac{1}{3}(x-2)^2 - 2$

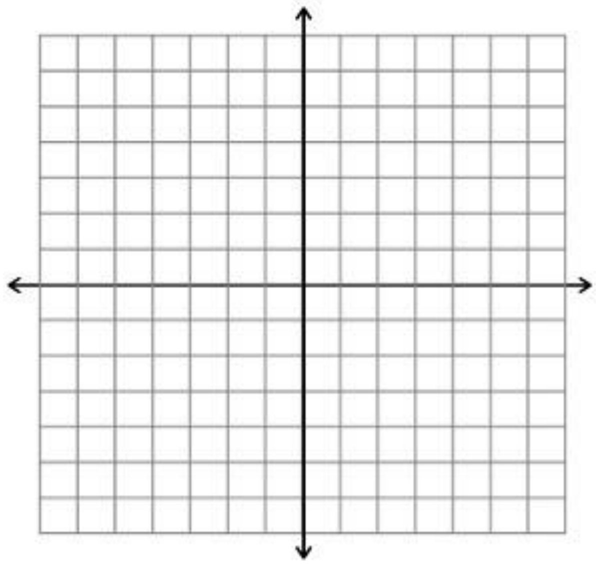


Using all of the examples we have discussed, what is another equation form for the parabola?

Explain some of the characteristics of a parabola that we get from this format:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x - 3)^2 - 4$



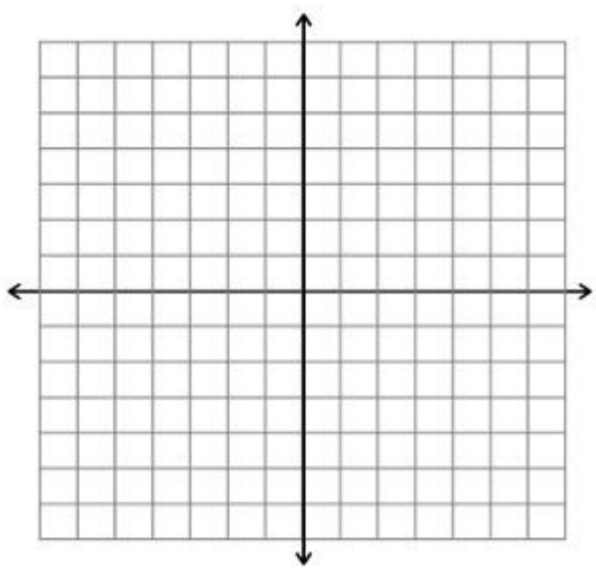
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x - 3)^2 + 8$



Vertex:

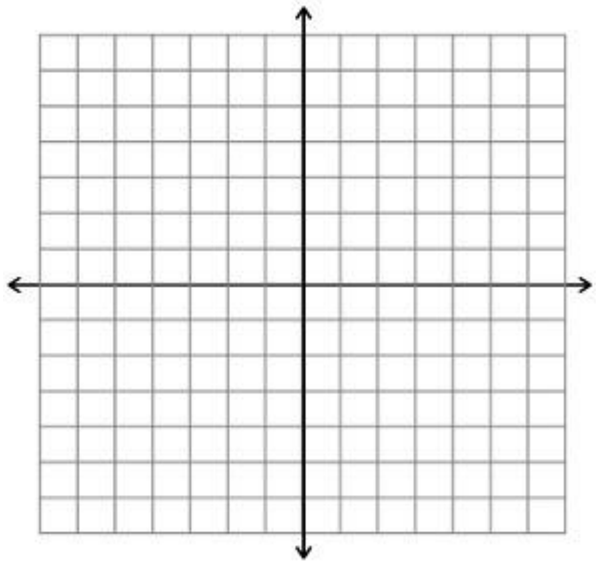
Axis of Symmetry:

X-intercepts:

Y-intercepts:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{4}(x+2)^2 - 3$



