

Study with the Prof: Derivative Rules

Problem Set 3: Difficult Derivatives

Do each problem on your own set of paper. Write out the problem as if you were doing it for an exam. Show all of your steps so that you can compare to the videos if you need to.

Using the derivative rules, find the derivative of the following functions:

1. $f(x) = \cos \sqrt{\sin^2 x + 3}$

2. $g(x) = \sin^2 x^7$

3. $y = \sqrt[3]{2x - \sin(4x)}$

4. $h(x) = (\tan x^4 + 3x)^7$

5. $y = \frac{\cos^2 x}{\tan x - 3x}$

6. $y = \sin^{-1} \sqrt{1 - x^2}$

7. $f(x) = \ln(\tan^{-1} x)$

8. $x^2 - \sqrt{5}xy + 3y^2 = 7$

9. $f(x) = (2 + x) \sin^{-1}(3x)$

10. $y^x = x^y$

11. $y^2 = \frac{x}{x+1}$

12. $y^2 = 2e^{-(\frac{1}{x})}$

13. $f(x) = (1 - x^3)e^{\sec^{-1} x}$

14. $y = (\ln x)^x$

15. $y = \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x}$

16. $y = \ln \sqrt{\frac{(x+4)^7}{(x-5)^8}}$

17. Find the normals to the curve $4x - y + xy = 0$ that are parallel to $4x + y = 0$.

18. Find an equation for the line that is tangent to the curve $y = x^2 + 3x + 1$ that goes through $(-1, -10)$.

19. What is the value of $\frac{d^{99}}{dx^{99}}(\sin x)$?

20. Find the second derivative of $y = x^2(x^4 - 1)^6$

21. Find the second derivative of $x^3 + y^3 = 125$

22. A 12-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 10-ft from the house, the base is moving at the rate of 4ft/sec. A) How fast is the top of the ladder sliding down the wall? B) At what rate is the area of the triangle formed by the ladder, wall, and ground changing? C) At what rate is the angle between the ladder and the ground changing?

23. Are there any points on the curve $y = x - e^{-x}$ where the slope is 1? If so, find them.

Answers

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| <ol style="list-style-type: none"> 1. $f'(x) = -\frac{\cos x \sin x \sin \sqrt{\sin^2 x + 3}}{\sqrt{\sin^2 x + 3}}$ 2. $g'(x) = 14x^6 \cos x^7 \sin x^7$ 3. $y' = \frac{2-4 \cos 4x}{3(2x-\sin 4x)^{\frac{2}{3}}}$ 4. $h'(x) = 7(4x^3 \sec^2(x^4) + 3)(\tan x^4 + 3x)^6$ 5. $y' = -\frac{2 \cos x \sin x}{\tan x - 3x} - \frac{\cos^2 x (\sec^2 x - 3)}{(\tan x - 3x)^2}$ 6. $y' = -\frac{x}{ x \sqrt{1-x^2}}$ 7. $f'(x) = \frac{1}{\tan^{-1} x (x^2+1)}$ 8. $y' = \frac{y\sqrt{5}-2x}{6y-x\sqrt{5}}$ 9. $f'(x) = \sin^{-1}(3x) + \frac{3(x+2)}{\sqrt{1-9x^2}}$ 10. $y' = \frac{xy \ln y - y^2}{xy \ln x - x^2}$ 11. $y' = \frac{1}{2y(x+1)^2}$ 12. $y' = \frac{e^{-\frac{1}{x}}}{x^2 y}$ | <ol style="list-style-type: none"> 13. $y' = \frac{(1-x^3)e^{\sec^{-1} x}}{ x \sqrt{x^2-1}} - 3x^2 e^{\sec^{-1} x}$ 14. $y' = (\ln x)^x \left(\ln(\ln x) \right) + \frac{1}{\ln x}$ 15. $y' = x e^{5x}$ 16. $y' = -\frac{x+67}{2(x-5)(x+4)}$ 17. $y = -4x - 15, y = -4x + 15$ 18. $y = -5x - 15, y = 7x - 3$ 19. $y^{(99)} = -\cos x$ 20. $y'' = 2(x^4 - 1)^4 (325x^8 - 86x^4 + 1)$ 21. $y'' = \frac{(-2xy^3 - 2x^4)}{y^5}$ 22. a) $-\frac{40}{\sqrt{44}} \text{ft/sec}$ b) $2\sqrt{44} - \frac{200}{\sqrt{44}} \text{sq ft / sec}$
c) -0.603 rad/sec 23. No |
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In case you need more review, here are other videos to watch (by problem). These videos will give you a much deeper review and/or more examples.

