Study with the Prof: Derivative Rules

Problem Set 2: Medium Difficulty Derivatives

Do each problem on your own set of paper. Write out the problem as if you were doing it for an exam. Show all of your steps so that you can compare to the videos if you need to.

Using either the formula associated with the definition of the derivative or the alternative formula, find the derivatives of the following functions. You should **not** use the derivative rules here, the point is to practice using the formula (and memorizing it if needed).

1. $f(x) = \frac{1}{x}$

2. $g(x) = \sqrt{x+1}$

Using the derivative rules, find the derivative of the following functions:

3. $f(x) = \frac{e^x + \sin x}{4x-1}$

4. $g(x) = (3x-4)^{20} \sin 5x$

5. $h(x) = \ln(\ln(\ln(4x)))$

6. $y = \sin\left(e^{4x^2-3}\right)$

7. $f(x) = \log_5(\sin(2x-1))$

8. $y = 3^{\cos(5x-1)}$

9. $f(x) = \left(\frac{3x+1}{e^x}\right)^{17}$

10. $y = \sin^{-1}(e^x)$

11. $f(x) = \log_3(\tan^{-1} x)$

12. $5xy - 2x + 3y = 8$

13. $4x^2 - 2y^2 + xy = \sin x$

14. $\sin xy = 3$

15. $f(x) = (3 + \sin(4x))^5$

16. $g(x) = \tan^2(e^{5x})$

17. Find the equation of the tangent and normal lines for $x^2 + y^2 = 4$ at the point $(0,2)$

18. Find the equation of the line tangent to $f(x) = \sqrt{x}$ at $x = 4$

19. Find the equation of the line tangent to $f(x) = x + \sqrt{x}$ that has slope 4

20. Find any points on $f(x) = 2x^3 - 3x^2 - 12x + 8$ where the tangent lines would be horizontal.

21. What is the difference between velocity and speed?

22. A rock is thrown off at 64ft cliff with an initial velocity of 32ft/s. The height (in feet) of the rock above the ground t seconds after the rock is thrown is given by $s(t) = -16t^2 + 32t + 64$. A) When does the rock reach its highest point? B) What is the height of the rock at the highest point? C) What is the velocity of the rock when it strikes the ground? D) When is the speed of the rock increasing?

23. Find the fourth derivative of the following function: $f(x) = \sin 3x - 4x^3 + 2x - e^x$

24. A spherical ball is inflated at the rate of 15 $\frac{cm^3}{min}$. At what rate is the diameter of the ball increasing when the ball has a diameter of 5 cm?

Answers

1) $f'(x) = -\frac{1}{x^2}$
2) $g'(x) = \frac{1}{2\sqrt{x+1}}$
3) $f'(x) = \frac{(\cos x + e^x)}{4x-1} - \frac{4(\sin x + e^x)}{(4x-1)^2}$
4) $g'(x) = 60(3x-4)^{19}\sin 5x + 5(3x-4)^{20}\cos 5x$
5) $h'(x) = \frac{1}{x\ln(4x)\ln(\ln(4x))}$
6) $y' = 8xe^{4x^2-3}\cos\left(e^{4x^2-3}\right)$
7) $f'(x) = \frac{2\cos(2x-1)}{\ln 5 \sin(2x-1)}$
8) $y' = -5\ln 3 \sin(5x-1)\ 3^{\cos(5x-1)}$
9) $f'(x) = \frac{-17(3x+1)^{17}+51(3x+1)^{16}}{e^{17x}}$
10) $y' = \frac{e^x}{\sqrt{1-e^{2x}}}$
11) $f'(x) = \frac{1}{\ln 3(x^2+1)\tan^{-1}x}$
12) $y' = -\frac{5y-2}{5x+3}$
13) $y' = \frac{y-\cos x+8x}{4y-x}$
14) $y' = -\frac{y}{x}$
15) $f'(x) = 20(\cos 4x)(\sin 4x+3)^4$
16) $g'(x) = 10e^{5x}\sec^2(e^{5x})\tan(e^{5x})$
17) $y = 2, x = 0$
18) $y = \frac{1}{4}x + 1$
19) $y = 4x + \frac{1}{12}$
20) $x = 2, x = -1$
21) speed = |velocity|
22) a) t=1 second, b) 64 ft c) -64 ft/sec d) after 1 second
23) $f^{(4)}(x) = 81\sin(3x) - e^x$
24) $\frac{dD}{dt} = \frac{6}{5\pi}$ cm/min

In case you need more review, here are other videos to watch (by problem). These videos will give you a much deeper review and/or more examples.

Question 1, 2 Definition formula versus alternative formula https://youtu.be/S1U5RpqCUos, alternative formula examples https://youtu.be/vt9nQQdXkEc, definition version of the formula https://youtu.be/BMIkWYo3C7w

Question 3, 4 Chain rule lesson https://youtu.be/FsxMB-5SDzk, basic derivative rules lesson (product rule, quotient rule) https://youtu.be/-gfcRvqT3po, examples of product rule and quotient rule https://youtu.be/lZy9UPWMQwE; examples of trig derivatives https://youtu.be/zsTWUdSeBXA

Question 5 Chain rule lesson https://youtu.be/FsxMB-5SDzk, examples of derivatives with logarithms https://youtu.be/zXk_LOxap0w

Question 6, 7, 8 Chain rule lesson https://youtu.be/FsxMB-5SDzk, basic derivative rules lesson (product rule, quotient rule) https://youtu.be/-gfcRvqT3po, examples of product rule and quotient rule https://youtu.be/lZy9UPWMQwE; examples of trig derivatives https://youtu.be/zsTWUdSeBXA, examples of derivatives with logarithms https://youtu.be/zXk_LOxap0w

Question 9 Chain rule lesson https://youtu.be/FsxMB-5SDzk, examples of the chain rule https://youtu.be/GqtWUdtQ-uo

Question 10, 11 Chain rule lesson https://youtu.be/FsxMB-5SDzk, inverse trig function derivatives lesson https://youtu.be/vLe18VKxlqI, inverse trig function derivatives examples https://youtu.be/nFhdOwIbeWM, examples of derivatives of general exponentials and logarithms https://youtu.be/zXk_LOxap0w

Question 12, 13, 14 Implicit differentiation lesson https://youtu.be/CTscFbd8ibk, implicit differentiation examples https://youtu.be/QMp3lIMaBY0, step-by-step example implicit differentiation with product rule https://youtu.be/1L7Da2gxwjM

Question 15, 16 Chain rule lesson https://youtu.be/FsxMB-5SDzk, examples of trig derivatives https://youtu.be/zsTWUdSeBXA

Question 17 Implicit differentiation lesson https://youtu.be/CTscFbd8ibk, implicit differentiation examples https://youtu.be/QMp3lIMaBY0, examples of tangent and normal lines https://youtu.be/TwNFtqmQikM

Question 18, 19, 20 Writing equations of tangent lines https://youtu.be/gW04qcAzWno

Question 21, 22 An overview of position, velocity, and acceleration https://youtu.be/fshNoVIkM34

Question 23 Higher derivatives https://youtu.be/-CqteLlElsE

Question 24 Related rates examples https://youtu.be/A-qLhhefNQQ, https://youtu.be/1aBOI8mAsZA, and https://youtu.be/9kos-oCV1_U