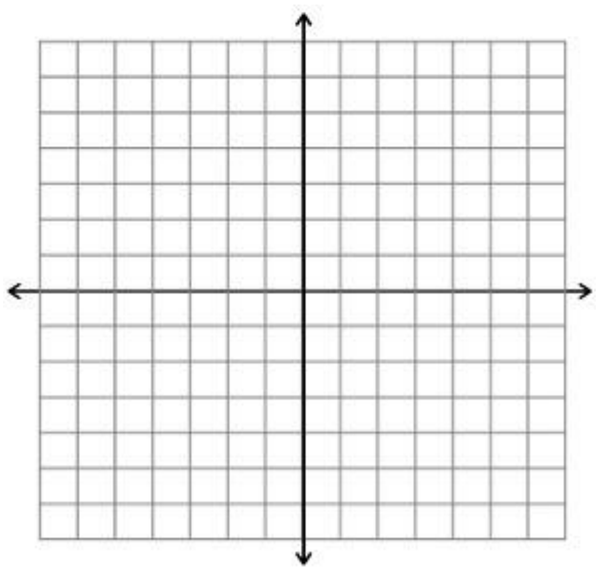
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -\frac{1}{3}(x-1)^2 + 3$



Vertex:

Axis of Symmetry:

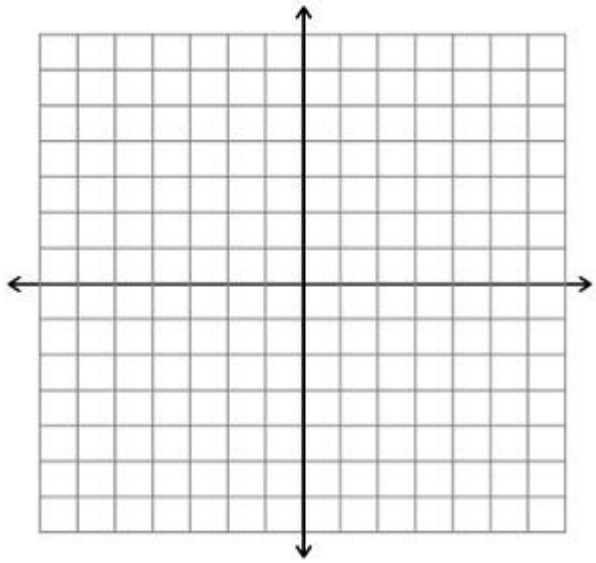
X-intercepts:

Y-intercepts:

Examples: Understanding How Many X-Intercepts There Are

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{2}(x+1)^2 - 8$



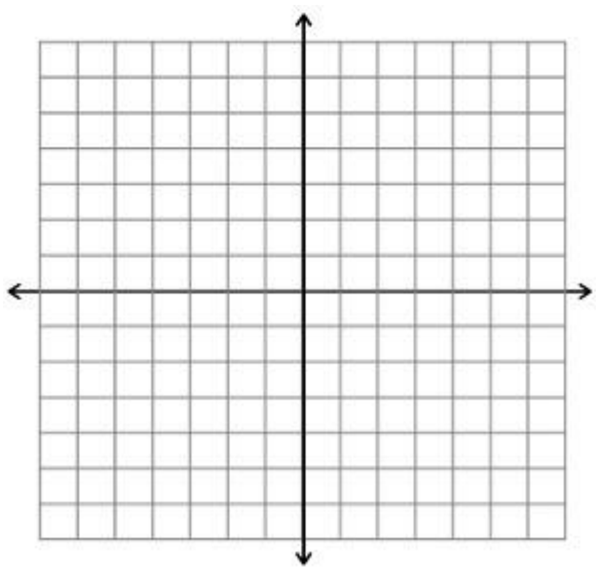
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2$



Vertex:

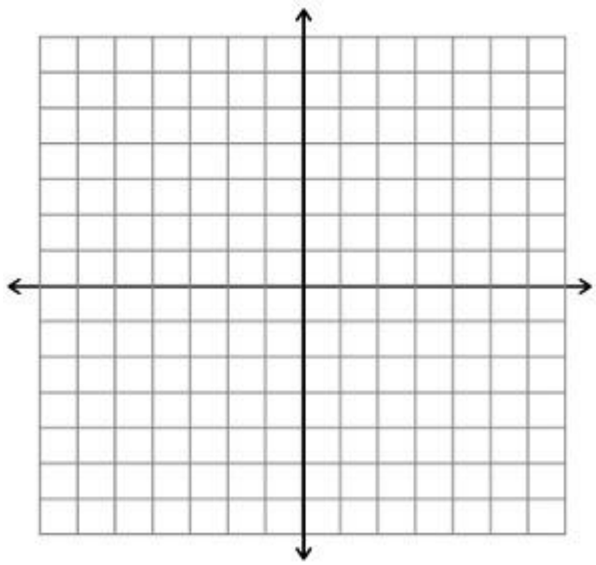
Axis of Symmetry:

