

Study with the Prof: Limits and Continuity

Problem Set 1: Basic Limit/Continuity Problems

Do each problem on your own set of paper. Write out the problem as if you were doing it for an exam. Show all of your steps so that you can compare to the videos if you need to.

1. Evaluate the following limits:

a. $\lim_{x \rightarrow 3} 2x(x - 1)$

b. $\lim_{x \rightarrow -1} \frac{x-2}{x+3}$

c. $\lim_{x \rightarrow 5} (3 + x)^{\frac{2}{3}}$

d. $\lim_{x \rightarrow \infty} \frac{5x^3 + 3x^2 - x + 7}{3x^3 + 2x - 8}$

e. $\lim_{x \rightarrow 0} (3 \sin x + 1)$

f. $\lim_{x \rightarrow 0} \frac{2-x+\sin x}{5 \cos x}$

g. $\lim_{x \rightarrow 0} \frac{x}{\sin x}$

h. $\lim_{x \rightarrow 1^+} \sqrt{\frac{x-1}{x+5}}$

2. Find the average rate of change of the functions over the given intervals.

a. $f(x) = x^2 + 1, [-2, 2]$

b. $g(x) = \sin x, \left[0, \frac{\pi}{2}\right]$

3. Using the difference quotient, find the slope of the curve at the given point and the equation of the tangent line at that point.

a. $y = 3 - x^2$ at $(1, 2)$

b. $y = \frac{1}{x}$ at $\left(2, \frac{1}{2}\right)$

4. Determine the intervals of continuity for the following functions:

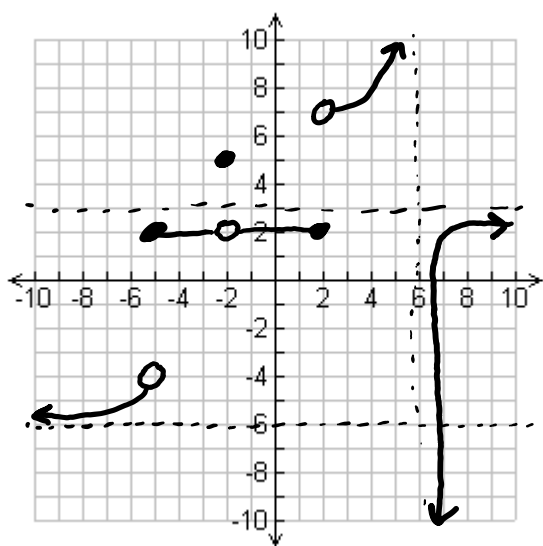
a. $y = 3x + 1$

b. $y = x^{\frac{3}{2}}$

c. $y = (3x + 1)^{\frac{2}{5}}$

d. $f(x) = \tan \frac{x}{2}$

5. Use the graph below of the function $f(x)$ to answer the following questions:



a. Find $\lim_{x \rightarrow -\infty} f(x)$

b. Find $\lim_{x \rightarrow -5^+} f(x)$

c. Find $\lim_{x \rightarrow -5^-} f(x)$

d. Find $\lim_{x \rightarrow -5} f(x)$

e. What type of continuity exists at $x = -5$?
Left? Right? Both sides? None?

f. Find $\lim_{x \rightarrow -2} f(x)$

g. What type of continuity exists at $x = -2$?
Left? Right? Both sides? None?

h. Find $\lim_{x \rightarrow 6^+} f(x)$

i. Find $\lim_{x \rightarrow 6^-} f(x)$

j. Find $\lim_{x \rightarrow \infty} f(x)$

Answers

1. a) 12 b) $-\frac{3}{2}$ c) 4 d) $\frac{5}{3}$ e) 1 f) $\frac{2}{5}$ g) 1 h) 0

2. a) 0 b) $\frac{2}{\pi}$

3. a) the slope is 2, the tangent is $y = 2x + 4$ b) the slope is $-\frac{1}{4}$, the tangent is $y = -\frac{1}{4}x + 1$

4. a) $(-\infty, \infty)$ b) $[0, \infty)$ c) $(-\infty, \infty)$ d) Anytime $\frac{x}{2}$ is an odd integer of π , ie $x = (2n - 1)\pi$ when n is an integer

5. a) -6 b) 2 c) -4 d) does not exist e) right f) 2 g) none h) ∞ i) $-\infty$ j) 3

What should you do if you missed a question?

Force yourself to do more problems. Even if you feel like you “just made a simple mistake.” If you are making simple mistakes, then you need to take some time to slow down, appreciate the smaller details, and get it right consistently.

Use your calculus book to study extra problems if you got something wrong! If you don't have a book, you can get a free one here: <https://openstax.org/details/books/calculus-volume-1>. Usually the odd practice problems have answers in the back of the book, or you can just use Google to find a calculator that will check your answer (seriously, there's a free calculator for literally everything these days on Google. Just try searching).

Below is some guidance of what I suggest you study for each problem if you got it wrong. I have also included links to some of my own videos if you want to watch some more examples.

1a, 1b, 1c: go to the basic limit laws section of your book and practice a handful of problems in the beginning of the homework section. Look for problems that look like the ones you practiced here. Continue to practice until you can consistently get 3 in a row correct. Other videos related to this problem: https://youtu.be/_ZBrYaYZYBQ or <https://youtu.be/mmX9qhY4Flg>

1d: Look for the section on limits as x goes to infinity, and try some of the beginning problems. Do at least 5. Continue to practice until you can consistently get 3 in a row correct. Other videos related to this problem: <https://youtu.be/bjRqRVI74JA> or https://youtu.be/Aa2G1I_cVRM

1e, 1f: Look for where trig limits were first introduced in your calculus book. This usually happens in one of the first sections on limits. Do the practice problems with trig in this section. Continue to practice until you get 3 in a row right. Other videos: <https://youtu.be/SOFylywNtrU>

1g: This is basically a theorem/fact that I wanted to make sure you remember. Some calculus classes don't do problems like this, so you might want to omit it (but if you are in my classes, we do lots of problems with this fact). If you want to learn more about why this is true, check out the proof: https://youtu.be/W_VnWrNcjwE

1h: Go to where one-sided limits are introduced in your book and practice some of beginning homework problems. Continue to practice until you get 3 in row consistently right. Other videos: <https://youtu.be/HtT3NZYPHcw>

2a, 2b: Not all calculus classes will talk about this, so omit if yours does not (my students learn this). Usually this is introduced in the first section of limits, so you can practice more problems like this in that section. Other videos: <https://youtu.be/K2CMtXDqjOY>

3a, 3b: Not all calculus classes will talk about this, so omit if yours does not (my students learn this). Usually this is introduced in the first section of limits, so you can practice more problems like this in that section. Other videos: <https://youtu.be/FTpirA3f9Nw>

4a, 4b, 4c, 4d: These problems will be discussed in any section on continuity. You should be able to find more problems like this in any calculus book. Continue to practice these types of problems until you get 3 in a row right consistently.

Other videos: <https://youtu.be/glgRZmdVjH0>

5a-j: These types of problems are usually discussed at the beginning of most limit homeworks, so you can look for graphs scattered throughout your homework problems across multiple limit sections. Other videos where I break down how to view these graphs: <https://youtu.be/glgRZmdVjH0>, <https://youtu.be/hmaCM8AVjr0>, https://youtu.be/FE_mM_Kdf8Q,

<https://youtu.be/kEWu6AdxWNw>, <https://youtu.be/kEWu6AdxWNw>, https://youtu.be/_ZBrYaYZYBQ