Question 1, 2, 3, 4 Chain rule lesson <https://youtu.be/FsxMB-5SDzk>, chain rule examples with trig functions https://youtu.be/W1dan_IdHV0

Question 5 Chain rule lesson <https://youtu.be/FsxMB-5SDzk>, chain rule examples with trig functions https://youtu.be/W1dan_IdHV0; examples of product rule and quotient rule <https://youtu.be/lZy9UPWMQwE>; examples of trig derivatives <https://youtu.be/zsTWUdSeBXA>

Question 6, 7 Chain rule lesson <https://youtu.be/FsxMB-5SDzk>, inverse trig function derivatives lesson <https://youtu.be/vLe18VKxlql>, inverse trig function derivatives examples <https://youtu.be/nFhdOwlbeWM>

Question 8 Implicit differentiation lesson <https://youtu.be/CTscFbd8ibk>, implicit differentiation examples <https://youtu.be/QMp3lIMaBYO>, step-by-step example implicit differentiation with product rule <https://youtu.be/1L7Da2gxwjM>

Question 9 Chain rule lesson <https://youtu.be/FsxMB-5SDzk>, inverse trig function derivatives lesson <https://youtu.be/vLe18VKxlql>, inverse trig function derivatives examples <https://youtu.be/nFhdOwlbeWM>

Question 10 Logarithmic differentiation lesson <https://youtu.be/qhdmF-1YJRI>, logarithmic differentiation examples <https://youtu.be/8XxLnvAi4b8>

Question 11, 12 Implicit differentiation lesson <https://youtu.be/CTscFbd8ibk>, implicit differentiation examples <https://youtu.be/QMp3lIMaBYO>, step-by-step example implicit differentiation with product rule <https://youtu.be/1L7Da2gxwjM>

Question 13 Examples of product rule and quotient rule <https://youtu.be/lZy9UPWMQwE>; chain rule lesson <https://youtu.be/FsxMB-5SDzk>, inverse trig function derivatives lesson <https://youtu.be/vLe18VKxlql>, inverse trig function derivatives examples <https://youtu.be/nFhdOwlbeWM>

Question 14 Logarithmic differentiation lesson <https://youtu.be/qhdmF-1YJRI>, logarithmic differentiation examples <https://youtu.be/8XxLnvAi4b8>

Question 15, 16 Chain rule lesson <https://youtu.be/FsxMB-5SDzk>, examples of product rule and quotient rule <https://youtu.be/lZy9UPWMQwE>; examples of trig derivatives <https://youtu.be/zsTWUdSeBXA>

Question 17 Finding tangent and normal lines when no point is given <https://youtu.be/Z9SMBawnDhU>

Question 18 Examples of writing equations of tangent lines <https://youtu.be/gW04qcAzWno>

Question 19, 20 Higher derivatives <https://youtu.be/-CqteLIIEsE>

Question 21 Second derivatives with implicit differentiation examples <https://youtu.be/-CqteLIIEsE>, step-by-step example with second derivatives and implicit differentiation <https://youtu.be/QH0OnHI5K1I>

Question 22 Related rates examples <https://youtu.be/A-qLhhefNQQ>, <https://youtu.be/1aBOI8mAsZA>, and https://youtu.be/9kos-oCV1_U

Question 23 Examples of writing equations of tangent lines <https://youtu.be/gW04qcAzWno>