X-intercepts:

Y-intercepts:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-2)^2 + 1$



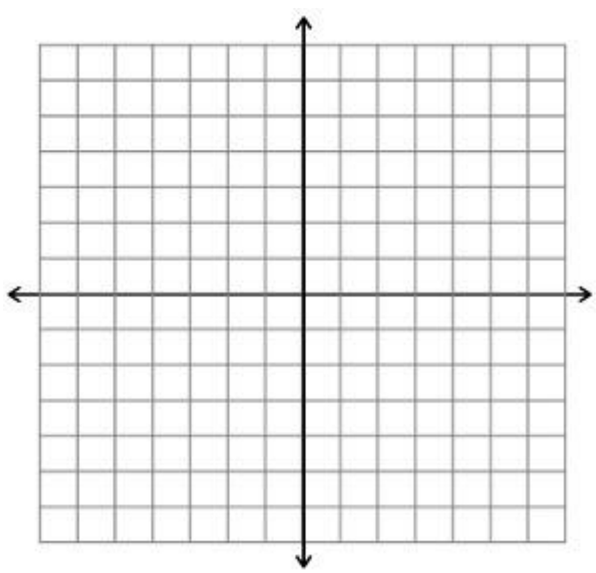
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -(x+3)^2 - 1$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

Lesson: Finding the Vertex of Quadratic Equations in the form $\boldsymbol{f(x) = ax^2 + bx + c}$

What is the vertex formula?

Show how to use the vertex formula for $f(x) = 3x^2 + 6x - 2$

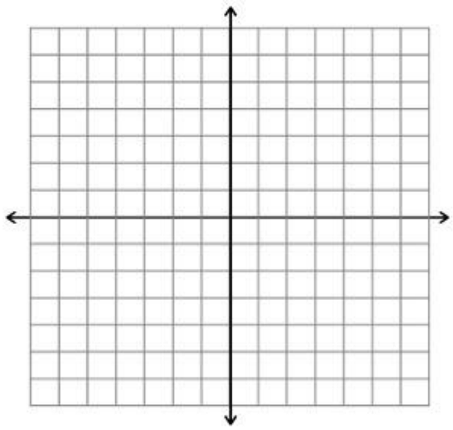
<u>Examples</u> Find the vertex of the following:

a. $f(x) = -x^2 + 4x - 2$

b. $f(x) = -\frac{1}{2}x^2 + 3x - 1$

Lesson: Putting Quadratics into the $\boldsymbol{f(x) = a(x-h)^2 + k}$ Form

Before you begin this lesson, you should already feel comfortable graphing an equation like $f(x) = 2(x-1)^2 + 3$. Graph this below:



Now show all the steps to put $f(x) = 2x^2 - 8x + 2$ into $f(x) = a(x-h)^2 + k$ form:

Examples Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = x^2 + 4x - 2$

b. $g(x) = x^2 + 6x + 10$

c. $h(x) = -x^2 + 2x - 4$

d. $y = 3x^2 + 6x - 12$

Examples: Putting Quadratics into the $\boldsymbol{f(x) = a(x-h)^2 + k}$ Form

<u>Examples</u> Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = \frac{1}{2}x^2 + x - 3$

b. $g(x) = -\frac{1}{3}x^2 + 4x - 2$

c. $y = x^2 + 3x + 2$

Not included (no notes file):

013 (Examples of Simplifying Rational Expressions)

014 (Examples of How to Use the Multiplicative Property of -1 to Simplify Rational Expressions)

015 (Examples of Finding Domains of Rational Expressions)

018 (Examples of Multiplying and Dividing Rational Expressions)

022 (Examples of Finding Common Denominators for Rational Expressions)